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Generalized Semi-closed Sets in a Space

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Abstract: In this paper the concept of generalized semi-closed sets and generalized semi open sets are introduced and some of its properties are investigated in a space considered by Aiexandroff A.D. [1] where arbitrary union of open sets may not be open.

Keywords: Generalised semi-closed set (g*s closed set), generalized semi open set (g*s open set).

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1. Introduction

Topological spaces have been generalized in many ways, for example, Alexandroff A.D. [1] weakened the union requirement, Mashhour et al. [14] omitted the intersection condition and so on. The space of Alexandroff occupies a prominent role in the literature. In 1987, Bhattacharya and Lahiri [8] introduce the class of semi-generalized closed sets in a topological space and obtain its various properties. S.P. Arya and T. Nour [3] defined generalized semi-closed sets in 1990. In this paper, generalized semi-closed sets and generalized semi open sets are introduced in the space considered by A.D. Alexandroff and some properties are investigated.

2. Preliminaries

Definition 2.1 ([1]). *An Alexandroff space (or σ-space, briefly space) is a set X together with a system F of subsets satisfying the following axioms*

(1). The intersection of a countable number of sets from F is a set in F.

(2). The union of a finite number of sets from F is a set in F.

(3). Φ and X are in F.

Members of F are called closed sets. Their complementary sets are called open sets. It is clear that instead of closed sets in the definition of an Alexandroff space, one may put open sets with subject to the conditions of countable sum ability, finite intersect ability and the condition that X and Φ should be open.

The collection of all such open sets will sometimes be denoted by τ and the Alexandroff space by (X, τ). When there is no confusion, (X, τ) will simply be denoted by X.

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Note 2.2. *In general τ is not a topology as can be easily seen by taking $X = R$ and τ as the collection of all $F\sigma$ sets in R. By a space we shall always mean an Alexandroff space.*

Definition 2.3 ([1]). *With every $M \subset X$ we associate its closure $\mathrm{cl}(M)$ the intersection of all closed sets containing M.*

Note that $\mathrm{cl}(M)$ is not necessarily closed. Throughout X stands for a space and unless otherwise stated, sets are always subsets of X. The letters R and Q stand respectively for the set of real numbers and the set of rational numbers.

Definition 2.4 ([8]). *A set A in a topological space is said to be semi generalized closed (sg-closed for short) if and only if $s\,\mathrm{cl}(A) \subset U$ whenever $A \subset U$ and U is semi open (where $s\,\mathrm{cl}(A)$ denote semiclosure of A).*

Definition 2.5 ([3]). *A set A in a topological space is said to be generalized semi-closed (gs-closed for short) if and only if $s\,\mathrm{cl}(A) \subset U$ whenever $A \subset U$ and U is open.*

3. Generalised Semi-closed Sets in a Space

Definition 3.1. *A set A is said to be a semi generalised closed (sg*-closed) set in a space if and only if there is a semi-closed set F containing A such that $F \subset U$ whenever $A \subset U$ and U is semi open.*

Definition 3.2. *A set A is said to be a generalised semi-closed (g*semi-closed) set if and only if there is a semi-closed set F containing A such that $F \subset U$ whenever $A \subset U$ and U is open.*

Remark 3.3. *Every semi-closed set is generalised semi-closed but the converse is not true as shown by the following example.*

Example 3.4. *Let $X = R - Q$ and $\tau = \{X, \Phi, Gi\}$ where Gi runs over all countable subsets of $R - Q$. Then (X, τ) is a space but not a topological space, for any $A \subset X$, $\mathrm{cl}(A) = A$, members of τ are the only semi open sets. Let B be the set of all irrational numbers in $(0, \infty)$. Then B is not semi-closed but B is generalised semi-closed, since X is the only open set containing B.*

Theorem 3.5. *A set A is generalized semi-closed (g*s closed) if and only if there is a semi-closed set F containing A such that $F - A$ does not contain any non-void closed set.*

Proof. Let A be g*s-closed. Then there is a semi-closed set F containing A such that $F \subset U$ whenever $A \subset U$ and U is open. Assume $F_1 \subset F - A$ and F_1 is closed. Since F_1^C is open and $A \subset F_1^C$ where c denotes the complement operator, it follows that $F \subset F_1^C$, i.e., $F_1 \subset F^C$ and so $F_1 \subset F \cap F^C = \Phi$. Hence the condition is necessary.
Conversely suppose that the given condition is satisfied. Let $A \subset U$ and U be open. If F is not subset of U, then $F \cap U^C$ is a non-void closed set contained in $F - A$, a contradiction. So A is g*s-closed. □

Corollary 3.6. *A g*s-closed set A is closed if and only if both $\mathrm{cl}(A)$ and $\mathrm{cl}(A) - A$ are closed.*

Proof. If A is both closed and g*s-closed then evidently $\mathrm{cl}(A) = A$ and $\mathrm{cl}(A) - A = \Phi$ are closed.
Conversely let A be a g*s-closed set such that both $\mathrm{cl}(A)$ and $\mathrm{cl}(A) - A$ are closed. Since A is g*s-closed, by Theorem 3, there is a semi-closed set F containing A such that $F - A$ does not contain any non-void closed set. Now since $\mathrm{cl}(A) - A$ is closed and $\mathrm{cl}(A) - A \subset F - A$, $\mathrm{cl}(A) - A = \Phi$, i.e., $A = \mathrm{cl}(A)$ and so A is closed. □

Theorem 3.7. *A set A is g*s-closed if and only if there is a semi-closed set F containing A such that $F \subset \ker(A) = \cap\{U; U$ is open and $U \supset A\}$. The proof is omitted.*

Theorem 3.8. *Union of two g*s-closed sets is g*s-closed if and only if union of two semi-closed sets is g*s-closed.*

Proof. Let the union of two g*s-closed sets is g*s-closed. Let A, B be semi-closed. Then A, B be g*s-closed and so $A \cup B$ is g*s-closed.

