Int. J. Math. And Appl., 6(2–A)(2018), 125–134
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Viscous Dissipation Effects on Unsteady MHD Flow Past an Impulsively Started Infinite Isothermal Vertcal Plate in The Presence of Radiation

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Abstract: In this paper, the effects of viscous dissipation on an unsteady MHD viscous incompressible fluid flow past an impulsively started infinite isothermal vertical plate in the presence of radiation has been presented. The fluid is considered gray, absorbing-emitting radiation but a non-scattering medium. Thermal radiation effects are simulated via a radiation parameter N, based on the Rossseland diffusion approximation. The momentum and thermal boundary layer equations are non-dimensionalized using appropriate transformations and then solved numerically subject to physically realistic boundary conditions by using the Ritz finite element method. The influence of physical parameters on the velocity profiles (u), temperature distribution (θ), skin-friction (τ), and the Nusselt number (Nu) are presented with the help of graphs and tables. The obtained results show that a decrease in the velocity and temperature as the radiation parameter is increased. The velocity u is decreased as the magnetic parameter increases and increases with increasing Grashof number and time parameter. The skin-friction increases with increase in the magnetic parameter and decreases with increasing Grashof number and Eckert number.

Keywords: MHD, magnetic parameter, radiation parameter, isothermal vertical plate, Ritz finite element method.

 Accepted on: 05.02.2018

1. Introduction

The problem of radiative-convection flows has been studied for its important in many areas of technology and applied physics including oxide melt materials processing, astrophysical fluid dynamics, plasma flows switch performance, MHD energy pumps operating at very high temperatures and hypersonic aerodynamics. Bestman and Adjepong [1] studied unsteady hydro-magnetic free-convection flow with radiative heat transfer in a rotating fluid. The radiation effects on transient free convection heat transfer flow past a hot vertical surface in porous media by Laplace transform technique was reported by Ghosh and Beg [2]. Thermal radiation effects on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity was presented by Mohamoud [3]. Chandrakala and Raj [4] have studied the magneto-hydrodynamic flow past an impulsively started infinite isothermal vertical plate in the presence of radiation by Laplace technique. Shanker [6] have investigated the radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption by finite element method. Finite element analysis of heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer was presented by Rao and Reddy [7]. MHD natural convection flow with radiative heat transfer past an impulsively

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moving plate with ramped wall temperature was studied by Seth [8]. Ghara [9] have presented the effects of radiation on MHD free convection flow past an impulsively moving vertical plate with ramped wall temperature. The effects of radiation on MHD flow thorough porous media past an impulsively started vertical plate with variable heat and mass transfer was presented by Rajput and Kumar [10].

The viscous dissipation effect is expected to be relevant for fluids with high values of dynamic viscosity as for high velocity flows. The viscous dissipation heat is important in the natural convective flows, when the field is of extreme size or at extremely low temperature or in high gravitational field. Gebhart [11] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal vertical plate with constant heat flux. Gebhart and Mollendorf [12] considered the effects of viscous dissipation for external natural convection flow over a surface. The viscous dissipation heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate was reported by Soundalgekar [13]. The computational analysis of coupled radiation-convection dissipation non-gray gas flow in a non-Darcy porous medium with the Keller-Box implicit difference scheme was presented by Takhar [14]. Zueco [15] has investigated radiation and viscous dissipation effects on MHD unsteady free convection over vertical porous plate by Network simulation method. Recently, Reddy [?] presented mass transfer effects on an unsteady MHD free convective flow of an incompressible viscous dissipative fluid past an infinite vertical porous plate by finite element method.

The object of the present paper is to study unsteady MHD flow past an impulsively started infinite isothermal vertical plate in the presence of radiation taking into account viscous dissipation effect. The problem is governed by the system of non-linear partial differential equations, whose exact solutions are difficult to obtain, if possible. So that, the Ritz finite element method has been adopted for its solution, which is more economical from a computational point of view. The influence of the material parameter on the velocity field, temperature distribution, skin-friction and rate of heat transfer have been presented and then discussed.

