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Slip Effects on Blood Flow in the Presence of a Bell-shaped Stenosed Artery: A Casson Fluid Model

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Abstract: In this a paper, one dimensional Casson fluid flow under the influence of bell shaped stenosed artery have been considered. Analytical and numerical techniques both are carried out to observe this situation. Analytical expressions for axial velocity, plug flow velocity, volumetric flow rate, pressure gradient and wall shear stress has been derived. Numerical techniques have been used to show the effects of yield stress, slip and no slip velocity on these flow variables. The results show that wall shear stress increases as the values of yield stress diminishes and increase in slip velocity reduces the wall shear stress. It is also observed that plug flow velocity increases as slip velocity increases, pressure gradient increases as the slip velocity increases whereas it reduces as yield stress diminishes and axial velocity increases as velocity slip increases.

MSC: 92B05.

Keywords: Casson Fluid, Yield Stress, slip velocity, plug flow velocity.

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1. Introduction

Atherosclerosis and cardiac diseases are becoming challenging problems in Indian and European countries. Mathematical modeling and simulations can provide more accurate physiological and pathological information to medical practiceners and biotechnological scientists to invent better diagnosis gear and improving the surgical process. Atherosclerosis is the special type of cardiovascular disease that primarily affects large and medium sized arteries. This is a fundamental cause of high blood pressure, Heart attack, stroke and hypertension and is found of the people over the age of 35. The deposition of cholesterol, calcium, fat and other harm full material narrows the artery and make hard and inflexible which is a leading cause of death in people over 40 and places a significant financial strain on health care systems. [1] formulated a problem to reading the power of the flow index behaviour, shear dependent nonlinear viscosity and the yield stress in flowing of blood by a stenosed artery. They assumed that blood acts Herschel-Bulkley fluid the diseased condition. [2] talk about a numerical model to explain the effects of shape and height of stenosis on the flow resistance for diverse values of yield stress, blood viscosity and volumetric flux during an axially symmetric and radially non-symmetric artery under stenotic condition if blood acts as Casson fluid. [3] studied the outcome of magnetic field intensity on axially non-symmetric but radially symmetric atherosclerotic artery with core region. They assumed that blood behaves like a Herschel-Bulkley. [4] explored the effect of an overlapping obstruction and velocity at the stenosed arterial wall on non-Newtonian flow of blood through an artery using power law fluid. They gave both analytical and numerical solutions for axial velocity of blood, volumetric

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flow rate, resistance to flow and wall shear stress in the stenotic area. [5] developed a numerical model to investigate flow of blood across a stenosed arterial segment in the view of the slip velocity at the wall of the artery by considering that blood performs approximating Herschel-Bulkley fluid. [6] projected a computational study of pulsatile blood flow by a axially symmetric catheterized stenosed artery having slip velocity at the obstructed arterial wall. They established the analytic expression for flow characteristics, velocity profiles, the flow resistance, the wall shear stress and the effective viscosity. [7] dealt with unsteady flow of blood by an obstructed artery in the existence of slip velocity and obtain a solution of governing equations by using perturbation method if low womerseley frequency parameter is taken. [4] explained a computational model to learn the flow of blood in a multiple stenosed artery by assuming combined effect of slip velocity and magnetic field intensity. They have derived the analytical expressions for wall shear stress, volumetric flow rate, axial velocity, and core velocity and represented results graphically in the view of blood as a Herschel-Bulkley fluid. [8] explored slip effects on unsteady non-Newtonian MHD blood flow during an inclined catheterized overlapping stenosed artery. They used power law fluid to imitate the rheological characteristics of the blood. [9] formulated a problem to show the effect of angle of elevation on unsteady two-dimensional flow of blood during a flexible stenosed artery. They assumed a generalized power law fluid having shear-thickening as well as shear-thinning properties. Siddiqui (2014) described that slip velocity and body acceleration are sensitive parameters for the flow characteristics of blood in stenotic situation. [10] presented a mathematical model for high viscosity blood flow by small arteries having multiple stenoses. They observed that the flow resistance ratio travels nearer to 1 as yield stress raises or when viscosity of blood and volumetric flux reduce and the level of these changes are maximum for yield stress and slightest for flux.

