International Journal of Mathematics And its Applications
Volume 4, Issue 1–D (2016), 67–72.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Further Study of ω-closed Sets

Research Article

O.Ravi[1]*, M.Rajakalaivanan[1] and I.Rajasekaran[1]

1 Department of Mathematics, P.M.Thevar College, Usilampatti, Madurai District, Tamil Nadu, India.

Abstract: In 1982, the notions of ω-closed sets and ω-open sets were introduced and studied by Hdeib [4]. In 2009, Noiri et al [6] introduced some generalizations of ω-open sets and investigated some properties of the sets. Moreover, they used them to obtain decompositions of continuity. Quite Recently Ravi et al [7] studied further generalizations of ω-open sets in topological spaces. In this paper we introduce new classes of functions in topological spaces and study the characterizations of such functions in detail. Finally we obtain some decomposition theorems.

MSC: AMS Classification : 54A05, 54C05, 54C08, 54C10.

Keywords: ω-open set, α-ω-open set, pre-ω-open set, β-ω-open set, ω-t-set, δ-ω-open set, semi*-ω-closed set.

1. Introduction

In 1982, the notion of ω-closed sets were introduced and studied by Hdeib [4]. This notion was shown to be productive and very useful. In his paper, he also defined a new kind of functions, namely ω-closed functions, which are strictly weaker than closed functions. He also proved that the Lindelöf property is preserved by counter images of ω-closed functions with Lindelöf counter images of points. The notion of contra continuous functions was introduced and investigated by Dontchev. A new class of functions called almost contra ω-continuous functions as a new generalization of contra continuity was introduced and studied by Ahmad Al-Omari and Mohd Salmi Md Noorani [2]. The notions of $g\omega$-closed sets and $g\omega$-continuous functions were introduced and studied by Khalid Y. Al-Zoubi [5] in 2005. In 2009, Noiri et al [6] introduced some generalizations of ω-open sets and investigated some properties of the sets. Moreover, they used them to obtain decompositions of continuity. Quite Recently, Ravi et al [7] introduced the notion of semi-ω-open sets in topological spaces. Further they studied the weaker and stronger forms of such classes. The properties of such classes were also studied in detail. Also, the notion of $g^\star\omega$-closed sets [8] was introduced by them in topological spaces. It is proved that this class lies between the class of ω-closed sets and the class of $g\omega$-closed sets. Fundamental properties and characterizations of $g^\star\omega$-closed sets are obtained. In this paper, we introduce new classes of functions in topological spaces and study the characterizations of such functions in detail. Finally we obtain some decomposition theorems.

2. Preliminaries

By a space (X, τ), we always mean a topological space (X, τ) with no separation properties assumed. If H $\subset$ X, cl(H) and int(H) will, respectively, denote the closure and interior of H in (X, τ).

* *E-mail: siingam@yahoo.com*

Definition 2.1 ([10]). *Let H be a subset of a space (X, τ), a point p in X is called a condensation point of H if for each open set U containing p, $U \cap H$ is uncountable.*

Definition 2.2 ([4]). *A subset H of a space (X, τ) is called ω-closed if it contains all its condensation points. The complement of an ω-closed set is called ω-open.*

It is well known that a subset W of a space (X, τ) is ω-open if and only if for each $x \in W$, there exists $U \in \tau$ such that $x \in U$ and $U - W$ is countable. The family of all ω-open sets, denoted by τ_ω, is a topology on X, which is finer than τ. The interior and closure operator in (X, τ_ω) are denoted by int_ω and cl_ω respectively.

Lemma 2.3 ([4]). *Let H be a subset of a space (X, τ). Then*

(1). H is ω-closed in X if and only if $H = cl_\omega(H)$.

(2). $cl_\omega(X \backslash H) = X \backslash int_\omega(H)$.

(3). $cl_\omega(H)$ is ω-closed in X.

(4). $x \in cl_\omega(H)$ if and only if $H \cap G \neq \emptyset$ for each ω-open set G containing x.

(5). $cl_\omega(H) \subset cl(H)$.

(6). $int(H) \subset int_\omega(H)$.