2. Mathematical Model

We consider unsteady magneto-hydrodynamic (MHD) flow of a viscous incompressible gray, absorbing-emitting fluid past an impulsively started infinite isothermal vertical plate taking viscous dissipation into account. The coordinate system is selected such that the $x'-$axis along the plate in the upward direction and the $y'-$axis is normal to the plate. The radiation heat flux in the $x'-$direction is considered negligible as compared to that in the $y'-$direction. A transverse magnetic field of uniform strength B_0 is applied normal to the plate. The induced magnetic field is neglected in comparison to the applied magnetic field. All the fluid properties are considered constant except the influence of density variation in the body force term. Under the Boussinesq's approximation, the unsteady boundary layer equations for momentum and energy (heat) conservation under these approximations in non-dimensional form are:

$$\frac{\partial u'}{\partial t'} = v\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T_\infty^{'}\right) - \frac{\sigma B_0^2}{\rho}u' \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

The appropriate boundary conditions at the wall and in the free stream are:

$$\begin{aligned} &t' \leq 0; \quad u' = 0, \quad T' = T_\infty^{'} \ \text{ for all } y' \geq 0, \\ &t' > 0; \quad u' = u_0, \quad T' = T_w^{'} \ \ at \ \ y' = 0, \\ &u' = 0, \quad T' \to T_\infty^{'} \ \text{ for } y' \to \infty \end{aligned} \tag{3}$$

where u' is the velocity component along the plate, g is the acceleration due to gravity, ν is the kinematic coefficient of viscosity, T' is the temperature of the fluid, T'_∞ is the temperature of the fluid far away from the plate, ρ is the density, C_p is the specific heat at constant pressure, k is the thermal conductivity, β is the volumetric coefficient of thermal expansion, $t'$is the dimensional time, q_r is the radiative heat flux, T'_w is the temperature at the plate and $u_0$is the velocity of the moving plate. The radiation flux on the basis of Rossel and approximation can be expressed as:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T'^4}{\partial y'} \tag{4}$$

where σ^* is the Stefan-Boltzmann constant and k^* is the spectral mean absorption coefficient of the medium. It is assumed that the temperature differences within the flow are sufficient small such that T'^4can be expressed as the linear function of temperatureT'. It can be established by expanding T'^4 in a Taylor series about a free stream temperature$T'_\infty$and neglecting higher-order terms, we obtain T'^4 as

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \tag{5}$$

By using equations (4) and (5), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T'^3_\infty}{3k^*}\frac{\partial^2 T'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{6}$$

Let us introduce the non-dimensional parameters:

$$u = \frac{u'}{u_0},\ y = \frac{u_0 y'}{\nu},\ t = \frac{u_0^2 t'}{\nu},\ \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ P_r = \frac{\mu C_p}{k},\ M = \frac{\sigma B_0^2 \nu}{u_0^2},\ E_c = \frac{u_0^2}{C_p(T'_w - T'_\infty)},\ G_r = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3},\ N = \frac{k^{\bullet} k}{4\sigma T'^3_\infty} \tag{7}$$

The governing system of equations (1) and (6) with the use of (7) is reduced to the following non-dimensional form:

$$\frac{\partial u}{\partial t} = G_r\theta + \frac{\partial^2 u}{\partial y^2} - Mu \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\left(1 + \frac{4}{3N}\right)\frac{\partial^2 \theta}{\partial y^2} + E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

where u is the dimensionless velocity, t is the dimensionless time, y is the dimensionless distance, G_r is the Grashof number, M is the magnetic parameter, θ is the dimensionless temperature, P_r is the Prandtl number, N is the radiation parameter and E_c is the Eckert number. The corresponding boundary conditions in non-dimensional form are:

$$\begin{aligned} &t \le 0;\quad u = 0,\quad \theta = 0 \quad \text{for all } y \ge 0,\\ &t > 0;\quad u = 1,\quad \theta = 1 \quad \text{at } y = 0,\\ &u = 0,\quad \theta \to 0 \quad \text{as } y \to \infty \end{aligned} \tag{10}$$

3. Solution of the Problem

By applying the Ritz finite element method to equation (8) over the element $(e), (y_j \le y \le y_k)$, we have

$$J^{(e)}(u) = \frac{1}{2}\int_{y_j}^{y_k}\left\{\left(\frac{\partial u^{(e)}}{\partial y}\right)^2 + M{u^{(e)}}^2 + 2u^{(e)}\frac{\partial u^{(e)}}{\partial t} - 2u^{(e)}G_r\theta\right\}dy = minimum \tag{11}$$