2. Mathematical Formulation of the Problem

Let us consider the bell-shaped stenosis geometry given by

$$\frac{R}{R_0} = 1 - \frac{\delta}{R_0} \exp\left(-\frac{k^2\varepsilon^2 z^2}{R_0^2}\right) \tag{1}$$

Here R_0 represents the radius of the artery in outer region of the stenosis, $R(z)$is the radius of the stenosed segment δ is the height of stenosis at middle point and k is the parametric constant and $\varepsilon = \frac{R_0}{L_0}$. Consider the geometry of stenosis as shown in figure

$$\frac{R}{R_0} = 1 - ae^{-bz^2} \tag{2}$$

Where $a = \frac{\delta}{R_0}$ and $b = \frac{m^2\varepsilon^2}{R_0^2}$.

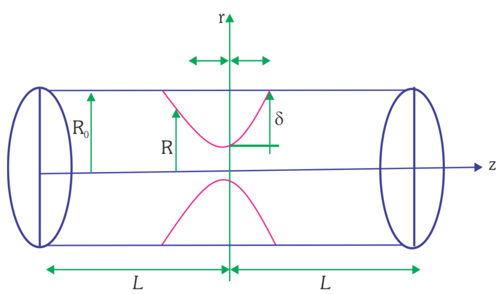

Figure 1. **Geometry of the arterial segment with stenosis**

Flowing of blood which is incompressible one has been assumed to behave like a Casson fluid. The equation of motion for laminar and incompressible, steady, fully developed, one dimensional flow of blood are given by

$$0 = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau) \tag{3}$$

$$0 = -\frac{\partial p}{\partial r} \tag{4}$$

Where τ represents the shear stress of blood behave like a Casson fluid and p is the pressure at any point. The constitutive equation for Casson fluid be written as

$$-\frac{\partial u}{\partial r} = f(\tau) = \frac{(\sqrt{\tau} - \sqrt{\tau_0})^2}{\mu}; \tau \geq \tau_0 \tag{5}$$

$$= 0; \tau \leq \tau_0 \tag{6}$$

Where u stands for the axial velocity of blood and $\tau_0 \geq 0$ is the yield stress and μ be the fluid viscosity. Boundary conditions are

$$\begin{aligned} &u = u_s \ at \ r = R(z) \\ &\tau \ \text{ is finite at } r = 0 \end{aligned} \tag{7}$$

Where u_s is the axial slip velocity. Using the boundary conditions (7) in equation (2), we get

$$\tau = -\frac{r}{2}\frac{\partial p}{\partial z} \tag{8}$$

The wall shear stress τ_R is given by

$$\tau_R = -\frac{R}{2}\frac{\partial p}{\partial z} \quad [\tau \ at \ r = R(z)] \tag{9}$$

The volumetric flow rate is given by. Now integrate (5), using equation (7), (8), the velocity function becomes

$$u(r) = u_s + \frac{P}{2\mu}\left[\frac{1}{2}(R^2 - r^2) + r_0(R - r) + \frac{4}{3}\sqrt{r_0}(R^{3/2} - r^{3/2})\right], \quad r_0 \leq r \leq R(z) \tag{10}$$

Where $P = -\frac{\partial p}{\partial z}$. And the expression for plug flow velocity is

$$u_0 = u_s + \frac{P}{2\mu}\left[\frac{R^2}{2} - \frac{17}{6}r_0^2 + \frac{4}{3}r_0^{1/2}R^{3/2} + r_0 R\right], \quad 0 \leq r \leq r_0 \tag{11}$$

The volumetric flow rate is defined as

$$Q = \int_0^R 2\pi r u \, dr = \int_0^{r_0} 2\pi r u \, dr + \int_{r_0}^R 2\pi r u \, dr$$

Which after using (10) and (11) becomes

$$Q = \pi R^2 u_s + \frac{\pi P R^4}{2\mu}\left[\frac{1}{4} + \frac{1}{3}\frac{r_0}{R} - \frac{4}{7}\sqrt{\frac{r_0}{R}} - \frac{19}{12}\left(\frac{r_0}{R}\right)^4\right]$$

$$Q = \pi R^2 u_s + \frac{\pi R^3 \tau_R}{\mu}\left[\frac{1}{4} + \frac{1}{3}\frac{\tau_0}{\tau_R} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_R}} - \frac{19}{12}\left(\frac{\tau_0}{\tau_R}\right)^4\right] \tag{12}$$

