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Isolate Domination in Total graphs

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Abstract: A dominating set S of a graph G is said to be an isolate dominating set of G if induced subgraph on S has at least one isolated vertex. An isolate dominating set S is said to be a minimal isolate dominating set if no proper subset of S is an isolate dominating set. The isolate domination number γ_0 is defined as the minimum cardinality of minimal isolate dominating set. In this paper, we investigate the isolate domination number of total graphs of certain classes of graphs.

MSC: 05C69.

Keywords: Domination number, Isolate domination number, Total graph.

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1. Introduction

Graph theory offers solutions to many practical problems like how to reach a particular place in minimum time at a minimum cost or how exams can be scheduled so that no student has two exams on the same day. In a graph G, a set $S \subseteq V$ is a dominating set of G if every vertex in $V - S$ is adjacent to some vertex in S. The domination number of a graph G i.e. $\gamma(G)$ is the minimum size of a dominating set of vertices in G [1]. The study of isolate domination was initiated by I. Sahul Hamid in 2013 [2]. We determine the isolate domination number of total graphs of certain classes of graphs. We consider simple, finite and undirected graphs for our study.

A dominating set S such that subgraph $< S >$ has atleast one isolate vertex is called an *isolate dominating set* [2]. An isolate domination set none of whose proper subset is an isolate dominating set is called the *minimal isolate domination set* [1]. The minimum cardinality of a minimal isolate dominating set is called *isolate domination number* $\gamma_0(G)$ for a graph G. The *total graph* $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G. The strctural properties of total graph are investigated in [3]. The study of isolate domination motivated us to introduce isolate domination in total graphs.

2. Main Results

In this section, we study isolate domination number γ_0 of total graphs of complete graphs, wheels, cycles and paths.

A complete graph K_n is a graph on n vertices with $\binom{n}{2}$ edges. Since every vertex is adjacent to every other vertex of K_n, $\gamma_0(K_n) = 1$. In the following theorem, we find $\gamma_0(T(K_n))$, the isolate domination number of a total graph of a K_n.

Theorem 2.1. $\gamma_0(T(K_n)) = \lceil \frac{n}{2} \rceil$, *for* $n \geq 2$.

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Proof. Let $v_1, v_2, ..., v_n$ be the vertices and $e_1, e_2, ..., e_{\binom{n}{2}}$ be the edges of K_n. Then $T(K_n)$ has the vertices $v_1, v_2, ..., v_n, e_1, e_2, ..., e_{\binom{n}{2}}$. Hence $|V(T(K_n))| = n+ \binom{n}{2}$. Since v_1 is adjacent to $v_2, v_3, ..., v_n$ in K_n, v_1 is also adjacent to those vertices in $T(K_n)$. Hence v_1 dominates $v_2, v_3, ..., v_n$ in $T(K_n)$. Also since $(n-1)$ $e_i's$ are incident on v_1 in K_n, v_1 is adjacent to $(n-1)e_i's$ in $T(K_n)$. Hence v_1 dominates $n-1$ $e_i's$ in $T(K_n)$.

Therefore there are $\binom{n}{2} - (n-1) = \binom{n-1}{2}$ $e_i's$ in $T(K_n)$ which need to be dominated. Let such $e_i's$ belongs to $E = \{e_1, e_2, ..., e_{\binom{n-1}{2}}\}$. Consider a vertex e_1 belongs to E in $T(K_n)$. Let $e_1 = \{v_p, v_q\}$ in K_n. Since v_1 already dominates v_p, v_q and edges incident on it, e_1 is adjacent to $n-3+n-3 = 2n-6$ $e_i's$ of E in $T(K_n)$. Hence e_1 dominates $2n-6+1 = 2n-5$ $e_i's$ of E in $T(K_n)$ including itself. Of the remaining $e_i's$ that are not dominated, consider a vertex e_2. It is adjacent to $n-5+n-5 = 2n-10$ $e_i's$ after excluding edges to e_1 and already dominated vertices. Hence e_2 dominates $2n-10+1 = 2n-9$ $e_i's$ including itself. This process continues till $2n-(4k+1) \geq 1$, where $k = 1, 2, ..., \frac{n-1}{2}$ for corresponding edge e_k, whenever K_n is of odd n. Also $2n-(4k+1) \geq 3$, where $k = 1, 2, ..., \frac{n-2}{2}$ for corresponding edge e_k, whenever K_n is of even n. Hence dominating set $S = \{v_1, e_1, ..., e_k\}$, where

$$k = \begin{cases} \frac{n-1}{2} & \text{when } n \text{ is odd.} \\ \frac{n-2}{2} & \text{when } n \text{ is even.} \end{cases}$$

