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On Contra Weakly μg-Continuous Function in Generalized Topological Spaces

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Abstract: In this paper we introduce some new class of functions called contra wμg-continuous functions and contra wμg-irresolute functions using wμg-closed sets in generalized topological space. We investigate their relationships with other existing functions in generalized topological space. Also we prove that composition of two contra wμg-continuous need not be contra wμg-continuous.

Keywords: wμg-closed sets, contra wμg-continuous functions, contra wμg-irresolute functions.

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1. Introduction

In 2002, generalized topological space (GTS), introduced by A. Császár [2] In 2009, W.K.Min [4] has introduced and studied the notion of (α, g') continuous functions, (π, g') continuous functions in generalized topological space. In 2011, D.Jayanthi [3] has introduced some contra continuous functions in generalized topological space. The purpose of this paper is to define a new class of continuous functions called contra wμg-continuous functions and contra wμg-irresolute functions in GTS.

2. Preliminaries

Let X be a non empty set, $\exp X$ denotes the power set of X . A generalized topology simply GT [2] on a non empty set X is a collection of subsets of X such that $\phi \in \mu$ and μ is closed under arbitrary union. Elements of μ are called μ-open sets.A subset A of X is said to be μ-closed if A^c is μ-open.Then the pair (X,μ) is called a Generalized Topological Space(GTS).If A is a subset of X, then c_μ(A) is the smallest μ-closed set containing A and i_μ(A) is the largest μ-open set contained in A.A space (X,μ) is said to be strong, if $X \in \mu$. Throughout this paper X and Y mean GTS's (X,μ_1) and (Y,μ_2) and the function $f : X \to Y$ denotes a single valued function of a space (X,μ_1) into a space (Y,μ_2).We recall the following definitions and results.

Definition 2.1 ([5])**.** *Let (X, μ) be a GTS, and $A \subseteq X$. Then A is said to be*

(1). μ-α-open if $A \subseteq i_\mu c_\mu i_\mu(A)$

(2). μ-π-open if $A \subseteq i_\mu c_\mu(A)$

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The complements of μ-α-open (resp.μ-π-open, μ-open) is said to be $\mu-\alpha$-closed(res.μ-π-closed,μ-closed).

Definition 2.2 ([5]). *A subset A of X is said to be weakly μg-closed set (simply $W\mu g$-closed) if $c_\mu i_\mu(A) \subseteq U$ whenever $A \subseteq U$ and U is μ- open. The complements of $W\mu g$-closed set is called $W\mu g$-open set. Let us denote $\mu(X)$ (reps.$\alpha\mu(X), \pi\mu(X), G\mu(X), W\mu GO(X)$) the class of all μ-open (resp.μ-α-open,μ-π-open,μg-open $W\mu g$-open)sets on X.*

Definition 2.3 ([5]). *A function $f : X \to Y$ is said to be (μ_1, μ_2) continuous functions, if $f^{-1}(U)$ is μ_1-open in X for every μ_2-open set U of Y.*

Definition 2.4. *[5] A function $f : X \to Y$ is said to be ($w\mu g - \mu_1, \mu_2$)-continuous if $f^{-1}(U)$ is $w\mu g$-open in X for every μ_2-open set U of Y.*

Definition 2.5 ([3]). *Let (X, μ_1) and (Y, μ_2) be GTS's. Then a function $f : X \to Y$ is said to be contra (μ_1, μ_2)-continuous if $f^{-1}(U)$ is μ-closed in X for every μ-open set U of Y.*

Definition 2.6 ([3]). *Let (X, μ_1) and (Y, μ_2) be GTS's. Then a function $f : X \to Y$ is said to be contra $(\alpha\mu_1, \mu_2)$-continuous if $f^{-1}(U)$ is μ-α-closed in X for every μ-open set U of Y.*

Definition 2.7 ([3]). *Let $(X, \mu_1)$and (Y, μ_2) be GTS's. Then a function $f : X \to Y$ is said to be contra $(\pi\mu_1, \mu_2)$-continuous if $f^{-1}(U)$ is μ-π-closed in X for every μ-open set U of Y.*

