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On Soft $g^{**}\mu$-Closed Sets in Soft Generalized Topological Spaces

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Abstract: In this paper we introduce some new class of sets called soft $g\mu$-closed sets,soft $g^*\mu$-closed sets and soft $g^{**}\mu$-closed sets in soft generalized topological spaces and some of their properties are established.Further, by using soft $g^{**}\mu$-closed sets we introduce soft $T_{\mu_{g^{**}}}$-space.

Keywords: Soft $g\mu$-closed sets, soft $g^*\mu$-closed sets, soft $g^{**}\mu$-closed sets and soft $T_{\mu_{g^{**}}}$-space.
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1. Introduction

In 1999 Molodtsov [4] was introduced the concept of soft set theory as a new mathematical tool for dealing with uncertain problems.Shabir and Naz [5] introduced the notion of soft topological spaces which is defined over a universe with a fixed set of parameters. In 2002, A.Császár [2] introduced the theory of generalized topological spaces. Soft Generalized Topology (**SGT**) is based on soft set theory. In 2014, Jyothis and Sunil [3] introduced the concept of Soft Generalized Topological Space and discussed some separation axioms in **SGTS**. Recently M.Vigneshwaran, K. Baby [6] were introduced soft β^*g$\alpha\mu$-closed sets in Soft Generalized Topology. The Soft Generalized Topology is different from Soft Topology by its axioms. The purpose of this paper is to introduce soft $g\mu$, $g^*\mu$, $g^{**}\mu$-closed sets in SGTS and investigate some of their properties.

2. Preliminaries

Let U be an initial universe and E be the set of all possible parameters with respect to U. Let P(U) denote the power set of U and A be a nonempty subset of E.

Definition 2.1 ([1])**.** *Let F_A be a soft set over U defined by the set of ordered pairs $F_A = \{(e, f_A(e))/e \in E, f_A(e) \in P(U)\}$, where f_A is a mapping given by $f_A\colon A \to P(U)$ such that $f_A(e) = \varphi$ if $e \notin A$. Here f_A is called an approximate function of the soft set F_A. The family of all these soft sets over U with E is denoted by $S(U)$ or $S(U)_E$.*

Definition 2.2 ([1])**.** *Let $F_G, F_H \in S(U)$. Then the soft union of soft sets F_G, F_H is defined by the approximate function $f_{G\cup H}(e) = f_G(e) \cup f_H(e)$.*

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Definition 2.3 ([1]). *Let $F_G, F_H \in S(U)$. Then the soft intersection of soft sets F_G, F_H is defined by the approximate function $f_{G\cap H}(e) = f_G(e) \cap f_H(e)$.*

Definition 2.4 ([1]). *Let $F_G \in S(U)$. Then the soft complement of F_G is defined by the approximate function $f_{G^c}(e) = (f_G(e))^c$ where $(f_G(e))^c$ is the complement of $f_G(e)$, that is $(f_G(e))^c = U \backslash f_G(e) \forall e \in E$.*

Definition 2.5 ([1]). *Let $F_G, F_H \in S(U)$. Then the soft difference of F_G and F_H, denoted by $F_G \backslash F_H$, is defined by the approximate function $f_{G\backslash H}(e) = f_G(e) \backslash f_H(e)$.*

Definition 2.6 ([3]). *Let $F_A \in S(U)$. A soft generalized Topology (SGT) on F_A, denoted by μ or μ_{F_A} is a collection of soft subsets of F_A having the following properties:*

- $F\phi \in \mu_{F_A}$
- *μ_{F_A} is closed under arbitrary union.*

*The pair (F_A, μ_{F_A}) is called a Soft Generalized Topological Spaces (**SGTS**). A space (F_A, μ_{F_A}) is said to be strong if $F_A \in \mu_{F_A}$. Every element of μ_{F_A} is called a soft μ-open set. Note that F_ϕ is a soft μ-open set.*