Let

$$u^{(e)} = \psi_j(y)u_j(t) + \psi_k(y)u_k(t) = \psi_j u_j + \psi_k u_k \tag{12}$$

be the linear piecewise approximation solution over the element, $(e), (y_j \leq y \leq y_k)$, where u_j, u_k are the values of u at the ends of the element (e) and ψ_j, ψ_k are the basis functions defined as:

$$\psi_j = \frac{y_k - y}{y_k - y_j}, \psi_k = \frac{y - y_j}{y_k - y_j}.$$

For the minimum, $\frac{\partial J^{(e)}}{\partial u^{(e)}} = 0$. Using equation (12) in (11), we obtain the element equation

$$\int_{y_j}^{y_k} \begin{bmatrix} \psi_j^{'}\psi_j^{'} & \psi_j^{'}\psi_k^{'} \\ \psi_k^{'}\psi_j^{'} & \psi_k^{'}\psi_k^{'} \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy + M \int_{y_j}^{y_k} \begin{bmatrix} \psi_j\psi_j & \psi_j\psi_k \\ \psi_k\psi_j & \psi_k\psi_k \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} dy + \int_{y_j}^{y_k} \begin{bmatrix} \psi_j\psi_j & \psi_j\psi_k \\ \psi_k\psi_j & \psi_k\psi_k \end{bmatrix} \begin{bmatrix} u_j^{\bullet} \\ u_k^{\bullet} \end{bmatrix} dy - G_r\theta \int_{y_j}^{y_k} \begin{bmatrix} \psi_j \\ \psi_k \end{bmatrix} dy = 0 \tag{13}$$

where the prime and the dot denotes the differentiation with respect to y and t, respectively. Simplification of Equation (13) yields

$$\frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{Ml^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j^{\bullet} \\ u_k^{\bullet} \end{bmatrix} - G_r\theta \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0 \tag{14}$$

In order to get the difference equation at the node i, we write the element equations for two consecutive elements $y_{i-1} \leq y \leq y_i$ and $y_i \leq y \leq y_{i+1}$ and assembling the resulting element equations, we get

$$\frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{Ml^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \\ u_{i+1} \end{bmatrix} + \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1}^{\bullet} \\ u_i^{\bullet} \\ u_{i+1}^{\bullet} \end{bmatrix} = G_r\theta \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{15}$$

Put row corresponding to the node i to zero in equation (15), we obtain the following difference schemes with $l^{(e)} = y_k - y_j = h$,

$$(u_{i-1}^{\bullet} + 4u_i^{\bullet} + u_{i+1}^{\bullet}) = \frac{1}{h^2}(6 - Mh^2)(u_{i-1} + u_{i+1}) + \frac{1}{h^2}(12 + 4Mh^2)u_i + 6G_r\theta \tag{16}$$

Applying the trapezoidal rule to equation (16), the following system of equations in the Crank-Nicholson method are obtained:

$$(1 - 3r + \frac{1}{2}rMh^2)u_{i-1}^{j+1} + (4 + 6r + 2rMh^2)u_i^{j+1} + (1 - 3r + \frac{1}{2}rMh^2)u_{i+1}^{j+1} = \Omega_{1i}^j \tag{17}$$

Similarly applying the same procedure to equation (9), we obtain

$$(P_1 - 3r)\theta_{i-1}^{j+1} + (4P_1 + 6r)\theta_i^{j+1} + (P_1 - 3r)\theta_{i+1}^{j+1} = \Omega_{2i}^j \tag{18}$$

where

$$\Omega_{1i}^j = (1 + 3r - \frac{1}{2}rMh^2)u_{i-1}^j + (4 - 6r - 2rMh^2)u_i^j + (1 + 3r - \frac{1}{2}rMh^2)u_{i+1}^j + 6kG_r\theta_i^j$$
$$\Omega_{2i}^j = (P_1 - 3r)\theta_{i-1}^j + (4P_1 + 6r)\theta_i^j + (P_1 - 3r)\theta_{i+1}^J + 6rE_cP_r(u_i^{j+1} - u_i^j)^2 \quad and \quad P_1 = \frac{3NP_r}{3N + 4}.$$

