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Existence and Sensitivity Analysis of Solutions for a System of Parametric General Nonlinear Variational Inequalities

Research Article

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Abstract: In this paper, we introduce and study a new class of system of parametric general nonlinear variational inequalities. Using the resolvent operator in Hilbert spaces, we establish the equivalence between the system of parametric general nonlinear variational inequalities and a fixed point problem. This equivalence is used to investigate the existence and uniqueness and sensitivity analysis of solutions for the system of parametric general nonlinear variational inequalities.

MSC: 47J20.

Keywords: The system of parametric general nonlinear variational inequalities, existence, sensitivity analysis.

1. Introduction

Variational inequality theory has become a very powerful tool in pure and applied mathematics. In 1996, Zhu and Mar-cotte introduced and investigated a class of system of variational inequalities in R^n. Afterwards, Liu, Hao, Lee and Kang [1], Nie, Liu, Kim and Kang [2], and Wu, Liu, Shim and Kang [3] studied the approximation and solvability of a few kinds of various systems of variational inequalities in Hilbert spaces. Recently, Dafermos [4] studied the sensitivity property of solutions of a parametric variational inequalities involving single-valued mapping in R^n. Afterwards, using the ideas of Dafermos, many researchers including Agrwal, Cho and Huang [5], Liu, Debnath, Kang and Ume [6], and others have established the sensitivity analysis of solutions of various types of variational inequalities and quasivariational inclusions in Hilbert spaces, respectively. Inspired and motivated by the results [1, 2, 7], in this paper , we introduce and study a new class of system of parametric general nonlinear variational inequalities. We show its equivalence with a fixed point problem and establish the existence and sensitivity analysis of solutions for the system of parametric general nonlinear variational inequalities involving strongly monotone, Lipschitz continuous and pseudoconractive. Our results extend and improve the corresponding results in [1, 2, 7, 8].

2. Preliminaries

Let H be a real Hilbert space with a norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$, respectively. Let P be a nonempty open subset of H in which the parameter λ take values. Let $M, N : H \times P \to H$ be any mappings and $\varphi : H \times P \to (-\infty, +\infty]$ be a proper

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convex lower semicontinuous functional, let ρ and β be positive constants and f and g be arbitrary elements in H. For each $\lambda \in P$, we consider the follow problem: find $x, y \in H$ such that

$$\begin{cases} \langle \rho(M(y,\lambda) - N(y,\lambda) - f) + x - y, u - x\rangle \geq \rho\phi(x,\lambda) - \rho\phi(u,\lambda) \\ \langle \beta(M(x,\lambda) - N(x,\lambda) - g) + y - x, u - y\rangle \geq \beta\phi(y,\lambda) - \beta\phi(u,\lambda) \end{cases} \forall u \in H, \tag{1}$$

which is known as the system of parametric general nonlinear variational inequalities. It is clear that the system of parametric general nonlinear variational inequalities (1) includes the system of variational inequalities in [2, 3] as special cases.

Lemma 2.1. *For a given $u \in H$, the element $z \in H$ satisfies the following inequality*

$$\langle u - z, v - u\rangle \geq \rho\varphi(u,\lambda) - \rho\varphi(v,\lambda),\ \forall v \in H$$

if and only if $u = J_\rho^{\varphi(\cdot,\lambda)}(z)$, where $J_\rho^{\varphi(\cdot,\lambda)} = (I + \rho\partial\varphi(\cdot,\lambda))^{-1}$ is the resolvent operator, I is the identity mapping and $\partial\varphi(\cdot,\lambda)$ stand for the subdifferential of the proper convex lower semicontinuous functional φ. Furthermore, $J_\rho^{\varphi(\cdot,\lambda)}$ is nonexpansive , that is,

$$\left\| J_\rho^{\varphi(\cdot\lambda)}(x) - J_\rho^{\varphi(\cdot\lambda)}(y) \right\| \leq \|x - y\|,\ \forall x, y \in H.$$