Remark 2.4. *For a subset of a space (X, τ), the following property holds: Every closed set is ω-closed but not conversely [1, 4].*

Definition 2.5 ([6]). *A subset H of a space (X, τ) is said to be*

1. *α-ω-open if $H \subset int_\omega(cl(int_\omega(H)))$;*
2. *pre-ω-open if $H \subset int_\omega(cl(H))$;*
3. *β-ω-open if $H \subset cl(int_\omega(cl(H)))$.*

Definition 2.6 ([6]). *A subset H of a space (X, τ) is called an ω-t-set if $int(H) = int_\omega(cl(H))$.*

Definition 2.7. *A space (X, τ) is called submaximal [3] if every dense subset is open.*

Definition 2.8 ([7]). *A subset H of a space (X, τ) is said to be*

(1). semi-ω-open if $H \subset cl(int_\omega(H))$.

(2). semi-ω-closed if $int(cl_\omega(H)) \subset H$.

The complement of a semi-ω-open set is called semi-ω-closed.

Theorem 2.9 ([7]). *Let H be a subset of a space (X, τ). Then H is α-ω-open if and only if it is semi-ω-open and pre-ω-open.*

Theorem 2.10 ([7]). *For a subset H of a submaximal space (X, τ), the following properties are equivalent.*

(1). H is semi-ω-open,

(2). H is β-ω-open.

Definition 2.11 ([7]). *A subset H of a space (X, τ) is said to be*

(1). δ-ω-open if $int_\omega(cl(H)) \subset cl(int_\omega(H))$.

(2). δ-ω-closed if $int(cl_\omega(H)) \subset cl_\omega(int(H))$.

The complement of a δ-ω-open set is called δ-ω-closed.

Definition 2.12 ([7]). *A subset H of a space (X, τ) is said to be β-ω-closed if $int(cl_\omega(int(H))) \subset H$. The complement of a β-ω-open set is called β-ω-closed.*

Theorem 2.13 ([7]). *For a subset H of a space (X, τ), the following properties are equivalent:*

(1). H is semi-ω-closed.

(2). H is β-ω-closed and δ-ω-closed.

Theorem 2.14 ([7]). *Let (X, τ) be a space. Then a subset of X is α-ω-open if and only if it is both δ-ω-open and pre-ω-open.*

Definition 2.15 ([7]). *A subset H of a space (X, τ) is said to be*

(1). semi*-ω-open if $H \subset cl_\omega(int(H))$.

(2). semi*-ω-closed if $int_\omega(cl(H)) \subset H$.

The complement of a semi*-ω-open set is called semi*-ω-closed.

Definition 2.16 ([7]). *A subset H of a space (X, τ) is said to be $\omega^\star$-t-set if $int_\omega(cl(H)) = int_\omega(H)$.*

Theorem 2.17 ([7]). *A subset H of a space (X, τ) is semi*-ω-closed if and only if H is a $\omega^\star$-t-set.*

Definition 2.18 ([7]). *A subset H of a space (X, τ) is said to be semi-ω-regular if H is semi-ω-open and a $\omega^\star$-t-set.*

Theorem 2.19 ([7]). *Let H be a subset of a space (X, τ). Then H is semi-ω-regular if and only if H is both β-ω-open and semi*-ω-closed.*

Definition 2.20 ([7]). *A subset H of a space (X, τ) is called ω-$\mathcal{R}$-closed if $H = cl(int_\omega(H))$.*

Theorem 2.21 ([7]). *Let H be a subset of a space (X, τ). Then the following properties are equivalent.*

(1). H is ω-$\mathcal{R}$-closed.

(2). H is semi-ω-open and closed.

(3). H is β-ω-open and closed.

3. Some Decomposition Theorems

Definition 3.1 ([6]). *A function $f : X \to Y$ is said to be pre-ω-continuous (resp. α-ω-continuous) if $f^{-1}(V)$ is pre-ω-open (resp. α-ω-open) in X for each open set V in Y.*

Definition 3.2 ([9]). *A function $f : X \to Y$ is said to be semi-ω-continuous (resp. β-ω-continuous, δ-ω-continuous) if $f^{-1}(V)$ is semi-ω-open (resp. β-ω-open, δ-ω-open) in X for each open set V in Y.*

Definition 3.3. *A function $f : X \to Y$ is said to be contra-semi-ω-continuous (resp. contra-β-ω-continuous, contra-δ-ω-continuous) if $f^{-1}(V)$ is semi-ω-closed (resp. β-ω-closed, δ-ω-closed) in X for each open set V in Y.*