Here, $r = \frac{k}{h^2}$ and h, k are mesh sizes along $y-$direction and time $t-$direction, respectively. Index i refers to the space and j refers to the time. In the equations (17) and (18), taking $i = 1(1)n$ and using boundary conditions (10), the following tri-diagonal systems of equations are obtained:

$$Au = A'$$
$$B\theta = B'$$

where $A$and $B$are tridiagonal matrices of order - n and whose elements are given by

$$a_{i,i} = 4 + 6r + 2rMh^2; b_{i,i} = 4P_1 + 6r; \qquad i = 1(1)n$$
$$a_{i-1,j} = a_{i,j-1} = 1 - 3r + \frac{1}{2}rMh^2; b_{i-1,j} = b_{i.j-1} = P_1 - 3r; \quad i = 2(1)n$$

Here u, θ and A', B' are the column matrices having n-components $u_i^{j+1}, \theta_i^{j+1}$ and u_i^j, θ_i^j respectively. The above tri-diagonal system equations are solved by using Thomas algorithm. The boundary condition $y \to \infty$ is approximated by $y_{\max} = 10$, which is sufficiently large for the velocity to approach convergence criterion. The numerical computations have been carried out for the velocity and temperature fields by using $C-$program. To judge the convergence and stability of the Ritz finite element method, the same program was run with slightly changed values of h and k, no significant change was observed in the values of velocity u and temperature θ. Hence, we conclude that the Ritz finite element method is convergent and stable.

The skin-friction at the plate in the direction of the flow is given by $\tau = \left(\frac{du}{dy}\right)_{y=0}$.

The rate of heat transfer in terms of Nusselt number is given by $Nu = -\left(\frac{d\theta}{dy}\right)_{y=0}$.

4. Results and Discussion

To achieve a perspective of the physics of the fluid flow, we have computed numerical values for velocity profiles (u), temperature distribution (θ), skin-friction (τ) and Nusselt number (Nu) for variations in the material parameters encountered in the problem. The computed numerical results are presented through the graphs and tables. These results show the effect of the material parameters on the quantities mentioned.

Temperature distribution: In Fig.1 we have plotted the temperature distribution for $P_r = 0.71$, as corresponds to air and $P_r = 7.00$, as corresponds to water at room temperature and one atmosphere pressure, highlighting the effect of radiation at time$t = 1.0$. It is observed that a decrease in the temperature and the temperature boundary layer as the radiation parameter N is increased and also as the Prandtl number P_r increases. The temperature is observed to decrease steeply and exponentially away from the plate. Fig.2 displays the effects of Eckert number (E_c) and time parameter (t) on the temperature profiles. It is seen that there is a rise in the temperature with increase in the Eckert number and the time parameter. Also, we notice that the temperature increased markedly as the time parameter increased. This trend is maintained at all locations in the flow regime.

Velocity Profiles: The effects of radiation parameter N on the velocity field u are presented in Figure 3 at time $t = 1.0$. As radiation parameter N increases from 3.0 to 5.0 and then 10.0, we observe that the velocity u decreases. Deceleration of the flow is therefore sustained at considerable distance from the plate towards the free stream as the radiation parameter is increased. The profiles decay monotonically for all values of N from the maximum at the plate to its minimum in the free stream. Figure 4 depicts the effects of Eckert number E_c on the velocity field u at time $t = 1.0$. It can be seen that the velocity of the fluid increases with increasing value of the Eckert number. We have plotted the velocity u in Figure 5 for variations in the magnetic parameter M at time $t = 1.0$. It is observed that the fluid velocity decreases as the magnetic parameter increases. Physically, this is due to the magnetic field exerts a retarding force on the free convective flow. In Figure 6 we have studied the effect of buoyancy via Grashof number (G_r) on the velocity field u at a time $t = 1.0$. The fluid velocity increases consistently, as the Grashof number G_r increased from 2.0 to 3.0 and then 4.0. The fluid flow is accelerated due to the enhancement in buoyancy forces corresponding to an increase in Grashof number i.e., free convection effects. Here, the positive values of G_r corresponds to cooling of the plate by natural convection. For large value of G_r (i.e. 4) there is an over-shoot near the moving plate (at~ 0.8) after which the velocity descends smoothly to its minimum, in the free stream. Fig.7 shows the effects of the time parameter (t) on the velocity profiles. It is clear that there is a reasonable increase in the velocity field with increasing time parameter. With time the fluid flow is therefore accelerated in the upward direction. Also, we observe that beyond about $t = 8.0$, the flow attains steady state condition with the flow velocity increasing with increase in the time parameter up to this point.