Definition 2.2. *Let $M : H \times P \to H$ be a mapping, M is said to be*

(1). $m-$ Lipschitz continuous in the first argument if there exists a constant $m > 0$ such that

$$\|M(x,\lambda) - N(y,\lambda)\| \leq m\|x - y\|,\ \forall x, y \in H, \lambda \in P$$

(2). $m-$ strongly monotone in the first argument if there exists a constant $m > 0$ such that

$$\langle x - y, M(x,\lambda) - M(y,\lambda)\rangle \geq m\|x - y\|^2,\ \forall x, y \in H, \lambda \in P;$$

(3). $m-$ pseudocontractive in the first argument if there exists a constant $m > 0$ such that

$$\langle x - y, M(x,\lambda) - M(y,\lambda)\rangle \leq m\|x - y\|^2,\ \forall x, y \in H, \lambda \in P.$$

3. Main Results

Lemma 3.1. *Let ρ and β be positive constants , and f and g be arbitrary elements in H and $\lambda \in P$. Then the following statements are equivalent to each other:*

(1). the system of parametric general nonlinear variational inequalities (1) has a solution $x, y \in H$;

(2). there exists $x, y \in H$ satisfying

$$x = J_\rho^{\varphi(\cdot,\lambda)}[y - \rho(M(y,\lambda) - N(y,\lambda) - f)],$$

$$y = J_\beta^{\varphi(\cdot,\lambda)}[x - \beta(M(x,\lambda) - N(x,\lambda) - g)];$$

(3). the mapping $F(\cdot,\lambda): H \to H$ *defined by*

$$\begin{aligned} F(u,\lambda) = J_{\rho}^{\phi(\cdot,\lambda)}[J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)) - \rho(M(J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) \\ - N(J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) - f)], \quad \forall u \in H. \end{aligned} \tag{2}$$

has a fixed point $x \in H$ *and* $y = J_{\beta}^{\varphi(\cdot,\lambda)}[x-\beta(M(x,\lambda) - N(x,\lambda)-g)]$.

Remark 3.2. *Lemma 2.1 in [2] are special cases of Lemma 3.1 in this paper.*

Theorem 3.3. *Let* $M : H \times P \to H$ *be Lipschitz continuous with constant* m *and strongly monotone with constant* a, $N : H \times P \to H$ *be Lipschitz continuous with constant* n *and pseudocontractive with constant* b. *Let* $\varphi : H \times P \to (-\infty, +\infty]$ *be a proper convex lower semicontinuous functional, let* ρ *and* β *be positive constants. If there exists a constant* θ *satisfying*

$$\theta = \sqrt{(1-2(a-b)\rho+(m+n)^2\rho^2} \times \sqrt{(1-2(a-b)\beta+(m+n)^2\beta^2} < 1 \tag{3}$$

then for any given $f, g \in H$, $\lambda \in P$, *the system of parametric general nonlinear variational inequalities* (1) *has a unique solutions* $x, y \in H$.

Proof. For each given $\lambda \in P$, we assert that $F(\cdot,\lambda) : H \to H$ defined by (2) is a contraction mapping. Since M is both m-Lipschitz continuous and a-strongly monotone and N is both n-Lipschitz continuous and b-pseudocontractive, it follows that