Definition 2.8 ([5]). *A function $f : X \to Y$ is said to be $w\mu g$-open if image of every $w\mu g$-open set is $w\mu g$-open*

3. Contra $W\mu g$-Continuous Function

Definition 3.1. *A function $f : X \to Y$ is said to be contra $W\mu g$-continuous if the inverse image of each μ-open in Y is $W\mu g$-closed in X.*

Example 3.2. *Let $X = Y = \{u, v, w\}$ and $\mu_1 = \mu_2 = \{\phi, X, \{u\}, \{v, w\}, \{u, w\}\}$. Then $W\mu gC(X) = \{\phi, X, \{u\}, \{v\}, \{w\}, \{v, w\}, \{u, v\}\}$. A function $f : X \to Y$ defined by $f(u) = u$, $f(v) = v = f(w)$. Then f is contra $W\mu g$-continuous.*

Remark 3.3. *$\mu(X) \subseteq \mu$-$\alpha(X) \subseteq \mu$-$\pi(X) \subseteq W\mu g(X)$.*

Theorem 3.4. *Let $f : X \to Y$ be a map. Then the following are equivalent*

(i). f is contra $W\mu g$- continuous.

(ii). The inverse image of each μ-closed in Y is $W\mu g$-open in X.

Proof. $(i) \Rightarrow (ii)$ Let f be contra $W\mu g$-continuous. Let F be a μ-closed set in Y.Then F^c is μ-open in Y. Since f is contra $W\mu g$- continuous, $f^{-1}(F^c)$ is $W\mu g$-closed in X. $(f^{-1}(F))^c$ is $W\mu g$-closed in X. Then $f^{-1}(F)$ is $W\mu g$-open in X.
$(ii) \Rightarrow (i)$ Let F be μ-open set in Y.F^c is μ-closed in Y. $f^{-1}(F^c)$ is $W\mu g$-open in X.$(f^{-1}(F))^c$ is $W\mu g$-open in X.Then $f^{-1}(F)$ is $W\mu g$-closed in X. Hence f is contra $W\mu g$-continuous. □

Theorem 3.5. *If $f : X \to Y$ is contra $W\mu g$-continuous, then for every $x \in X$ and each μ-closed subset F of Y containing $f(x)$ there exists $U \in W\mu gO(X)$ and $f(U) \subseteq F$.*

Proof. Let $x \in X$, F be a μ-closed set in Y and $f(x) \in F$. Since F is a contra $W\mu g$-continuous, $f^{-1}(F)$ is $W\mu g$-open in X and $x \in f^{-1}(F)$. Let $U = f^{-1}(F)$ and $x \in U$. Then U is $W\mu g$-open and $f(U) \subseteq F$. □

Theorem 3.6. *Every contra (μ_1, μ_2)-continuous is contra $W\mu g$-continuous.*

Proof. Let F be a μ-open set in Y. Since f is contra (μ_1, μ_2)-continuous, $f^{-1}(F)$ is μ-closed in X. Thus, $f^{-1}(F)$is $W\mu g$-closed in X. Hence f is contra $W\mu g$-continuous. The converse of the theorem need not be true as seen from the following example. □

Example 3.7. *Let $X = Y = \{u, v, w\}$ and $\mu_1 = \mu_2 = \{\phi, X, \{u\}, \{u, v\}, \{v, w\}\}$. Then $W\mu gC(X) = \{\phi, X, \{u\}, \{v\}, \{w\}, \{v, w\}, \{u, w\}\}$. A function $f : X \to Y$ defined by $f(u) = u$, $f(v) = w$, $f(w) = v$. Clearly f is contra $W\mu g$-continuous but not contra (μ_1, μ_2)-continuous, since $f^{-1}\{u, v\} = \{u, w\}$ which is not μ-closed.*

Theorem 3.8. *Every contra $(\alpha\mu_1, \mu_2)$-continuous is contra $W\mu g$-continuous.*