Skin-friction: The numerical values of the skin-friction (τ) for variations in the material parameters are presented in table 1 and 2. From table 1, it is seen that the skin-friction increases with increasing radiation parameter and decreases with an increase in the Grashof number and time parameter. From table 2, we observe that an increase in the magnetic parameter increases the skin-friction and with increasing Eckert number and time parameter decreases the skin-friction.

Nusselt number: The numerical values of the heat transfer coefficient in terms of Nusselt number (Nu) are presented in table 3. It is observed that an increase in the Prandtl number and radiation parameter increases in the Nusselt number whereas an increase in the Eckert number and time parameter decreases the Nusselt number. Also, we notice that the rate of heat transfer is more for water than air.

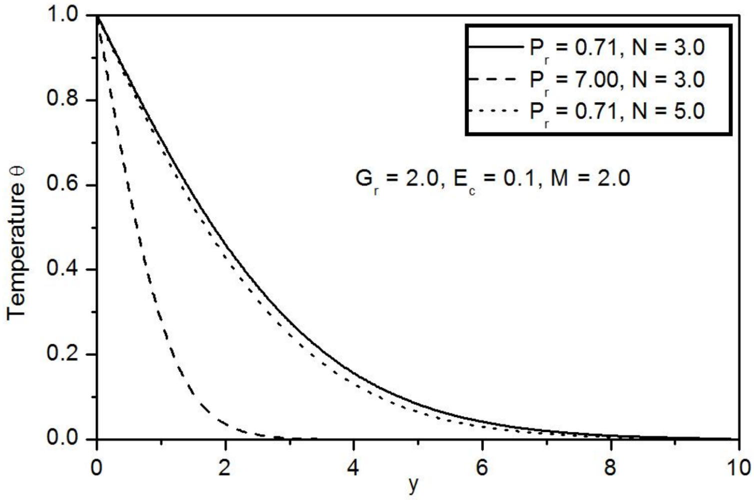

Figure 1. **Effects of P_r and N on temperature profiles at $t = 1.0$**

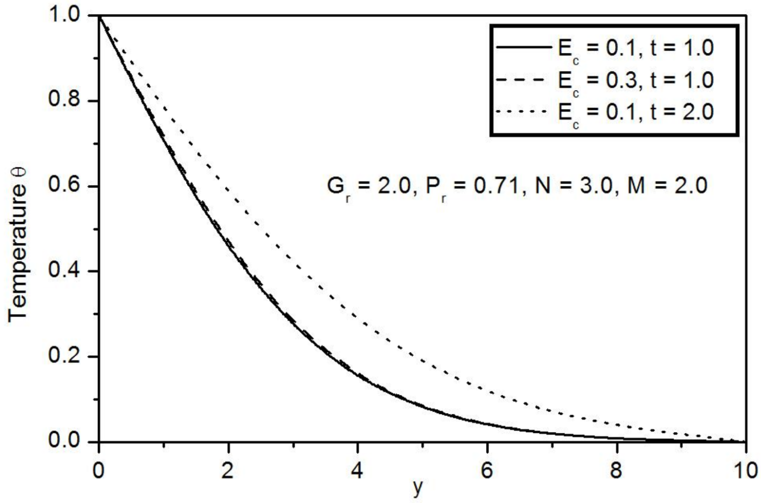

Figure 2. **Effects of E_c and time t on temperature profiles.**

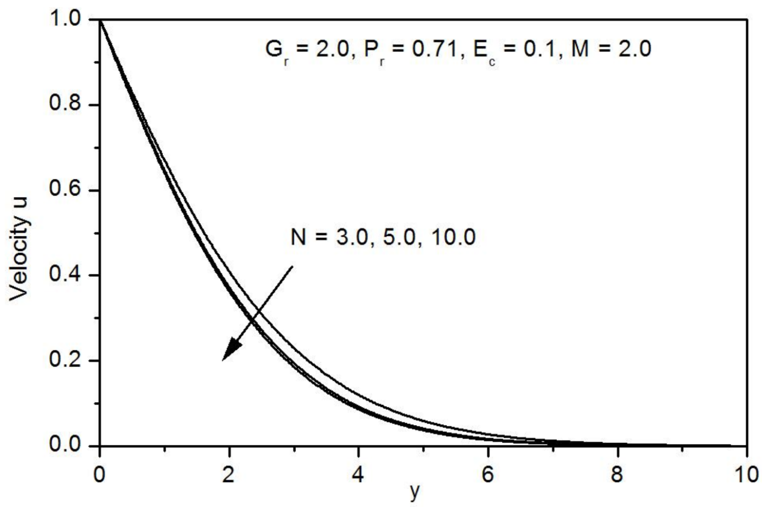

Figure 3. **Effect of radiation parameter** (N) **on the velocity profiles at** $t = 1.0$.

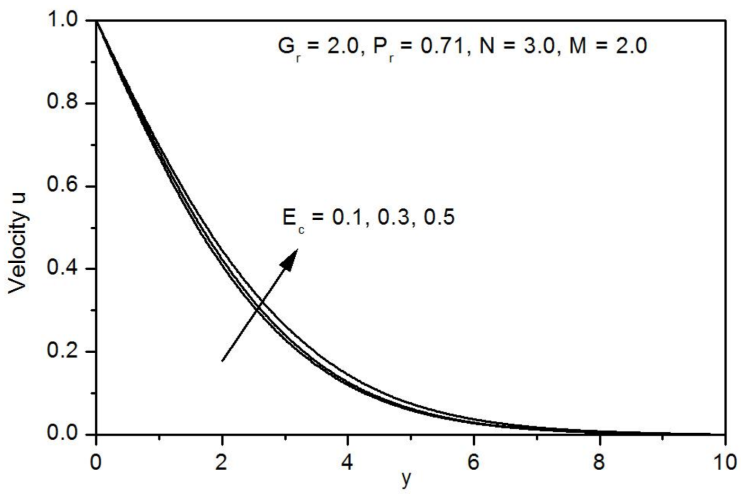

Figure 4. **Effect of Eckert number** (E_c) **on the velocity profiles at** $t = 1.0$.

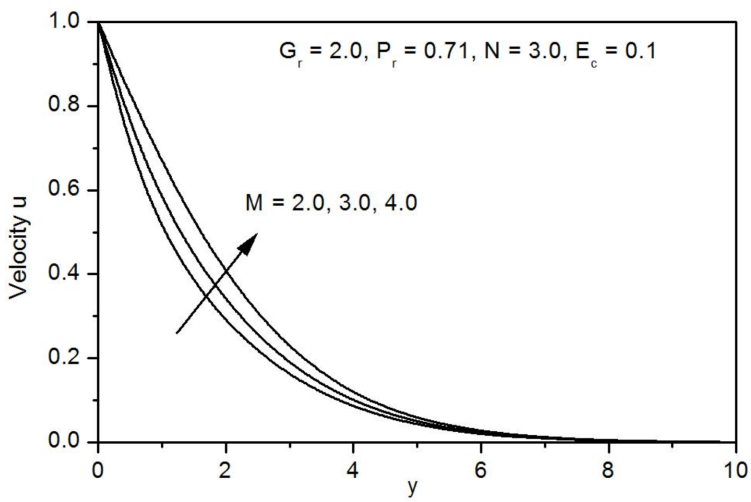

Figure 5. **Effect of magnetic parameter** (M) **on the velocity profiles at** $t = 1.0$.

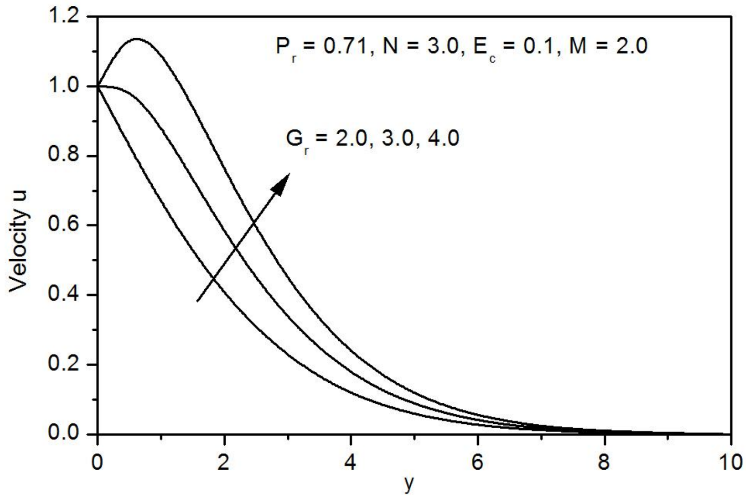

Figure 6. **Effect of Grashof number (G_r) on the velocity profiles at $t = 1.0$.**