$$\begin{aligned}
\|F(u,\lambda)-F(v,\lambda)\|^2 &= \Big\| J_{\rho}^{\phi(\cdot,\lambda)}[J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)) - \rho(M(J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) \\
&- N(J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda)-f)] - J_{\rho}^{\phi(\cdot,\lambda)}[J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)) \\
&- \rho(M(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda) - N(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda)-f)]\|^2 \\
&\le \Big\| J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)) - J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)) - \rho[M(J_{\beta}^{\phi(\cdot,\lambda)}(u \\
&- \beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) - M(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda)] + \rho[N(J_{\beta}^{\phi(\cdot,\lambda)}(u \\
&- \beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) - N(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda)]\Big\|^2 \\
&= \Big\| J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)) - J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g))\Big\|^2 \\
&- 2\rho\Big\langle J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)) - J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)), M(J_{\beta}^{\phi(\cdot,\lambda)}(u \\
&- \beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) - M(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda)\ \Big\rangle \\
&+ 2\rho\Big\langle J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)) - J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)), N(J_{\beta}^{\phi(\cdot,\lambda)}(u \\
&- \beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) - N(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda)\ \Big\rangle \\
&+ \rho^2\Big\| M(J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(u,\lambda)-g)),\lambda) \\
&- M(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda) - N(J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)),\lambda) \\
&+ N(J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g)),\lambda)\,\|^2 \\
&\le (1-2(a-b)\rho+(m+n)^2\rho^2)\Big\| J_{\beta}^{\phi(\cdot,\lambda)}(u-\beta(M(u,\lambda)-N(u,\lambda)-g)) \\
&- J_{\beta}^{\phi(\cdot,\lambda)}(v-\beta(M(v,\lambda)-N(v,\lambda)-g))\|^2 \\
&\le (1-2(a-b)\rho+(m+n)^2\rho^2)[\|u-v\|^2 - 2\beta\langle u-v, M(u,\lambda)-M(v,\lambda)\rangle + 2\beta\langle u-v, N(u,\lambda) \\
&- N(v,\lambda)\rangle + \beta^2\|M(u,\lambda)-M(v,\lambda)-N(u,\lambda)+N(v,\lambda)\|^2] \\
&\le (1-2(a-b)\rho+(m+n)^2\rho^2)(1-2(a-b)\beta+(m+n)^2\beta^2)\|u-v\|^2 \qquad (4)
\end{aligned}$$

for all $u, v \in H$. By virtue of (3) and (4), we get that

$$\|F(u,\lambda) - F(v,\lambda)\| \le \theta \|u - v\|, \ \forall u, v \in H \tag{5}$$

That is, $F(\cdot,\lambda)$ is a contraction mapping and hence it has a unique fixed point $x \in H$ for each given $\lambda \in P$. Set $y = J_\beta^{\phi(\cdot,\lambda)}[x - \beta(M(x,\lambda) - N(x,\lambda) - g)]$. It follows from Lemma 3.1 that the system of parametric general nonlinear variational inequalities (1) has a solution $x, y \in H$. Now we claim that (x, y) is the unique solution of the system of parametric general nonlinear variational inequalities (1). In fact, if $(u, v) \in H \times H$ is also a solution of the system of parametric general nonlinear variational inequalities (1), by Lemma 3.1, we know that $u = F(u,\lambda)$ and $v = J_\beta^{\phi(\cdot,\lambda)}[u - \beta(M(u,\lambda) - N(u,\lambda) - g)]$. It follows from the uniqueness of fixed point of $F(\cdot,\lambda)$ that $u = x$ and hence $v = J_\beta^{\phi(\cdot,\lambda)}[u - M(u,\lambda) - N(u,\lambda) - g)] = y$. This completes the proof. □

Theorem 3.4. *Let the conditions of Theorem 3.1 be satisfied. Assume that M and N are continuous (resp. uniformly continuous or Lipschitz continuous) with respect to the second argument. Suppose that there exists ζ satisfying*

$$\left\| J_\rho^{\phi(\cdot\lambda)}(z) - J_\rho^{\phi(\cdot\overline{\lambda})}(z) \right\| \le \zeta \left\| \lambda - \overline{\lambda} \right\|, \forall z \in H, \lambda, \overline{\lambda} \in P, \tag{6}$$

Then the solutions of the system of parametric general nonlinear variational inequalities (1) are continuous (resp. uniformly continuous or Lipschitz continuous).