When $\frac{\tau_0}{\tau_R} \ll 1$, it is possible to approximate equation 12 (after ignoring the last term and replacing 1/3 by 16/49 in the second term) as

$$Q = \pi R^2 u_s + \frac{\pi R^3 \tau_R}{\mu}\left[\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_R}}\right]^2 \tag{13}$$

Solving the quadratic for $\sqrt{\tau_R}$ in Equation (13), followed by substitution of equations (8) and (9) in to this solution and rearrangement of the result, yields

$$\frac{dp}{dz} = -\frac{2}{R}\left[\frac{8}{7}\sqrt{\tau_0} + 2\sqrt{\frac{\mu(Q-\pi R^2 u_s)}{\pi R^3}}\right]^2 \tag{14}$$

Again using equation (14) in equation (9) yields

$$\tau_R = \frac{64}{49}\tau_0 + \frac{4\mu(Q-\pi R^2 u_s)}{\pi R^3} + \frac{32}{7}\sqrt{\frac{\mu(Q-\pi R^2 u_s)\tau_0}{\pi R^3}} \tag{15}$$

When there is no stenosis present in artery i.e. $R(z) = R_0$, the expression for wall shear stress becomes

$$\tau_N = \frac{64}{49}\tau_0 + \frac{4\mu(Q-\pi R_0^2 u_s)}{\pi R_0^3} + \frac{32}{7}\sqrt{\frac{\mu(Q-\pi R_0^2 u_s)\tau_0}{\pi R_0^3}} \tag{16}$$

Now the dimensionless term for wall shear stress may be put as

$$\bar{\tau}_R = \frac{\tau_R}{\tau_N} = \frac{\frac{64}{49}\tau_0 + \frac{4\mu(Q-\pi R^2 u_s)}{\pi R^3} + \frac{32}{7}\sqrt{\frac{\mu(Q-\pi R^2 u_s)\tau_0}{\pi R^3}}}{\frac{64}{49}\tau_0 + \frac{4\mu(Q-\pi R_0^2 u_s)}{\pi R_0^3} + \frac{32}{7}\sqrt{\frac{\mu(Q-\pi R_0^2 u_s)\tau_0}{\pi R_0^3}}}$$

$$\bar{\tau}_R = \frac{\frac{64}{49}\tau_0 + \frac{4\mu}{\pi R_0^3}\left\{Q - \pi R_0^2\left(\frac{R}{R_0}\right)^2 u_s\right\}\frac{1}{(R/R_0)^3} + \frac{32}{7R_0^{3/2}}\sqrt{\frac{\mu\left\{Q-\pi R_0^2\left(\frac{R}{R_0}\right)^2 u_s\right\}\tau_0}{\pi (R/R_0)^3}}}{\frac{64}{49}\tau_0 + \frac{4\mu(Q-\pi R_0^2 u_s)}{\pi R_0^3} + \frac{32}{7}\sqrt{\frac{\mu(Q-\pi R_0^2 u_s)\tau_0}{\pi R_0^3}}} \tag{17}$$

The wall shear stress at the middle of stenosis is

$$\hat{\tau}_R = \frac{\frac{64}{49}\tau_0 + \frac{4\mu}{\pi R_0^3}\left\{Q - \pi R_0^2\left(1-\frac{\delta}{R_0}\right)^2 u_s\right\}\frac{1}{\left(1-\frac{\delta}{R_0}\right)^3} + \frac{32}{7R_0^{3/2}}\sqrt{\frac{\mu\left\{Q-\pi R_0^2\left(\frac{R}{R_0}\right)^2 u_s\right\}\tau_0}{\pi\left(1-\frac{\delta}{R_0}\right)^3}}}{\frac{64}{49}\tau_0 + \frac{4\mu(Q-\pi R_0^2 u_s)}{\pi R_0^3} + \frac{32}{7}\sqrt{\frac{\mu(Q-\pi R_0^2 u_s)\tau_0}{\pi R_0^3}}} \tag{18}$$