Hence

$$|S| = \begin{cases} \frac{n-1}{2} + 1 = \frac{n+1}{2} & \text{when } n \text{ is odd.} \\ \frac{n-2}{2} + 1 = \frac{n}{2} & \text{when } n \text{ is even.} \end{cases}$$

Therefore,

$$\gamma_0(T(K_n)) = \begin{cases} \frac{n+1}{2} & \text{when } n \text{ is odd.} \\ \frac{n}{2} & \text{when } n \text{ is even.} \end{cases}$$

In general, $\gamma_0(T(K_n)) = \lceil \frac{n}{2} \rceil$. □

In the above case we observe that $\gamma(T(Kn)) = \gamma_0(T(K_n))$.

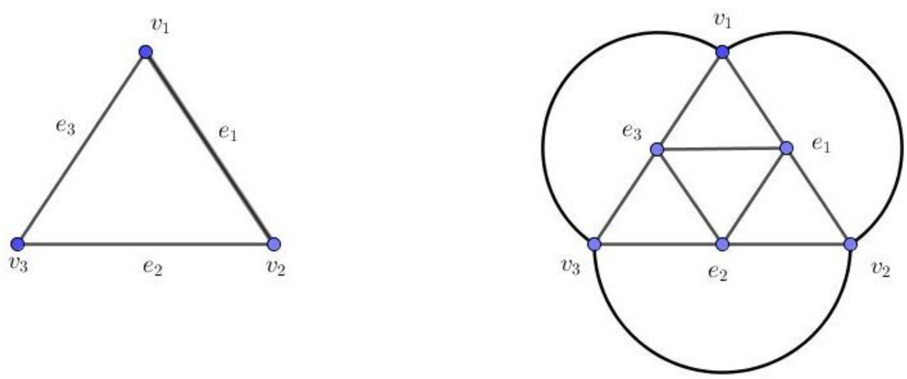

Figure 1: K_3 and $T(K_3)$

Example 2.2. *Consider K_3 with vertex set $V = \{v_1, v_2, v_3\}$ and edge set $E = \{e_1, e_2, e_3\}$. $T(K_3)$ has vertex set as $V(T(K_3)) = \{v_1, v_2, v_3, e_1, e_2, e_3\}$ and $|V(T(K_3))| = 6$. As seen in figure 1.*

v_1 will dominate $\{v_1, v_2, v_3, e_1, e_3\}$ and e_2 will dominate itself. Therefore $S = \{v_1, e_2\}$. $|S| = 2$. Hence $\gamma_0(T(K_3)) = \lceil \frac{3}{2} \rceil = 2$.

Wheel W_n on $n+1$ vertices has a central vertex which is adjacent to other n vertices. The isolate domination number of W_n is determined 1 in [2]. Total graph of a wheel is graph has a vertex set as $n+1$ $v_i{}'s$ and $2n$ $e_i{}'s$. In the following theorem we find $\gamma_0(T(W_n))$, the isolate domination number of total graph of wheel.

Theorem 2.3. *$\gamma_0(T(W_n)) = \lceil \frac{n}{3} \rceil + 1$, for $n \geq 3$.*

Proof. Let $v_1, v_2, ..., v_{n+1}$ be the vertices of wheel W_n, where v_1 is the central vertex of W_n. Let $e_1, e_2, ..., e_{2n}$ be the edges of W_n. Let $e_1, e_2, ..., e_n$ be the edges with end vertices as peripheral vertices and $e_{n+1}, e_{n+2}, ..., e_{2n}$ the edges with v_1 as a one end vertex and other end vertex to be $v_2, v_3, ..., v_{n+1}$ respectively. Hence total number of vertices in total graph $T(W_n)$ is $|V(T(W_n))| = n+1+2n = 3n+1$. Now v_1 is adjacent to $v_2, v_3, ..., v_{n+1}$ and $e_{n+1}, e_{n+2}, ..., e_{2n}$ in $T(W_n)$. Therefore v_1 dominates $n+n+1 = 2n+1$ vertices including itself. Therefore $3n+1-(2n+1) = n$ vertices need to be dominated in $T(W_n)$. The vertices that need to be dominated in $T(W_n)$ are $e_1, e_2, ..., e_n$. Each of these $e_i{}'s$ as edges of W_n have two edges $e_i{}'s$ incident on their end vertices. Hence in $T(W_n)$, each such e_i is dominating two $e_j{}'s$ and itself. Hence $\lceil \frac{n}{3} \rceil$ $e_i{}'s$ are required to dominate n $e_i{}'s$ and v_1 dominates the remaining. Hence dominating set $S = \{v_1, e_1, e_2, ..., e_{\lceil \frac{n}{3} \rceil}\}$ and $|S| = \lceil \frac{n}{3} \rceil + 1$. Therefore $\gamma_0(T(W_n)) = \lceil \frac{n}{3} \rceil + 1$. □