Definition 2.7 ([3]). *Let (F_A, μ_{F_A}) be a soft generalized topological space over F_A and F_G be a soft set over F_A .Then $c_\mu(F_G)$ is the smallest soft μ-closed set over F_A which contains F_G and $i_\mu(F_G)$ is the largest soft μ-open set which is contained in F_G.*

Definition 2.8 ([6]). *Let (F_A, μ_{F_A}) be a SGTS. A soft subset F_G of F_A is said to be soft $\alpha\mu$-closed if $c_\mu(i_\mu(c_\mu(F_G)) \subseteq F_G$.*

Definition 2.9 ([6]). *Let (F_A, μ_{F_A}) be a SGTS and $\alpha, \beta \in F_A$ such that $\alpha \neq \beta$. If there exists soft μ-open sets F_G and F_H such that $\alpha \in F_G$ and $\beta \notin F_G$ or $\beta \in F_H$ and $\alpha \notin F_H$, then (F_A, μ_{F_A}) is called a soft generalized μ-T_0 space.*

3. Soft $g\mu$-closed Sets, Soft $g^*\mu$-closed Sets and Soft $g^{**}\mu$-closed Sets in SGTS

Definition 3.1. *A soft subset $F_G \in S(U)$ is called a soft $g\mu$-closed in SGTS (F_A, μ_{F_A}) if $c_\mu(F_G) \subseteq F_E$ whenever $F_G \subseteq F_E$ and F_E is soft μ-open.*

Definition 3.2. *A soft subset $F_G \in S(U)$ is called a soft $g^*\mu$-closed in SGTS (F_A, μ_{F_A}) if $c_\mu(F_G) \subseteq F_E$ whenever $F_G \subseteq F_E$ and F_E is soft $g\mu$-open.*

Definition 3.3. *Let (F_A, μ_{F_A}) be a soft generalized topological space over F_A and F_G be a soft set over F_A. Then $c_{\mu_{g^*}}(F_G)$ is the smallest soft $g^*\mu$-closed set over F_A which contains F_G and $i_{\mu_{g^*}}(F_G)$ is the largest soft $g^*\mu$-open set which is contained in F_G.*

Definition 3.4. *A soft subset $F_G \in S(U)$ is called a soft $g^{**}\mu$-closed in SGTS (F_A, μ_{F_A}) if $c_{\mu_{g^*}}(F_G) \subseteq F_E$ whenever $F_G \subseteq F_E$ and F_E is soft $g^*\mu$-open.*

Definition 3.5. *A soft subset $F_G \in S(U)$ is said to be soft $g^{**}\mu$-open if $(F_G)^c$ is soft $g^{**}\mu$-closed.*

Remark 3.6. *The union (intersection) of two soft $g^{**}\mu$-closed sets is not a soft $g^{**}\mu$-closed set.*

Example 3.7. *Let $U = \{a, b, c\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$; $\mu = \{F_\phi, F_1, F_2, F_3, F_4, F_5, F_7\}$, where*

$$F_\phi = \{(e_1, \phi), (e_2, \phi)\},$$

$$F_1 = \{(e_1, \{b\}), (e_2, \{b\})\},$$
$$F_2 = \{(e_1, \{c\}), (e_2, \{c\})\},$$
$$F_3 = \{(e_1, \{a, b\}), (e_2, \{a, b\})\},$$
$$F_4 = \{(e_1, \{b, c\}), (e_2, \{b, c\})\},$$
$$F_5 = \{(e_1, \{a, c\}), (e_2, \{a, c\})\},$$
$$F_6 = \{(e_1, \{a\}), (e_2, \{a\})\},$$
$$F_7 = \{(e_1, \{a, b, c\}), (e_2, \{a, b, c\})\},$$