Definition 3.4. *A function $f : X \to Y$ is said to be semi*-ω-continuous (resp. $\omega^\star$-t-continuous, ω-$\mathcal{R}$-continuous) if $f^{-1}(V)$ is semi*-ω-closed (resp. $\omega^\star$-t-set, ω-$\mathcal{R}$-closed) in X for each closed set V in Y.*

Definition 3.5. *A function $f : X \to Y$ is said to be contra-semi-ω-continuous (resp. contra-β-ω-continuous, contra-α-ω-continuous, contra-pre-ω-continuous, contra-δ-ω-continuous) if $f^{-1}(V)$ is semi-ω-open (resp. β-ω-open, α-ω-open, pre-ω-open, δ-ω-open) in X for each closed set V in Y.*

Theorem 3.6. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is α-ω-continuous.

(2). f is semi-ω-continuous and pre-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.9. □

Theorem 3.7. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is contra-α-ω-continuous.

(2). f is contra-semi-ω-continuous and contra-pre-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.9. □

Theorem 3.8. *Let X be a submaximal space. For a function $f : X \to Y$, the following are equivalent:*

(1). f is semi-ω-continuous.

(2). f is β-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.10. □

Theorem 3.9. *Let X be a submaximal space. For a function $f : X \to Y$, the following are equivalent:*

(1). f is contra-semi-ω-continuous.

(2). f is contra-β-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.10. □

Theorem 3.10. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is semi-ω-continuous.

(2). f is β-ω-continuous and δ-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.13. □

Theorem 3.11. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is contra-semi-ω-continuous.

(2). f is contra-β-ω-continuous and contra-δ-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.13. □

Theorem 3.12. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is α-ω-continuous.

(2). f is δ-ω-continuous and pre-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.14. □

Theorem 3.13. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is contra-α-ω-continuous.

(2). f is contra-δ-ω-continuous and contra-pre-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.14. □

Definition 3.14. *A function $f : X \to Y$ is said to be semi-ω-regular continuous if $f^{-1}(V)$ is semi-ω-regular in X for each closed set V in Y.*

Theorem 3.15. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is semi*-ω-continuous.

(2). f is $\omega^\star$-t-continuous.

Proof. This is an immediate consequence of Theorem 2.17. □

Theorem 3.16. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is semi-ω-regular continuous.

(2). f is contra-β-ω-continuous and semi*-ω-continuous.

Proof. This is an immediate consequence of Theorem 2.19. □

Theorem 3.17. *For a function $f : X \to Y$, the following are equivalent:*

(1). f is ω-$\mathcal{R}$-continuous.

(2). f is contra-semi-ω-continuous and continuous.

(3). f is contra-β-ω-continuous and continuous.

Proof. This is an immediate consequence of Theorem 2.21. □

References

[1] A.Al-Omari and M.S.M.Noorani, *Regular generalized ω-closed sets*, Intern. J. Math. Math. Sci., 2007(2007), Article ID 16292.

[2] A.Al-Omari and M.S. M.Noorani, *Contra-ω-continuous and Almost contra-ω-continuous*, Intern. J. Math. Math. Sci., 2007(2007), Article ID 40469.

[3] J.Dontchev, *On submaximal spaces*, Tamkang J. Math., 26(1995), 253-260.

[4] H.Z.Hdeib, *ω-closed mappings*, Revista Colomb. De Matem., 16(1982), 65-78.

[5] Khalid Y.Al-Zoubi, *On generalized ω-closed sets*, Intern. J. Math. Math. Sci., 13(2005), 2011-2021.

[6] T.Noiri, A.Al-Omari and M.S.M.Noorani, *Weak forms of ω-open sets and decompositions of continuity*, Eur. J. Pure Appl. Math., 2(2009), 73-84.

[7] O.Ravi, I.Rajasekaran, S.Satheesh Kanna and M.Paranjothi, *New generalized classes of τ_ω*, Eur. J. Pure Appl. Math., Accepted.

[8] O.Ravi, M.Paranjothi, I.Rajasekaran and S.Murugesan, *On ω-open and $g^\star\omega$-closed sets*, Journal of New Theory.

[9] O.Ravi, M.Paranjothi, I.Rajasekaran and S.Satheesh Kanna, *ω-open sets and Decompositions of continuity*, Bulletin of International Mathematical Virtual Institute, 6(2)(2016), 143-155.

[10] S.Willard, *General Topology*, Addison-Wesley, Reading, Mass, USA, (1970).