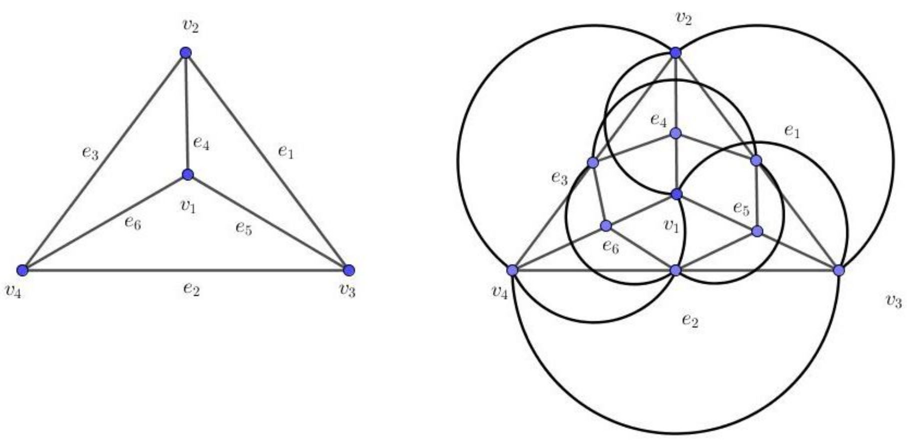

Figure 2: W_3 and $T(W_3)$

Example 2.4. *Consider W_3 with vertex set $V = \{v_1, v_2, v_3, v_4\}$ and edge set $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Then $V(T(W_3)) = \{v_1, v_2, v_3, v_4, e_1, e_2, e_3, e_4, e_5, e_6\}$. Here v_1 dominates $\{v_2, v_3, v_4, e_1, e_2, e_3\}$ and remaining $e_i{}'s$ are dominated by e_2. Therefore $S = \{v_1, e_2\}$ and $|S| = 2$. $\gamma_0(T(W_3)) = \lceil \frac{3}{3} \rceil + 1 = 2$.*

Cycle C_n is a closed path on n vertices. The isolate domination number of cycle is proved to be $\lceil \frac{n}{3} \rceil$ in [2]. In the following theorem, we study $\gamma_0(T(C_n))$, the isolate domination number of total graphs of cycles.

Theorem 2.5. *$\gamma_0(T(C_n)) = \lceil \frac{2n}{5} \rceil$, for $n \geq 3$*

Proof. Let $v_1, v_2, ..., v_n$ be the vertices and $e_1, e_2, ..., e_n$ be the edges of cycle C_n. Therefore number of vertices in total graph of C_n is $|V(T(C_n))| = n + n = 2n$. Now each v_i is adjacent to two $v_j{}'s$. Also there are two edges incident on each v_i in C_n. Hence v_i is adjacent to two $v_j{}'s$ and two $e_j{}'s$ in $T(C_n)$. Hence v_i dominates five vertices in $T(C_n)$ including itself. As each e_i has two edges incident on its end vertices in C_n, hence e_i is adjacent to two $v_j{}'s$ and two $e_j{}'s$ in $T(C_n)$. Hence e_i dominates five vertices in $T(C_n)$. Hence $\lceil \frac{2n}{5} \rceil$ vertices are required to dominate all the vertices. Hence dominating set $S = \{v_i, e_j, ..., v_x \text{ or } e_y\}$ where $\{1 \leq i, j, x, y \leq n\}$ and $|S| = \lceil \frac{2n}{5} \rceil$. Hence $\gamma_0(T(C_n)) = \lceil \frac{2n}{5} \rceil$. □