3. Numerical Results and Discussion

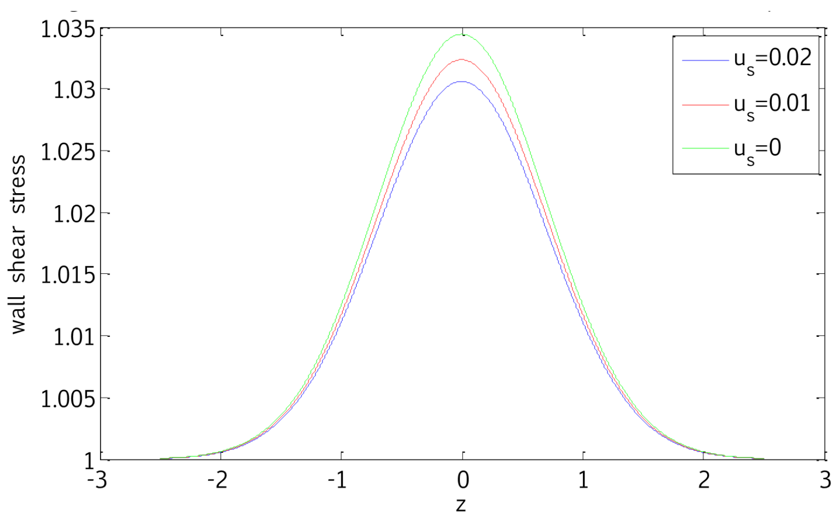

Figure 2. **Wall shear stress vs axial distance for different values of slip velocity**

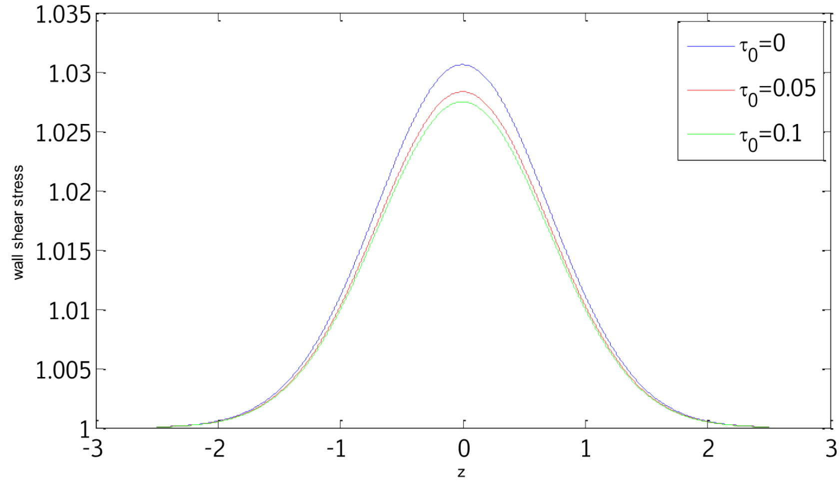

Figure 3. **Wall shear stress vs axial distance for different values of yield stress**

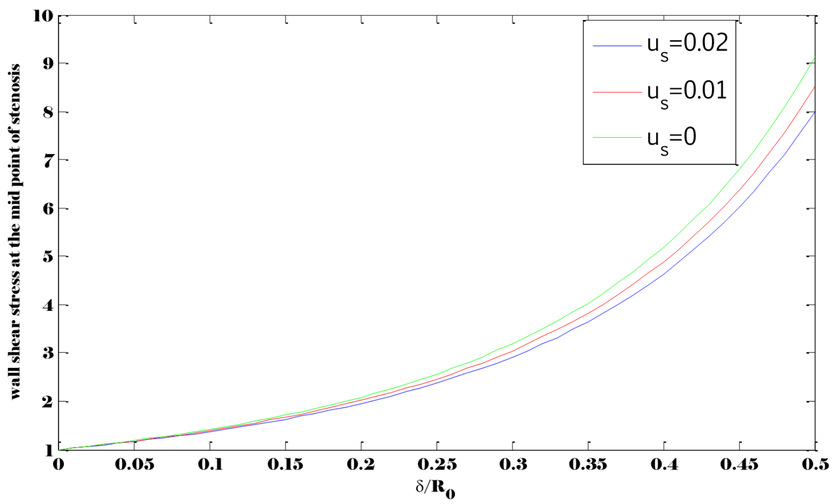

Figure 4. **Wall shear stress at the point of stenosis vs axial distance for different values of slip velocity**