Proof. Let F be a μ-open set in Y. Since f is contra $(\alpha\mu_1, \mu_2)$-continuous, $f^{-1}(F)$ is μ-α-closed in X. This implies $f^{-1}(F)$ is $W\mu g$-closed in X. Hence f is contra $W\mu g$-continuous. The converse of the theorem need not be true as seen from the following example. □

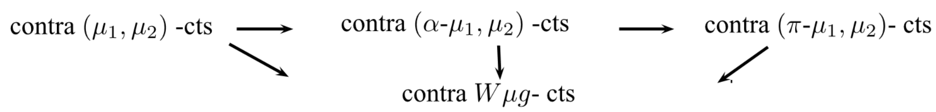
Example 3.9. *Let $X = Y = \{u, v, w\}$ and $\mu_1 = \mu_2 = \{\phi, X, \{u\}, \{v\}, \{u, v\}\}$. Then $W\mu gC(X) = \{\phi, X, \{u\}, \{w\}, \{v, w\}, \{u, w\}\}$. A function $f : X \to Y$ defined by $f(u) = u$, $f(v) = w$, $f(w) = v$. Clearly f is contra $W\mu g$-continuous but not contra $(\alpha\mu_1, \mu_2)$-continuous, since $f^{-1}\{u\} = \{u\}$ which is not μ-α-closed in X.*

Theorem 3.10. *Every contra $(\pi\mu_1, \mu_2)$-continuous is contra $W\mu g$-continuous.*

Proof. Let F be a μ open set in Y. Since f is contra $(\pi\mu_1, \mu_2)$-continuous, $f^{-1}(F)$ is μ-preclosed in X. This implies $f^{-1}(F)$ is $W\mu g$-closed in X. Hence f is contra $W\mu g$-continuous. The converse of the theorem need not be true as seen from the following example. □

Example 3.11. *Let $X = Y = \{u, v, w\}$ and $\mu_1 = \mu_2 = \{\phi, X, \{u\}, \{v\}, \{u, v\}\}$. Then $W\mu gC(X) = \{\phi, X, \{u\}, \{w\}, \{v, w\}, \{u, w\}\}$. A function $f : X \to Y$ defined by $f(u) = u = f(w)$, $f(v) = w$. Clearly f is contra $W\mu g$-continuous but not contra $(\pi\mu_1, \mu_2)$-continuous, since $f^{-1}\{u\} = \{u\}$ which is not π-μ-closed in X.*

Remark 3.12. *According to the above discussion we have the following relation*



$A \to B$ means A implies B but not conversely

Definition 3.13. *A space X is called a $Tw\mu g$ space if every $W\mu g$-closed set is μ-closed.*

Theorem 3.14. *If a function $f : X \to Y$ is contra $W\mu g$-continuous and X is $Tw\mu g$ space, then f is contra (μ_1, μ_2)-continuous.*

Proof. Let F be a μ open set in Y. Since f is contra $W\mu g$-continuous, $f^{-1}(F)$ is $W\mu g$-closed in X. Since X is a $Tw\mu g$ space, $f^{-1}(F)$ is μ-closed in X. Hence f is contra (μ_1, μ_2)-continuous. □

Theorem 3.15. *If X is $Tw\mu g$ space. Then then the following are equivalent.*

(i). f is contra (μ_1, μ_2)-continuous.

(ii). f is contra $W\mu g$-continuous.

Proof. (i) $\Rightarrow$ (ii) Let F be an open in Y. Since f is contra (μ_1,μ_2)-continuous, $f^{-1}(F)$ is μ-closed in X. Then, $f^{-1}(F)$ is $W\mu g$-closed in X. Hence f is contra $W\mu g$-continuous.