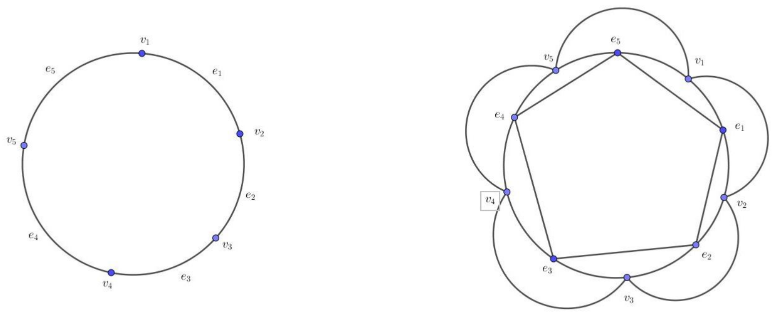

Figure 3: C_5 and $T(C_5)$

Example 2.6. *Consider C_5 with vertex set $V = \{v_1, v_2, v_3, v_4, v_5\}$, edge set $E = \{e_1, e_2, e_3, e_4, e_5\}$. Then $V(T(C_5)) = \{v_1, v_2, v_3, v_4, v_5, e_1, e_2, e_3, e_4, e_5\}$ as seen in figure 3 and $|V(T(C_5))| = 10$. v_1 dominates $\{v_1, v_2, v_5, e_1, e_5\}$ and e_3 dominates $\{v_3, v_4, e_2, e_3, e_4\}$. Hence $S = \{v_1, e_3\}$ and $|S| = 2$. $\gamma_0(T(C_5)) = \lceil \frac{2\times 5}{5} \rceil = 2$.*

We can make an observation that, $\gamma_0(T(C_n)) = \dfrac{|V(T(C_n))|}{\triangle + 1}$ where $\triangle$ is the maximum degree of $T(C_n)$. P_n is the path on n vertices. Isolate domination number of a paths is proved to be $\lceil \frac{n}{3} \rceil$ in [2]. In the following theorem, we study $\gamma_0(T(P_n))$, the isolate domination number of total graph of paths.

Theorem 2.7. *$\gamma_0(T(P_n)) = \lceil \frac{2n-1}{5} \rceil$, for $n \geq 2$.*

Proof. Let $v_1, v_2, ..., v_n$ be the vertices and $e_1, e_2, ..., e_{n-1}$ be the edges of path P_n. Hence $v_1, v_2, ..., v_n, e_1, ..., e_{n-1}$ are the vertices in total graph of P_n. Therefore $|V(T(P_n))| = n + n - 1 = 2n - 1$. Each v_i is adjacent to two $v_j's$ and since two edges are incident on it in $T(P_n)$, it is adjacent to two $v_j's$ and two $e_j's$ in $T(P_n)$ for $2 \leq i \leq n-1$. Hence v_i dominates five vertices including itself in $T(P_n)$. Similarly each e_i has two $e_j's$ incident on its vertices in P_n, it is adjacent to two $v_j's$ and two $e_j's$ in $T(P_n)$ for $2 \leq i \leq n-2$. Hence e_i dominates five vertices including itself in $T(P_n)$. Continuing in the same way $\lceil \frac{2n-1}{5} \rceil$ number of vertices will required to dominate $2n-1$ vertices in $T(P_n)$. Hence dominating set $S = \{v_i, e_j, ..., v_x$ or $e_y\}$ where $\{1 \leq i, x \leq n\}$ and $\{1 \leq j, y \leq n-1\}$ and $|S| = \lceil \frac{2n-1}{5} \rceil$. Hence $\gamma_0(T(P_n)) = \lceil \frac{2n-1}{5} \rceil$. □

Example 2.8. *Consider P_7 with vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and edge set $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. Then $V(T(P_7)) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, e_1, e_2, e_3, e_4, e_5, e_6\}$ as seen in figure 4 and $|V(T(P_7))| = 13$. Here v_2 dominates $\{v_2, v_1, v_3, e_1, e_2\}$, e_4 dominates $\{e_4, v_4, v_5, e_3, e_5\}$, v_7 dominates $\{v_7, v_6, e_6\}$. Therefore $S = \{v_2, e_4, v_7\}$ and $|S| = 3$. $\gamma_0(T(P_7)) = \lceil \frac{(2\times 7)-1}{5} \rceil = 3$.*

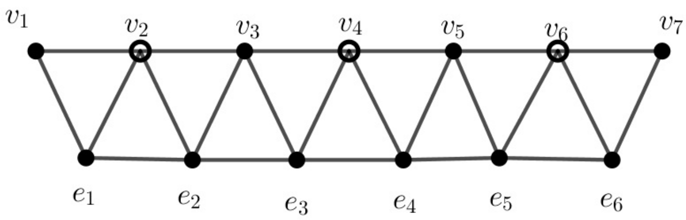

Figure 4: $T(P_7)$

In this case we observe that $\gamma_0(T(P_n)) = \dfrac{|V(T(P_n))|}{\triangle + 1}$ where $\triangle$ is the maximum degree of $T(P_n)$.

3. Conclusion

In this paper, we have discussed isolate domination in total graphs of complete graphs, wheels, cycles and paths. In future, our focus will be on doing comparative study of total domination and isolate domination in graphs.

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