Take $F_G = \{(e_1, \{a\}), (e_2, \{a, b\})\}$, $F_H = \{(e_1, \{a\}), (e_2, \{a, c\})\}$ *are soft* $g^{**}\mu$*-closed sets. But* $f_G(e) \cup f_H(e) = \{(e_1, \{a\}), (e_2, \{a, b, c\})\}$ *is not a soft* $g^{**}\mu$*-closed set. And, Take* $F_G = \{(e_1, \{a, b\}), (e_2, \{b, c\})\}$, $F_H = \{(e_1, \{b, c\}), (e_2, \{a, c\})\}$ *are soft* $g^{**}\mu$*-closed sets. But* $f_G(e) \cap f_H(e) = \{(e_1, \{b\}), (e_2, \{c\})\}$ *is not a soft* $g^{**}\mu$*-closed set.*

Remark 3.8. *We can see from the following example that a soft* $g^{**}\mu$*-closed set is independent of soft* $\alpha\mu$*-closed.*

Example 3.9. *Let* $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$ *and* $A = \{e_1, e_2\}$. *The set of soft sets over* U *with the parameter* E *is given by*

$$F_\phi = \{(e_1, \phi), (e_2, \phi)\},$$
$$F_1 = \{(e_1, \phi), (e_2, \{h_1\})\},$$
$$F_2 = \{(e_1, \phi), (e_2, \{h_2\})\},$$
$$F_3 = \{(e_1, \phi), (e_2, \{h_1, h_2\})\},$$
$$F_4 = \{(e_1, \{h_1\}), (e_2, \phi)\},$$
$$F_5 = \{(e_1, \{h_1\}), (e_2, \{h_1\})\},$$
$$F_6 = \{(e_1, \{h_1\}), (e_2, \{h_2\})\},$$
$$F_7 = \{(e_1, \{h_1\}), (e_2, \{h_1, h_2\})\},$$
$$F_8 = \{(e_1, \{h_2\}), (e_2, \phi)\},$$
$$F_9 = \{(e_1, \{h_2\}), (e_2, \{h_1\})\},$$
$$F_{10} = \{(e_1, \{h_2\}), (e_2, \{h_2\})\},$$
$$F_{11} = \{(e_1, \{h_2\}), (e_2, \{h_1, h_2\})\},$$
$$F_{12} = \{(e_1, \{h_1, h_2\}), (e_2, \phi)\},$$
$$F_{13} = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1\})\},$$
$$F_{14} = \{(e_1, \{h_1, h_2\}), (e_2, \{h_2\})\},$$
$$F_{15} = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1, h_2\})\},$$
$$\mu = \{F_\phi, F_2, F_4, F_6, F_7, F_{13}, F_{15}\};$$
$$\mu^c = \{F_\phi, F_2, F_8, F_9, F_{11}, F_{13}, F_{15}\}$$

Here soft $\alpha\mu$*-closed sets* $= \{F_\phi, F_1, F_2, F_3, F_8, F_9, F_{10}, F_{11}, F_{13}, F_{15}\}$. *Soft* $g^{**}\mu$*-closed sets* $= \{F_\phi, F_1, F_2, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}\}$.

Theorem 3.10. *Every soft* μ*-closed set is soft* $g^*\mu$*- closed.*

Proof. Let F_G be a soft μ-closed set. Suppose $F_G \subseteq F_E$ and F_E is soft $g\mu$-open. Since F_G is a soft μ-closed set, $c_\mu(F_G) = F_G$. This implies $c_\mu(F_G) \subseteq F_E$. Hence F_G is soft $g^*\mu$-closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.11. *Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$; $\mu = \{F_\phi, F_1, F_3, F_6, F_7\}$; $\mu^c = \{F_4, F_8, F_9, F_{13}, F_{15}\}$. Soft $g^*\mu$-closed sets= $\{F_4, F_8, F_9, F_{12}, F_{13}, F_{14}, F_{15}\}$. Here $\{F_{12}, F_{14}\}$ are not soft μ-closed sets.*