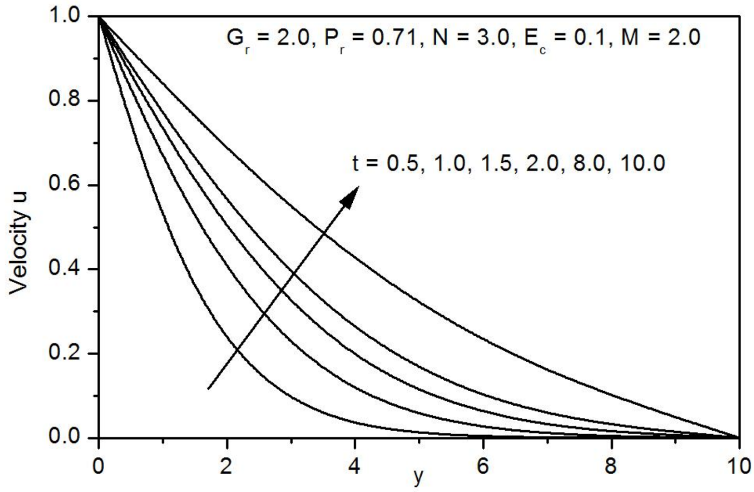

Figure 7. **Effect of time parameter (t) on the velocity profiles.**

N	$G_r = 2.0;\ t = 1.0$	$G_r = 2.0;\ t = 2.0$	$G_r = 3.0;\ t = 1.0$	$G_r = 3.0;\ t = 2.0$
3	0.346598	0.233168	0.003580	- 2.166262
5	0.364358	0.247446	0.030202	- 0.144824
10	0.379888	0.260090	0.053488	- 0.125856

Table 1. **Effects of $N, G_r$and time $t$on the skin-friction (τ).**

E_c	$M = 2.0;\ t = 1.0$	$M = 2.0;\ t = 2.0$	$M = 3.0;\ t = 1.0$	$M = 3.0; t = 2.0$
0.1	0.346598	0.233168	0.617520	0.547204
0.3	0.338216	0.224644	0.611620	0.541586
0.5	0.329906	0.216302	0.605760	0.536054

Table 2. **Effects of E_c, M and time t on the skin-friction (τ).**

P_r	E_c	$N=3.0;\ t=1.0$	$N=3.0;\ t=2.0$	$N=5.0;\ t=1.0$	$N=5.0;\ t=2.0$
0.71	0.1	0.303560	0.218088	0.324314	0.232970
	0.3	0.289048	0.206608	0.309312	0.220928
	0.5	0.274866	0.195594	0.294622	0.209346
7.00	0.1	0.867646	0.609552	0.925982	0.655286
	0.3	0.737026	0.457394	0.802946	0.505218
	0.5	0.604458	0.313998	0.677604	0.361288

Table 3. **Effects of P_r, E_c, N and time t on the Nusselt number (Nu).**

5. Conclusions

The boundary layer equations of the flow have been examined for the effects of viscous dissipation on MHD flow past an impulsively started infinite isothermal vertical plate in the presence of radiation. We conclude that

(1). The velocity of a fluid flow increases with increase in the Eckert number, Grashof number and time parameter and decreases with increase in the magnetic parameter and radiation parameter.

(2). The flow attains steady state condition with the flow velocity increasing with increase in the time parameter (Figure 7).

(3). There is fall in the temperature due to the increase in the Prandtl number and the radiation parameter.

(4). There is a rise in the temperature as the Eckert number and time parameter increases.

(5). The skin-friction decreases with increasing Grashof number, Eckert number and time parameter and increases with increase in the magnetic parameter and radiation parameter.

(6). The Nusselt number increases with increase in the Prandtl number and radiation parameter and decreases with Eckert number and time parameter.

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