Proof. Let $F(\cdot,\lambda)$ be defined by (2). It follows from Theorem 3.1 that for any $\lambda \in P$ there exists a unique $x, y \in H$ denoted by $x(\lambda)$ and $y(\lambda)$ such that they are the solution of the system of parametric general nonlinear variational inequalities (1). Hence for each $\lambda, \overline{\lambda} \in P$, we get that

$$\begin{aligned}
x(\lambda) &= F(x(\lambda),\lambda), \ \ x(\overline{\lambda}) = F(x(\overline{\lambda}),\overline{\lambda}),\\
y(\lambda) &= J_\beta^{\phi(\cdot,\lambda)}[x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) - g)],\\
y(\overline{\lambda}) &= J_\beta^{\phi(\cdot,\overline{\lambda})}[x(\overline{\lambda}) - \beta(M(x(\overline{\lambda}),\overline{\lambda}) - N(x(\overline{\lambda}),\overline{\lambda}) - g)],\\
\left\|x(\lambda) - x(\overline{\lambda})\right\| &\le \left\|F(x(\lambda),\lambda) - F(x(\lambda),\overline{\lambda})\right\| + \left\|F(x(\lambda),\overline{\lambda}) - F(x(\overline{\lambda}),\overline{\lambda})\right\|,
\end{aligned}$$

$$\begin{aligned}
\left\|F(x(\lambda),\lambda) - F(x(\lambda),\overline{\lambda})\right\| \le &\Big\| J_\rho^{\phi(\cdot,\lambda)}[J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) \ - g)) - \rho(M(J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda)\\
&- N(x(\lambda),\lambda) - g)),\lambda) - N(J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) - g)),\lambda) - f)]\\
&- J_\rho^{\phi(\cdot,\overline{\lambda})}[J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) - g)) - \rho(M(J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda)\\
&- N(x(\lambda),\lambda) - g)),\lambda) - N(J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) - g)),\lambda) - f)] \ \|\\
&+ \Big\| J_\rho^{\phi(\cdot,\overline{\lambda})}[J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) \ - g)) - \rho(M(J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda)\\
&- N(x(\lambda),\lambda) - g)),\lambda) - N(J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) - g)),\lambda) - f)]\\
&- J_\rho^{\phi(\cdot,\overline{\lambda})}[J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\overline{\lambda}) - N(x(\lambda),\overline{\lambda}) - g)) - \rho(M(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\overline{\lambda})\\
&- N(x(\lambda),\overline{\lambda}) - g)),\overline{\lambda}) - N(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\overline{\lambda}) - N(x(\lambda),\overline{\lambda}) - g)),\overline{\lambda}) - f)] \ \|\\
&\le \zeta \left\|\lambda - \overline{\lambda}\right\| + \Big\| J_\beta^{\phi(\cdot,\lambda)}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda), \lambda) - g)) - J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\lambda)\\
&- N(x(\lambda),\lambda) - g)) \ \| + \Big\| J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x \ (\lambda),\lambda) - g)) - J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)\\
&- \beta(M(x(\lambda),\overline{\lambda}) - N \ (x(\lambda),\overline{\lambda}) \Big\| - \rho(\Big\| M(J_\beta^{\phi(\cdot,\lambda)} \ (x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) - g)),\lambda)\\
&- M(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\lambda) - N(x(\lambda),\lambda) - g)),\lambda) \ \| + \| M \ (J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\lambda)\\
&- N(x(\lambda),\lambda) - g)),\lambda) - M(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda) - \beta(M(x(\lambda),\overline{\lambda}) - N(x(\lambda),\overline{\lambda}) - g)),\lambda) \ \|
\end{aligned}$$