Conversely, let the union of two semi-closed sets be g*s-closed. Let A_1, A_2 be g*s-closed. Then there exist semi-closed sets F_1, F_2 containing A_1, A_2 respectively such that $F_i \subset O_i$ and O_i is open for $i = 1, 2$. Let, $A_1 \cup A_2 \subset O$ where O is open. Then $F_1 \cup F_2 \subset O$. Since $F_1 \cup F_2$ is g*s-closed, there is a semi-closed set F containing $F_1 \cup F_2$ such that $F \subset O$. Hence $A_1 \cup A_2 \subset F \subset O$ and this is true for all open set O containing $A_1 \cup A_2$. This proves the theorem. □

Note 3.9. *Intersection of two g*s-closed sets is not necessarily g*s-closed as can be seen from the following example.*

Example 3.10. *Consider the space (X, τ) of Example 1. Let α belongs to X, Consider all the sets of the form $X - \{\beta\}$, $\beta \neq \alpha$, $\beta \in X$. Then all the sets of the form $X - \{\beta\}$ are g*s closed but $\cap\{X - \{\beta\};\ \beta \neq \alpha\} = \{\alpha\}$, which is not g*s closed.Let A_1, A_2 are the sets of all irrational numbers in $(0, 1)$ and $(1, 2)$ along with $\sqrt{5}$ respectively then clearly, A_1, A_2 are g*s closed but $A_1 \cap A_2 = \{\sqrt{5}\}$ is not so.*

Theorem 3.11. *If A is g*s-closed and $A \subset B \subset \mathrm{cl}(A)$, then B is g*s-closed.*

Proof. Let $B \subset U$ and U is open. Then $A \subset U$. Since A is g*s-closed, there is a semi-closed set F containing A such that $F \subset U$. Now $F \supset s\,\mathrm{cl}(A) \supset B$ and this shows that B is also g*s-closed. □

Theorem 3.12. *Let $B \subset A$ where A is open and g*s closed. Then B is g*s-closed relative to A if and only if B is g*s-closed.*

Proof. Since A is g*s-closed, there is a semi-closed set F containing A such that $F \subset U$ whenever $A \subset U$, U is open. Now since $A \subset A$ and A is open. So $F \subset A$, i.e., $A = F$ and so A is semi-closed. Now let B be g*s-closed. Then there is a semi-closed set F_1 witnessing the g*s-closeness of B. Now since A is open and $B \subset A$, $F_1 \subset A$. Also if $B \subset U'$, U' is open in A, then U' is open in X and so $F_1 \subset U'$. This shows that B is g*s closed in A.

Conversely let B be g*s-closed in A. Then there is a semi-closed set F_2 in A witnessing the g*s-closeness of B in A. Since A is semi-closed, F_2 is also semi-closed in X. Further if $B \subset U_1$, U_1 open in X, then $B \subset U_1 \cap A$ where $U_1 \cap A$ is open in A and so $F_2 \subset U_1 \cap A \subset U_1$. This completes the proof of the theorem. □

Note 3.13. *An open g*s-closed set is semi-closed.*

Corollary 3.14. *Let A be g*s closed set which is open. Then $A \cap F$ is g*s closed if F is g*s closed.*

Theorem 3.15. *If every subset of X is g*s closed then s. $o(X) = s.c(X)$ where $s.o(X)$ and $s.c(X)$ respectively denote collection of all semi open and semi-closed sets in X.*

The converse of the above Theorem is not true in a space.

Example 3.16. *Let $X = R - Q$ and $\tau = \{\Phi, X, Gi, Ai\}$ where Gi runs over all countable subsets of $R - Q$ and Ai runs over all co countable subsets of $R - Q$. Then (X, τ) is a space but not a topological space. Since in this space for any subset A, $\mathrm{cl}(A) = A$, we have $s.o(X) = \tau$. But $\tau = c(\tau) =$ the collection of all closed sets and so $s.o(X) = \tau = c(\tau) = s.c(X)$. As in example 1, let B be the set of all irrational numbers in $(0, \alpha)$. Then clearly B does not belongs to $s.c(X)$. B is not g*s closed, since $\ker(B) = B$.*

4. Generalized Semi Open Set

Definition 4.1. *A set A is said to be generalised semi open (g*s open) iff A^C is g*s closed.*

Theorem 4.2. *A set is g*s open if and only if there is a semi open set O contained in A such that $F \subset O$ whenever F is closed and $F \subset A$.*

Proof. Let A be g*s open. Then A^C is g*s closed and so there is a semi-closed set F_1 containing A^C such that $F_1 \subset O'$ whenever $A^C \subset O'$ and O' is open. Then $O = F_1^C$ is semi open and $O \subset A$. Now F is closed and $F \subset A$, implies $A^C \subset F^C$ where F^C is open and $F_1 \subset F^C$ (Since A^C is g*s closed). So $F \subset F_1^C = O$.

The converse can be similarly proved. □

Remark 4.3. *Arbitrary union of g*s open sets is g*s open in a topological space. But this is not true in a space. In example 2, A_1, A_2 are g*s closed, so, $X - A_1$, $X - A_2$ are g*s open but $(X - A_1) \cup (X - A_2)$ is not so.*

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