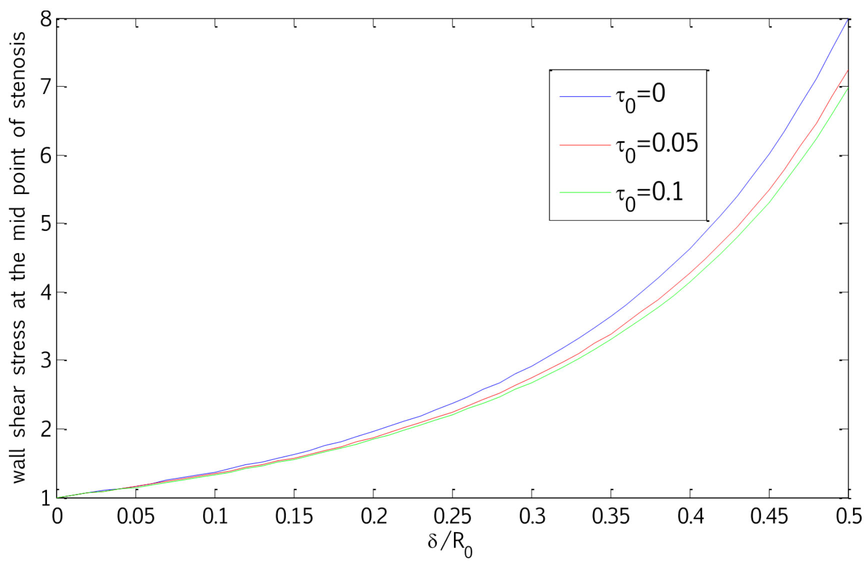

Figure 5. **Wall shear stress at the mid point of stenosis vs axial distance for different values of yield stress**

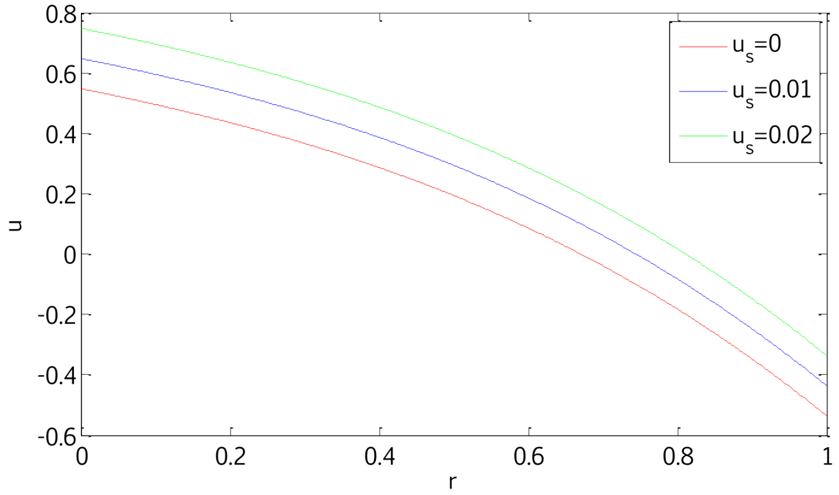

Figure 6. **Axial velocity vs radial distance for different values of slip velocity**

