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Edge Domination in Some Special Classes of Fuzzy Graphs

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Abstract: In this paper we find the exact edge domination number of some special classes of fuzzy graphs like complete fuzzy graph, star fuzzy graph complete bipartite fuzzy graph etc.

Keywords: Edge domination, complete fuzzy graph, star fuzzy graph, regular fuzzy graph.

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1. Introduction

The introduction of fuzzy set theory by Zadeh in 1965 [6] gave a remarkable beginning not only in the field of mathematics, but to all the fields of science and engineering. The concept of fuzzy graphs has been introduced by Rosenfeld. Domination in fuzzy graphs is one of the widest fields which have witness a tremendous growth recently. Somasundram introduced domination in fuzzy graphs [4]. Another parallel concept on domination called edge domination in fuzzy graphs was introduced by Velammal and Thiagarajan [5]. Nagoorgani and Prasanna Devi defined edge domination in fuzzy graphs and discussed about edge independence number of fuzzy graphs in [2]. Ramya and Lavanya also defined edge domination in fuzzy graphs just by using the concept of adjacency alone and studied few properties related to it [3]. In this paper as a continuation of our work we discuss in detail and present the exact edge domination number of some special classes of fuzzy graphs.

1.1. Preliminaries

Definition 1.1 ([1]). *A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that for all $u, v \in V$, $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ and μ is a symmetric fuzzy relation on σ. Here $\sigma(v)$ and $\mu(u, v)$ represent the membership values of the vertex u and of the edge (u, v) respectively.*

Definition 1.2 ([1]). *The underlying crisp graph of a fuzzy graph $G = (\sigma, \mu)$ is denoted by $G^* = (\mu^*, \sigma^*)$ where $\sigma^* = \{u \in v|\sigma(v) > 0\}$ and $\mu^* = \{(u, v) \in VXV|\mu(u, v) > 0$.*

Definition 1.3 ([1]). *A fuzzy graph $G = (\sigma, \mu)$ is a strong fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v) \ \ \forall \ \ (u, v) \in \mu^*$.*

Definition 1.4 ([1]). *A fuzzy graph is said to be a complete fuzzy graph if $\mu(u, v) = \sigma(u) \wedge \sigma(v) \ \ forall \ \ u, v \in \sigma^*$.*

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Definition 1.5 ([1]). *A fuzzy graph $G = (V, \sigma, \mu)$ is said to be regular if $d(v) = k$, a positive real number, for all $v \in V$.*

Definition 1.6 ([3]). *Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, E). A subset X of $V \times V$ is called an edge dominating set of G if every edge not in X is incident to some edge in X.*

Definition 1.7 ([3]). *A subset X of $V \times V$ is said to be a minimal edge dominating set if no proper subset of X is an edge dominating set of G*

Definition 1.8 ([3]). *The minimum cardinality of an edge dominating set is called as the edge domination number of G and is denoted by $\gamma'(G)$.*

Definition 1.9 ([3]). *An edge dominating set X of a fuzzy graph G is said to be an independent edge dominating set if its edges are not adjacent.*

2. Edge Domination in Special Classes of Fuzzy Graphs

Theorem 2.1. *The edge domination number of a complete fuzzy graph is given by $\gamma'_{K_n} = \gamma_1 + \gamma_2 + \cdots \gamma_N$, where $N = \left[\frac{n}{2}\right]$ and $\gamma_1, \gamma_2, \cdots, \gamma_N$ are first N minimum vertex membership values.*

Proof. We know that in a complete graph K_n with n vertices the edge domination number is given by $n/2$ if n is even and $(n-1)/2$ if n is odd. In a complete fuzzy graph K_n all the edges are strong edges and hence by the definition the membership values of the edges will be equal to the minimum membership value of the corresponding vertices, the edge domination number of a fuzzy graph is defined as the cardinality of the minimum edge dominating set. Let $N = \left[\frac{n}{2}\right]$; $\gamma_1 = \wedge_{\forall v_i \in V}\{\sigma(v_i)\} = \sigma(v_{k_1})$ (say). Let $V_1 = V - v_{k_1}$, then γ_2 is given by $\gamma_2 = \wedge_{\forall v_i \in V_1}\{\sigma(v_i)\} = \sigma(v_{k_2})$ (say). Let $V_2 = V - v_{k_2}$, then γ_3 is given by $\gamma_3 = \wedge_{\forall v_i \in V_2}\{\sigma(v_i)\} = \sigma(v_{k_3})$ (say). Proceeding similarly, we get $\gamma_N = \wedge_{\forall v_i \in V_{N-1}}\{\sigma(v_i)\} = \sigma(v_{k_N})$ (say). Thus the edge domination number is given by $\gamma\,'_{K_n} = \gamma_1 + \gamma_2 + \cdots \gamma_N$, where $N = \left[\frac{n}{2}\right]$. ☐

Example 2.2. *The edge dominating set of the complete fuzzy graph K_5 given below is $\{ac, ce\}$ and the edge domination number is $\gamma'_{K_n} = 0.2 + 0.3 = 0.5$ (Figure 1).*

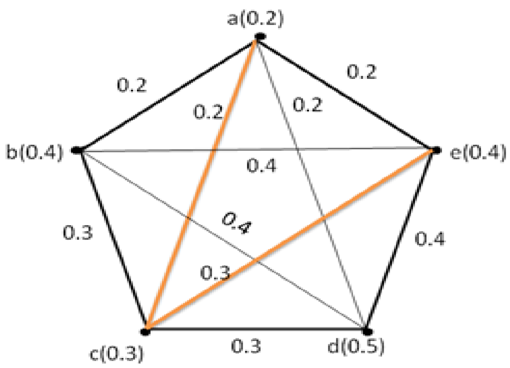

Figure 1. **Complete fuzzy graph K_5**

Here, $N = \left[\frac{n}{2}\right] = \left[\frac{5}{2}\right] = 2$; $\gamma_1 = \wedge_{\forall v_i \in V}\{\sigma(v_i)\} = \sigma(a) = 0.2$. *Let* $V_1 = V - a$; $\gamma_2 = \wedge_{\forall v_i \in V_1}\{\sigma(v_i)\} = \sigma(c) = 0.3$. *Hence edge domination number is given by* $\gamma_1 + \gamma_2 = 0.5$.