Theorem 3.12. *Every soft μ-closed set is soft $g^{**}\mu$-closed.*

Proof. Let F_G be a soft μ-closed set .Suppose $F_G \subseteq F_E$ and soft F_E is $g^*\mu$-open.Since F_G is a soft μ- closed set, $c_\mu(F_G) = F_G$. This implies $c_\mu(F_G) \subseteq F_E$. Hence F_G is soft $g^{**}\mu$-closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.13. *Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$; $\mu = \{F_\phi, F_6, F_7, F_{12}, F_{14}, F_{15}\}$; $\mu^c = \{F_\phi, F_1, F_3, F_8, F_9, F_{15}\}$. Soft $g^{**}\mu$-closed sets=$\{F_\phi, F_1, F_2, F_3, F_8, F_9, F_{10}, F_{11}, F_{13}, F_{15}\}$. Here $\{F_2, F_{10}, F_{11}, F_{13}\}$ are not soft μ-closed sets.*

Theorem 3.14. *Every soft $g^*\mu$-closed set is soft $g\mu$-closed.*

Proof. Let F_G be a soft $g^*\mu$-closed set. Suppose $F_G \subseteq F_E$ and F_E is soft μ-open. Since every soft μ-open is soft $g\mu$- open. Then $c_\mu(F_G) \subseteq F_E$. Hence F_G is soft $g\mu$-closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.15. *Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$; $\mu = \{F_\phi, F_1, F_3, F_6, F_7\}$; $\mu^c = \{F_4, F_8, F_9, F_{13}, F_{15}\}$. Soft $g\mu$-closed sets= $\{F_4, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}\}$. Here $\{F_{10}, F_{11}\}$ are not soft $g^*\mu$-closed set.*

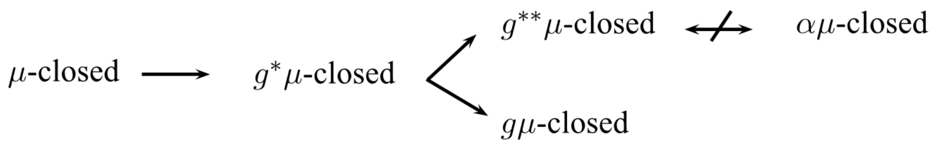
Theorem 3.16. *Every soft $g^*\mu$-closed set is soft $g^{**}\mu$-closed.*

Proof. Let F_G be a soft $g^*\mu$-closed set. Suppose $F_G \subseteq F_E$ and F_E is soft $g^*\mu$-open. Since every soft $g^*\mu$-open is soft $g\mu$-open. Then $c_\mu(F_G) \subseteq F_E$. Also we have $c_{\mu_{g^*}}(F_G) \subseteq c_\mu(F_G)$. Thus $c_{\mu_{g^*}}(F_G) \subseteq F_E$. Hence F_G is soft $g^{**}\mu$-closed. $\square$

The converse of the above theorem need not be true as seen from the following example.

Example 3.17. *Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$; $\mu = \{F_\phi, F_4, F_6, F_{10}, F_{14}\}$; $\mu^c = \{F_1, F_5, F_9, F_{11}, F_{15}\}$. Soft $g^{**}\mu$-closed sets= $\{F_1, F_3, F_5, F_7, F_8, F_9, F_{11}, F_{12}, F_{13}, F_{15}\}$. Here $\{F_3, F_7, F_8, F_{12}\}$ are not soft $g^*\mu$-closed.*

Remark 3.18. *According to the above discussion, we have the following relation.*



$A \to B$ means A implies B but not conversely, $A \not\leftrightarrow B$ means A and B are independent of each other.