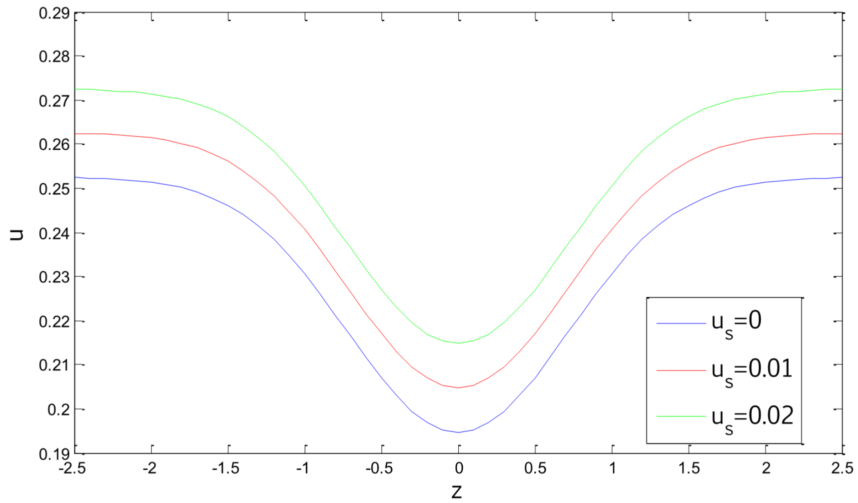

Figure 7. **Axial velocity vs axial distance for different values of slip velocity**

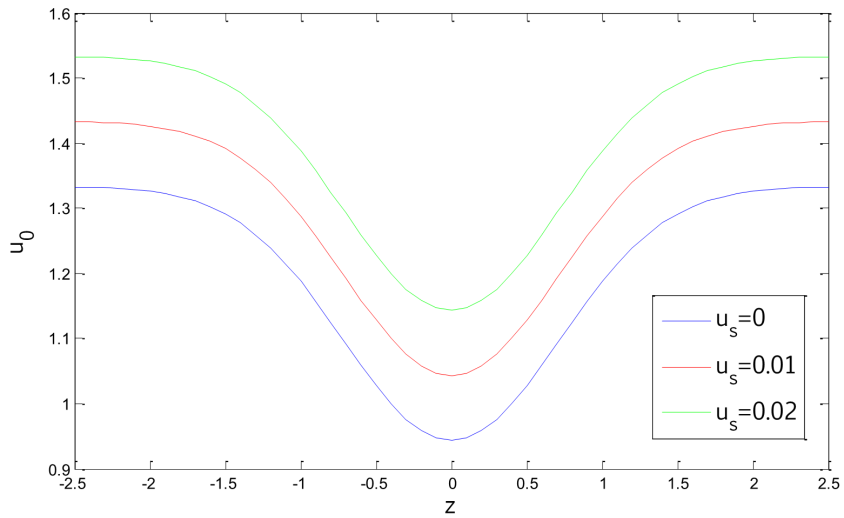

Figure 8. **Plug flow velocity vs axial distance for different values os slip velocity**