Theorem 2.3. *The edge domination number of a fuzzy star graph is given by $\gamma\,'_S = \wedge_{\forall (x,y) \in G}\{\mu(x, y)\}$.*

Proof. The edge domination set of a crisp star graph consists of only one edge, because in a star graph each edge will be adjacent to all the other edges, hence the edge dominating set of a fuzzy star graph also consist of only one edge whose membership value is minimum and that minimum membership value is its edge domination number. That is $\gamma\,'_S = \wedge_{\forall(x,y)\in G}\{\mu(x,y)\}$. □

Example 2.4.

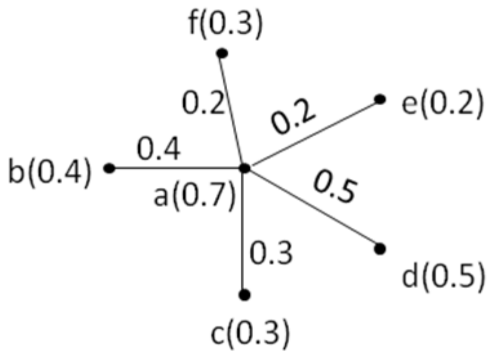

Figure 2. **Fuzzy Star graph $S_{1,5}$**

The edge domination number of the star graph $S_{1,5}$ is given by $\gamma\,'_{S_{1,5}} = 0.2$.

Theorem 2.5. *The edge domination number of a complete bipartite fuzzy graph $K_{m,n}$ is given by $\gamma'_{K_{m,n}} = \gamma_1 + \gamma_2 + \cdots \gamma_M$, where $M = min(m,n)$ and $\gamma_1, \gamma_2, \cdots, \gamma_N$ are first M minimum vertex membership values.*

Proof. We know that the edge domination number of a crisp complete bipartite graph $K_{m,n}$ is the $\min(m,n)$. In a fuzzy complete bipartite graph all the edges are strong edges and hence by the definition the membership values of the edges will be equal to the minimum membership value of the corresponding vertices. $M = \min(m,n)$; $\gamma_1 = \wedge_{\forall v_i \in V}\{\sigma(v_i)\} = \sigma(v_{k_1})$ (say). Let $V_1 = V - v_{k_1}$, then γ_2 is given by $\gamma_2 = \wedge_{\forall v_i \in V_1}\{\sigma(v_i)\} = \sigma(v_{k_2})$ (say). Let $V_2 = V - v_{k_2}$, then γ_3 is given by $\gamma_3 = \wedge_{\forall v_i \in V_2}\{\sigma(v_i)\} = \sigma(v_{k_3})$ (say). Proceeding similarly, we get $\gamma_N = \wedge_{\forall v_i \in V_{M-1}}\{\sigma(v_i)\} = \sigma(v_{k_M})$ (say). Thus the edge domination number is given by $\gamma\,'_{K_{m,n}} = \gamma_1 + \gamma_2 + \cdots \gamma_M$, where $M = min(m,n)$. Thus we have the edge domination number of fuzzy complete bipartite graph is the sum of first M vertices with minimum membership values. □

Example 2.6.

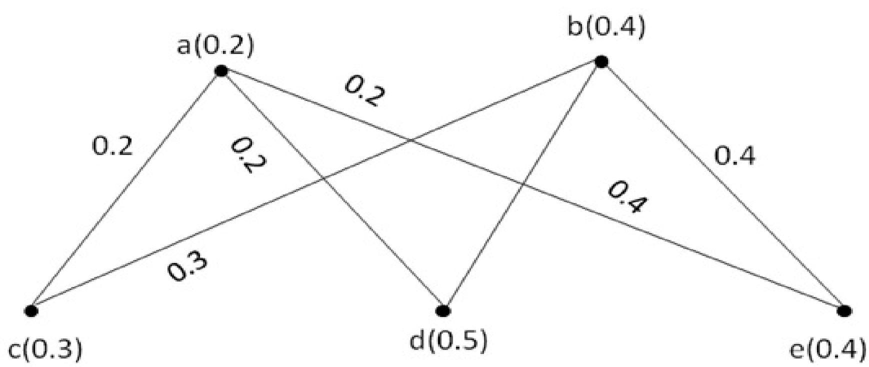

Figure 3. **Fuzzy complete bipartite graph**

Theorem 2.7. *Let C_{3n} (n is even) be a regular fuzzy cycle of degree $a + b$ then the edge domination number is given by* $\frac{n}{6}(a+b)$.

Proof. We know that in a crisp graph of C_{3n} the number of edges in the edge dominating set is given by $n/3$. We have considered a regular fuzzy cycle of degree $a+b$, that is the edge membership values are a and b alternatively. Thus the edge domination number is given by $\frac{n}{6}(a+b)$. □

3. Conclusion

Thus in this paper we have made an attempt to give the generalized method to find the exact fuzzy edge domination number of certain special classes of fuzzy graphs. This work can be extended to other types of fuzzy graphs available.

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