(ii) $\Rightarrow$ (i) Let F be a μ-open set in Y. Then $f^{-1}(F)$ be an $W\mu g$-closed set in X. Since X is a T$w\mu g$ space, $f^{-1}(F)$ is μ-closed in X. Hence f is contra (μ_1,μ_2)-continuous. □

Definition 3.16. *A function $f:(X,\mu_1)\rightarrow(Y,\mu_2)$ is called contra $W\mu g$-irresolute if $f^{-1}(U)$ is $W\mu g$-open in X for every $W\mu g$-closed set U of Y.*

Example 3.17. *Let $X = Y = \{u,v,w\}$ and $\mu_1 = \mu_2 = \{\phi,X,\{u\},\{u,v\},\{v,w\}\}$. Then $W\mu gO(X)$=$\{\phi,X,\{u\},\{v\},\{u,v\},\{v,w\},\{u,w\}\}$ and $W\mu gC(Y) = \{\phi,X,\{u\},\{v\},\{w\},\{v,w\},\{u,w\}\}$. A function $f:X\rightarrow Y$ defined by $f(u)=u$, $f(v)=v=f(w)$. Then f is contra $W\mu g$-irresolute.*

Theorem 3.18. *Every contra $W\mu g$-irresolute is contra $W\mu g$-continuous.*

Proof. It is obvious. □

Remark 3.19. *The converse of the above theorem is not true as seen from the following example.*

Example 3.20. *Let $X = Y = \{u,v,w\}$ and $\mu_1 = \mu_2 = \{\phi,X,\{u\},\{u,v\},\{v,w\}\}$. Then $W\mu gO(X) = \{\phi,X,\{u\},\{v\},\{u,v\},\{v,w\},\{u,w\}\}$ and $W\mu gC(Y) = \{\phi,X,\{u\},\{v\}\{w\},\{v,w\},\{u,w\}\}$. A function $f:X\rightarrow Y$ defined by $f(u)=u$, $f(v)=w$, $f(w)=v$. Then f is contra $W\mu g$-continuous which is not contra $W\mu g$-irresolute, since $f^{-1}(\{v\})=\{w\}$ is not $W\mu g$-open in X.*

Definition 3.21 ([5]). *A function $f:(X,\mu_1)\rightarrow(Y,\mu_2)$ is called $W\mu g$-irresolute if $f^{-1}(U)$ is $W\mu g$-open in X for every $W\mu g$-open set U of Y.*

Theorem 3.22. *If $f:X\rightarrow Y$ is contra $W\mu g$-continuous and irresolute. Then $f^{-1}(F)\subseteq i_\mu c_\mu(f^{-1}(c_\mu(F)))$ for every subset F in Y.*

Proof. Let $F\subseteq Y$. Then $c_\mu(F)$ is μ closed. Since every μ closed set is $W\mu g$-closed set, $c_\mu(F)$ is $W\mu g$-closed set. Since f is contra $W\mu g$-continuous, $f^{-1}(c_\mu(F))$ is $W\mu g$-open. Also, $f^{-1}(c_\mu(F))$ is μ closed. Then $(f^{-1}(c_\mu(F)))\subseteq i_\mu c_\mu(f^{-1}(c_\mu(F)))$. Therefore $f^{-1}(F)\subseteq(f^{-1}(c_\mu(F)))\subseteq i_\mu c_\mu(f^{-1}(c_\mu(F)))$. Hence $f^{-1}(F)\subseteq i_\mu c_\mu(f^{-1}(c_\mu(F)))$. □

Theorem 3.23. *If $f:X\rightarrow Y$ is a continuous map and $F\subseteq i_\mu c_\mu(f^{-1}(c_\mu(f(F))))$ for all μ-closed set F. Then f is contra $W\mu g$-continuous.*

Proof. Let F be a μ-closed set in Y. Since f is a continuous map, $f^{-1}(F)$ is μ closed set in X. By hypothesis, $f^{-1}(F)\subseteq i_\mu c_\mu(f^{-1}(c_\mu(f(f^{-1}(F)))))\subseteq i_\mu(c_\mu(f^{-1}(c_\mu(F))))$. Thus $f^{-1}(F)\subseteq i_\mu(c_\mu(f^{-1}(c_\mu(F))))$. Therefore $f^{-1}(F)$ is $W\mu g$-open in X. Hence f is contra $W\mu g$-continuous. □

Remark 3.24. *The composition of two contra $W\mu g$-continuous functions need not be contra $W\mu g$-continuous as seen from the following example.*