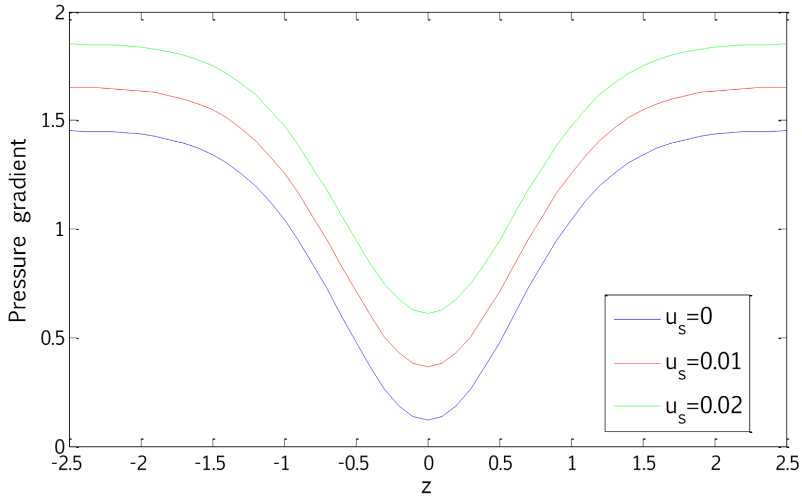

Figure 9. **Pressure gradient vs axial distance values of slip velocity**

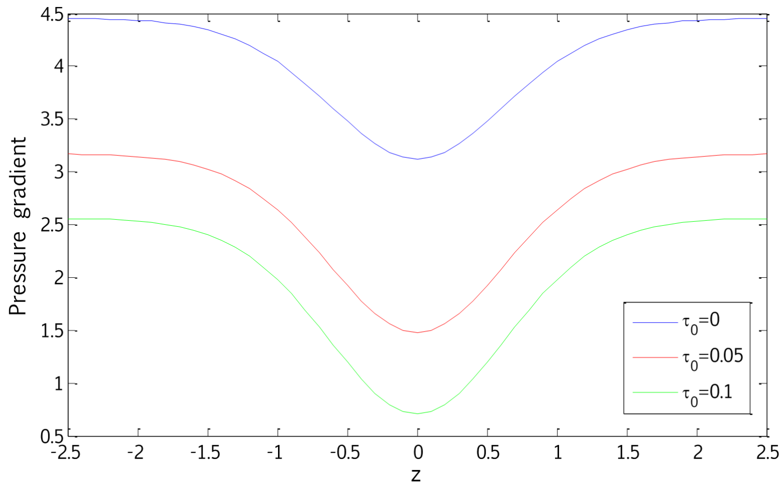

Figure 10. **Pressure gradient vs axial distance for different values of yield stress**

The analytic expression (10)-(17) of various flow characteristics are computed here graphically to study the influence of stenosis on the flow. In this study the values of sensitive parameters for different flow characteristics are taken as $\tau_0 = 0.0, 0.05, 0.1$ and $u_s = 0.0, 0.01, 0.02$. The computational results for wall shear stress, velocity profile, plug flow velocity and pressure gradient obtained from present study are presented graphically. The aim of nearby exploration is to show the combined outcome of slip velocity, stenosis and yield stress of the fluid on the balanced flow of blood through an artery having bell-shaped stenosis acting as Casson fluid. The results are obtained and discussed after computing the flow variables at different value of yield stress τ_0 and slip velocity u_s by fixing other parameters occurred in the flow analytic expression. Figures (1) and (2) depict the difference of the wall shear stress with the axial distance for diverse values of slip velocities and yield stress respectively. It is found that wall shear allotment in the stenotic region raises with the axial distance in the upstream of the stenosis gullet and attain the greatest amount at the stenosis of gorge. It is also noted that wall shear stress has increasing trend as the values of yield stress τ_0 diminishes as per study of [5]. Alternatively an enhancement in slip velocity shrinks the wall shear stress as in [1]. The variations of wall shear stress at the mid of stenosis with stenosis height with different values of slip velocities and yield stress have been shown in figures (3) and (4). Since velocity profiles give a complete depiction of the flow field, it is of interest to study their pattern. A comparison of velocity profiles using equation (10) for the case of slip and no slip parameters is shown in Figures (5) and (6). It is observed that axial velocity amplifies as velocity slip lifts. Also it is found from figure (5) that axial velocity declines as radial distance moves up and becomes 0 at the radial distance 1 as obtained by [2, 3]. Figure (6) shows the deviation of axial velocity with the axial distance. The effect of slip velocity on the plug flow velocity has been shown in the figure (7). The obtained results shows that plug flow velocity increases as slip velocity increases. This result is similar to the results of [1]. Figures (8) and (9) show the distinction of pressure gradient with the axial distance for dissimilar values of slip velocities and yield stress respectively. Here it is observed that pressure gradient increases as the slip velocity increases whereas it reduces as yield stress diminishes.

4. Conclusion

The present computational study for blood flow through an artery in the presence of bell shaped stenosis and with the assumption that blood acts as Casson fluid brings out many interesting facts for clinical purposes. In many diseases such as atherosclerosis, cardiovascular diseases and hypertension cause high blood pressure the agreeability and stiffness of red blood cells higher than their normal value. In such unwell conditions blood behaves like non-Newtonian fluid. Therefore axial velocity, plug flow velocity, pressure gradient and wall shear stress give accurate results in comparison assuming blood as Newtonian fluid. Also, this study revels that slip velocity and yield stress both are sensitive parameters for these flow characteristics. We can control various cardiovascular diseases if adjusting these parameters by exertion some external effect like medicines, applying appropriate magnetic effect and body acceleration etc.

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