Theorem 3.19. *If F_G is soft μ-open and soft $g\mu$-closed, then F_G is soft $g^{**}\mu$-closed.*

Proof. Given F_G is soft μ-open and soft $g\mu$-closed. Suppose $F_G \subseteq F_E$ and F_E is soft $g^*\mu$-open. Since F_G is soft μ-open, $F_G \subseteq F_G$. By our hypothesis, $c_\mu(F_G) \subseteq F_G$. This implies $c_\mu(F_G) \subseteq F_E$. Then $c_{\mu_{g^*}}(F_G) \subseteq c_\mu(F_G) \subseteq F_E$. Hence F_G is soft $g^{**}\mu$-closed. $\square$

Theorem 3.20. *A subset F_G of a SGTS (F_A, μ_{F_A}) is soft $g^{**}\mu$-closed then $c_{\mu_{g^*}}(F_G)\backslash F_G$ contains no non empty soft $g^*\mu$-closed sets.*

Proof. Let F_H be a soft $g^*\mu$-closed set such that $F_H \subseteq c_{\mu_{g^*}}(F_G)\backslash F_G$. Then $F_G \subseteq F_A\backslash F_H$ where F_G is soft $g^{**}\mu$-closed and $F_A\backslash F_H$ is soft $g^*\mu$-open.By the definition of soft $g^{**}\mu$-closed, we have $c_{\mu_{g^*}}(F_G) \subseteq F_A\backslash F_H$. This implies $F_H \subseteq F_A\backslash c_{\mu_{g^*}}(F_G)$. Thus $F_H \subseteq [c_{\mu_{g^*}}(F_G)] \cap [F_A\backslash c_{\mu_{g^*}}(F_G)] = F_\phi$. That is $F_H = F_\phi$. Hence $c_{\mu_{g^*}}(F_G)\backslash F_G$ contains no non empty soft $g^*\mu$-closed sets. □

The converse of the above theorem need not be true as seen from the following example.

Example 3.21. *Let $U = \{h_1, h_2\}$, $E = \{e_1, e_2, e_3\}$ and $A = \{e_1, e_2\}$; $\mu = \{F_\phi, F_4, F_6, F_{10}, F_{14}, F_{15}\}$; $\mu^c = \{F_\phi, F_1, F_5, F_9, F_{11}, F_{15}\}$. Now, $c_{\mu_{g^*}}(F_{12})\backslash F_{12} = F_{13}\backslash F_{12} = F_1$ which is a soft $g^*\mu$-closed set. Hence $c_{\mu_{g^*}}(F_{12})\backslash F_{12}$ contains a nonempty soft g^*-μ-closed set. Further, F_{12} is a soft $g^{**}\mu$-closed set.*

Theorem 3.22. *If soft $g^{**}\mu$-closed subset F_G of a SGTS (F_A, μ_{F_A}) be such that $c_{\mu_{g^*}}(F_G)\backslash F_G$ is soft $g^*\mu$-closed, then F_G is soft $g^*\mu$-closed.*

Proof. Given F_G is a soft $g^{**}\mu$-closed subset such that $c_{\mu_{g^*}}(F_G)\backslash F_G$ is a soft $g^*\mu$-closed set. Now, $c_{\mu_{g^*}}(F_G)\backslash F_G$ is a soft $g^*\mu$-closed subset to itself. By Theorem 3.19, $c_{\mu_{g^*}}(F_G)\backslash F_G = F_\phi$. Therefore $c_{\mu_{g^*}}(F_G) = F_G$. Hence F_G is soft $g^*\mu$-closed. □

Theorem 3.23. *Let F_G be a soft $g^{**}\mu$-closed set in a SGTS (F_A, μ_{F_A}) and $F_G \subseteq F_B \subseteq c_{\mu_{g^*}}(F_G)$. Then F_B is a soft $g^{**}\mu$-closed set.*

Proof. Let F_H be a soft $g^*\mu$-open set in (F_A, μ_{F_A}) such that $F_B \subseteq F_H$. Then $F_G \subseteq F_H$. Since F_G is soft $g^{**}\mu$-closed, $c_{\mu_{g^*}}(F_G) \subseteq F_H$. Now, $c_{\mu_{g^*}}(F_B) \subseteq c_{\mu_{g^*}}(c_{\mu_{g^*}}(F_G)) \subseteq c_{\mu_{g^*}}(F_G) \subseteq F_H$. Hence F_B is soft $g^{**}\mu$-closed. □