Example 3.25. *Let $X = Y = Z = \{u,v,w\}$ and $\mu_1=\mu_2=\mu_3=\{\phi,X,\{u\},\{u,v\},\{v,w\}\}$. Then $W\mu gO(X) = \{\phi,X,\{u\},\{v\},\{u,v\},\{v,w\},\{u,w\}\}$ and $W\mu gC(Y)=\{\phi,X,\{u\},\{v\},\{w\},\{v,w\},\{u,w\}\}$. A function $f:X\rightarrow Y$ defined by $f(u)=u$, $f(v)=w$, $f(w)=v$ and $g:Y\rightarrow Z$ defined by $g(u)=v$, $g(v)=u$, $g(w)=v$. Then clearly f and g are contra $W\mu g$-continuous. But $gof:X\rightarrow Z$ is not contra $W\mu g$-continuous, since $(gof)^{-1}\{v,w\}=f^{-1}(g^{-1}\{v,w\})=f^{-1}\{u,w\}=\{u,v\}$ which is not $W\mu g$-closed in X.*

Theorem 3.26. *Let $f : X \to Y$ be surjective, $W\mu g$-irresolute and $W\mu g$-open and $g : Y \to Z$ be any function then gof is contra $W\mu g$-continuous if and only if g is contra $W\mu g$-continuous.*

Proof. Suppose gof is contra $W\mu g$-continuous. Let F be a closed set in Z. Then $(gof)^{-1}(F)$=$f^{-1}(g^{-1}(F))$ is $W\mu g$-open in X. Since f is $W\mu g$-open and surjective $f(f^{-1}(g^{-1}(F))$ is $W\mu g$-open in Y (i.e) $g^{-1}(F)$ is $W\mu g$-open in Y. Hence g is contra $W\mu g$-continuous.

Conversely, suppose that g is contra $W\mu g$-continuous. Let F be a closed set in Z. Then $g^{-1}(F)$ is $W\mu g$-open in Y. Since f is $W\mu g$-irresolute, $f^{-1}(g^{-1}(F))$ is $W\mu g$-open in X. Hence gof is contra $W\mu g$-continuous. □

Theorem 3.27. *Let $f : X \to Y$ and $g : Y \to Z$ be a function then,*

(i). If g is $W\mu g$-irresolute and f is contra $W\mu g$-irresolute, then gof is contra $W\mu g$-irresolute.

(ii). If g is contra $W\mu g$-irresolute and f is $W\mu g$-irresolute, then gof is contra $W\mu g$-irresolute.

Proof.

(i). Let F be a $W\mu g$-closed in Z. Since g is $W\mu g$-irresolute, $g^{-1}(F)$ is $W\mu g$-closed in Y. Then $f^{-1}(g^{-1}(F))$ is $W\mu g$-open in X (since f is contra $W\mu g$-irresolute). That is $(gof)^{-1}(F)$ is $W\mu g$-open in X . This implies gof is contra $W\mu g$-irresolute.

(ii). Let F be closed in Z. Since g is contra $W\mu g$-irresolute, $g^{-1}(F)$ is $W\mu g$-open in Y. Then $f^{-1}(g^{-1}(F))$ is $W\mu g$-open in X(since f is $W\mu g$-irresolute). That is, $(gof)^{-1}(F)$is $W\mu g$-open in X. This implies gof is contra $W\mu g$-irresolute. □

Theorem 3.28. *If $f : X \to Y$ is contra $W\mu g$-irresolute and $g : Y \to Z$ is $W\mu g$-continuous, then gof is contra $W\mu g$-continuous.*

Proof. Let F be a closed set in Z. Since g is $W\mu g$-continuous, $g^{-1}(F)$ is $W\mu g$-closed in Y. Then $f^{-1}(g^{-1}(F))$is $W\mu g$-open in X (since f is contra $W\mu g$-irresolute). Then $(gof)^{-1}(F)$ is $W\mu g$-open in X. This implies gof is contra $W\mu g$-irresolute. □

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