$$+\left\|M(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\overline{\lambda})-N(x(\lambda),\overline{\lambda})-g)),\lambda)-M(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\overline{\lambda})-N(x(\lambda),\overline{\lambda})-g)),\overline{\lambda})\right\|$$
$$+\|N(J_\beta^{\phi(\cdot,\lambda)}(x(\lambda)-\beta(M(x(\lambda),\lambda)-N(x(\lambda),\lambda)-g)),\lambda)-N(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\lambda)-N(x(\lambda),\lambda)-g)),\lambda)\|$$
$$+\|N(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\lambda)-N(x(\lambda),\lambda)-g)),\lambda)-N(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\overline{\lambda})-N(x(\lambda),\overline{\lambda})-g)),\lambda)\|$$
$$+\|N(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\overline{\lambda})-N(x(\lambda),\overline{\lambda})-g)),\lambda)-N(J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\overline{\lambda})-N(x(\lambda),\overline{\lambda})-g)),\overline{\lambda})\|)$$
$$\leq[2+(m+n)\rho]\zeta\left\|\lambda-\overline{\lambda}\right\|+[1+(m+n)\rho]\beta\left(\left\|M(x(\lambda),\lambda)-M(x(\lambda),\overline{\lambda})\right\|+\left\|N(x(\lambda),\lambda)-N(x(\lambda),\overline{\lambda})\right\|\right)$$
$$+\rho\left(\left\|M(z,\lambda)-M(z,\overline{\lambda})\right\|+\left\|N(z,\lambda)-N(z,\overline{\lambda})\right\|\right)$$

where $z=J_\beta^{\phi(\cdot,\overline{\lambda})}(x(\lambda)-\beta(M(x(\lambda),\overline{\lambda})-N(x(\lambda),\overline{\lambda})-g))$. It follows from (5) that

$$\left\|F(x(\lambda),\overline{\lambda})-F(x(\overline{\lambda}),\overline{\lambda})\right\|\leq\theta\left\|x(\lambda)-x(\overline{\lambda})\right\|,$$

$$\left\|x(\lambda)-x(\overline{\lambda})\right\|\leq(1-\theta)^{-1}\{[2+(m+n)\rho]\zeta\left\|\lambda-\overline{\lambda}\right\|+[1+(m+n)\rho]\beta(\left\|M(x(\lambda),\lambda)-M(x(\lambda),\overline{\lambda})\right\|$$
$$+\left\|N(x(\lambda),\lambda)-N(x(\lambda),\overline{\lambda})\right\|+\rho\left(\left\|M(z,\lambda)-M(z,\overline{\lambda})\right\|+\left\|N(z,\lambda)-N(z,\overline{\lambda})\right\|\right)\tag{7}$$
$$\left\|y(\lambda)-y(\overline{\lambda})\right\|\leq\left\|J_\beta^{\phi(\cdot,\lambda)}[x(\lambda)-\beta(M(x(\lambda),\lambda)-N(x(\lambda),\lambda)-g)]-J_\beta^{\phi(\cdot,\overline{\lambda})}[x(\lambda)-\beta(M(x(\lambda),\lambda)-N(x(\lambda),\lambda)-g)]\right\|$$
$$+\left\|J_\beta^{\phi(\cdot,\overline{\lambda})}[x(\lambda)-\beta(M(x(\lambda),\lambda)-N(x(\lambda),\lambda)-g)]-J_\beta^{\phi(\cdot,\overline{\lambda})}[x(\overline{\lambda})-\beta(M(x(\overline{\lambda}),\overline{\lambda})-N(x(\overline{\lambda}),\overline{\lambda})-g)]\right\|\tag{8}$$
$$\leq\zeta\left\|\lambda-\overline{\lambda}\right\|+[1+(m+n)\beta]\left\|x(\lambda)-x(\overline{\lambda})\right\|+\beta\left(\left\|M(x(\lambda),\lambda)-M(x(\lambda),\overline{\lambda})\right\|+\left\|N(x(\lambda),\lambda)-N(x(\lambda),\overline{\lambda})\right\|\right)$$

It follows from (7), (8) and the continuities of M and N (resp. uniformly continuous or Lipschitz continuous) with respect to the second argument that the solutions of the system of parametric general nonlinear variational inequalities (1) are continuous (resp. uniformly continuous or Lipschitz continuous). This completes the proof. □

Remark 3.5. *Theorem 3.1, 3.2 extend, improve and unify Theorem 2.1 in [2] and Theorem 3.4 in [7].*

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