Theorem 3.24. *A subset F_G of a SGTS (F_A, μ_{F_A}) is soft $g^{**}\mu$-open if and only if $F_H \subseteq i_{\mu_{g^*}}(F_G)$ whenever $F_H \subseteq F_G$ and F_H is soft $g^*\mu$-closed.*

Proof. Let F_H be a soft $g^*\mu$-closed set contained in F_G. Then $(F_H)^c$ is a soft $g^*\mu$-open set containing $(F_G)^c$. Since $(F_G)^c$ is soft $g^{**}\mu$-closed, $c_{\mu_{g^*}}(F_G)^c \subseteq (F_H)^c$. This implies $F_H \subseteq i_{\mu_{g^*}}(F_G)$.
Conversely, suppose that $F_H \subseteq i_{\mu_{g^*}}(F_G)$, where $F_H \subseteq F_G$ and F_H is soft $g^*\mu$-closed. This implies $(F_H)^c$ is a soft $g^*\mu$-open set containing $(F_G)^c$. Then $(i_{\mu_{g^*}}(F_G))^c \subseteq (F_H)^c$. That is $c_{\mu_{g^*}}(F_G)^c \subseteq (F_H)^c$. Hence F_G is soft $g^{**}\mu$-open. □

Theorem 3.25. *A subset F_G is soft $g^{**}\mu$-open in a SGTS (F_A, μ_{F_A}) then $F_U = F_A$ whenever F_U is soft $g^*\mu$-open and $i_{\mu_{g^*}}(F_G) \cup (F_G)^c \subseteq F_U$.*

Proof. Given F_G is soft $g^{**}\mu$-open. Suppose F_U is soft $g^*\mu$-open and $i_{\mu_{g^*}}(F_G) \cup (F_G)^c \subseteq F_U$. Now, $(F_U)^c \subseteq i_{\mu_{g^*}}(F_G)^c \cap (F_G)$. That is, $(F_U)^c \subseteq [c_{\mu_{g^*}}(F_A\backslash F_G)]\backslash[F_A\backslash F_G]$. Since $(F_U)^c$ is soft $g^*\mu$-closed and $F_A\backslash F_G$ is soft $g^{**}\mu$-closed. By Theorem 3.19, $(F_U)^c = F_\phi$. Then $F_U = F_A$. □

Theorem 3.26. *For any point $\mathbf{a}$ of a strong SGTS (F_A, μ_{F_A}) ,$F_A\backslash\{\mathbf{a}\}$ is soft $g^{**}\mu$-closed or soft $g^*\mu$-open.*

Proof. Suppose $F_A\backslash\{\mathbf{a}\}$ is not soft $g^*\mu$-open. Then F_A is the only soft $g^*\mu$-open set containing $F_A\backslash\{\mathbf{a}\}$. This implies $c_{\mu_{g^*}}(F_A\backslash\{\mathbf{a}\}) \subseteq F_A$. Hence $F_A\backslash\{\mathbf{a}\}$ is soft $g^{**}\mu$-closed. □

Definition 3.27. *A space (F_A, μ_{F_A}) is said to be $T_{\mu_{g^{**}}}$-space if every soft $g^{**}\mu$-closed set is soft μ-closed.*

Theorem 3.28. *If a strong SGTS (F_A, μ_{F_A}) is a soft $T_{\mu_{g^{**}}}$-space, then every singleton is soft μ-open or soft $g^*\mu$-closed.*

Proof. Assume that $\{\mathbf{a}\}$ is not soft $g^*\mu$-closed. Then by Theorem 3.25, $F_A \backslash \{\mathbf{a}\}$ is soft $g^{**}\mu$-closed. Since F_A is a soft $T_{\mu_{g^{**}}}$-space, $F_A \backslash \{\mathbf{a}\}$ is soft μ-closed. Hence $\{\mathbf{a}\}$ is soft μ-open. □

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