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k- Harmonic Mean Labeling of Some Cycle Related Graphs

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Abstract: Somasundaram and Ponraj [12] have introduced the notion of mean labeling of graphs. R. Ponraj and D. Ramya introduced Super mean labeling of graphs in [5]. S. Somasundaram and S.S. Sandhya introduced the concept of Harmonic mean labeling in [6] and studied their behavior in [7–9]. In this paper, we investigate k-harmonic mean labeling of some graphs.

Keywords: Harmonic mean labeling, harmonic mean graph, k-harmonic mean labeling, k-harmonic mean graph.

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1. Introduction

By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [3]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J.A. Gallian (2016) can be found in [1]. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). Somasundaram and Ponraj [12] have introduced the notion of mean labeling of graphs. R. Ponraj and D. Ramya introduced Super mean labeling of graphs in [5]. S. Somasundaram and S.S. Sandhya introduced the concept of Harmonic mean labeling in [6] and studied their behavior in [7–9]. In this paper, we investigate k-harmonic mean labeling of some graphs.

1.1. Basic Definitions

Definition 1.1. *Let G be a (p, q) graph. A function f is called a harmonic mean labeling of a graph G if $f : V(G) \to \{1, 2, 3, ..., q+1\}$ is injection and the induced edge function $f^* : E(G) \to \{1, 2, ..., q\}$ defined as*

$$f^*(e = uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \text{ or } \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$$

is bijective. The graph which admits harmonic mean labeling is called harmonic mean graph.

Definition 1.2. *Let G be a (p, q) graph. A function f is called a k-harmonic mean labeling of a graph G if $f : V(G) \to \{k, k+1, k+2, ..., k+q\}$ is injection and the induced edge function $f^* : E(G) \to \{k, k+1, k+2, ..., k+q-1\}$ defined as*

$$f^*(e = uv) = \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \text{ or } \left\lfloor \frac{2f(u)f(v)}{f(u)+f(v)} \right\rfloor$$

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is bijective. The graph which admits k-harmonic mean labeling is called k-harmonic mean graph.

Definition 1.3. *The product $P_2 \times P_n$ is called a Ladder and it is denoted by L_n.*

Definition 1.4. *A Triangular ladder $TL_n, n \geq 2$ is a graph obtained from a ladder L_n by adding the edges u_iv_{i+1} for $1 \leq i \leq n-1$, where u_i and v_i $1 \leq i \leq n-1$, are the vertices of L_n such that $u_1, u_2, ..., u_n$ and $v_1, v_2, ..., v_n$ are two paths of length n in L_n*

Definition 1.5. *A Triangular snake T_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$.*

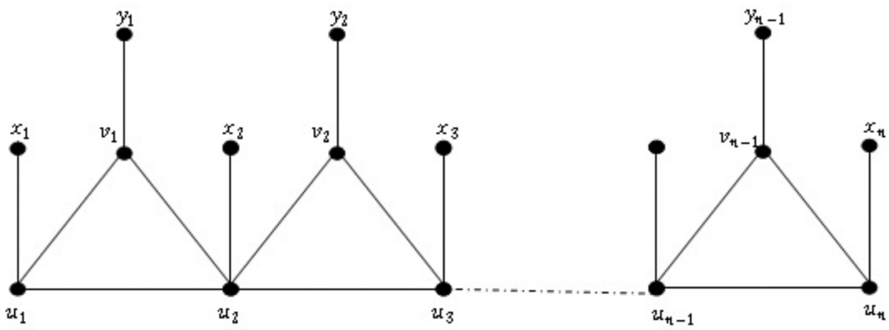
Definition 1.6. *A Quadrilateral snake Q_n is obtained from a path $u_1, u_2, ..., u_n$ by joining u_i and u_{i+1} to new vertices v_i and w_i for $1 \leq i \leq n-1$ respectively and then joining v_i and w_i.*

Definition 1.7. *If G has order n, the corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.*

2. Main Results

Theorem 2.1. *$T_n \odot K_1, (n \geq 2)$ is k-harmonic mean graph for all $k \geq 1$.*

Proof. Let $V(T_n \odot K_1) = \{u_i, x_i; 1 \leq i \leq n, v_i, y_i; 1 \leq i \leq n\}$; $E(T_n \odot K_1) = \{u_iu_{i+1}, u_iv_i, u_{i+1}v_i, v_iy_i; 1 \leq i \leq n-1; u_ix_i, 1 \leq i \leq n\}$ be denoted as in the following figure



First we label the vertices as follows. Define a function $f : V(T_n \odot K_1) \to \{k, k+1, k+2, ..., k+q\}$ by

$$f(u_i) = k + 5i - 4, \ for \ \ 1 \leq i \leq n$$

$$f(v_i) = k + 5i - 3, \ for \ \ 1 \leq i \leq n-1$$

$$f(x_1) = k$$

$$f(x_2) = k + 4$$

$$f(x_i) = k + 5i - 5, \ for \ \ 3 \leq i \leq n$$

$$f(y_i) = k + 5i - 2, \ for \ \ 1 \leq i \leq n-1.$$

Then the induced edge labels are

$$f^*(u_1u_2) = k + 3$$

$$f^*(u_iu_{i+1}) = k + 5i - 2,\ \ for\ \ 2 \leq i \leq n - 1$$
$$f^*(u_ix_i) = k + 5i - 5,\ \ for\ \ 1 \leq i \leq n$$
$$f^*(u_iv_i) = k + 5i - 4,\ \ for\ \ 1 \leq i \leq n - 1$$
$$f^*(u_{i+1}v_i) = k + 5i - 1,\ \ for\ \ 1 \leq i \leq n - 1$$
$$f^*(v_1y_1) = k + 2$$
$$f^*(v_iy_i) = k + 5i - 3,\ \ for\ \ 2 \leq i \leq n - 1.$$

The above defined function f provides k-harmonic mean labeling of the graph. Hence $T_n \odot K_1, (n \geq 2)$ is k-harmonic mean graph for all $k \geq 1$. □

Example 2.2. *500-harmonic mean labeling of $T_6 \odot K_1$ is as follows:*

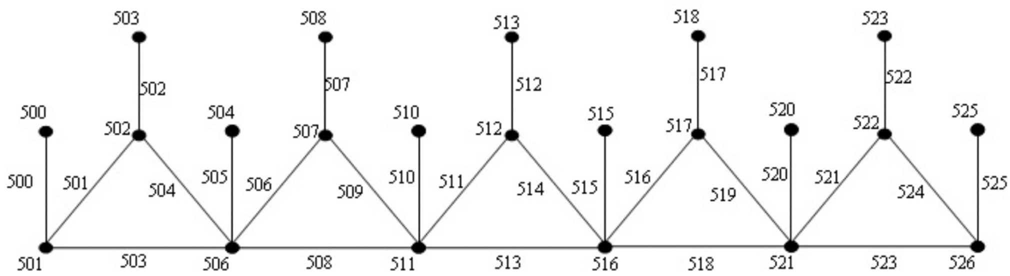
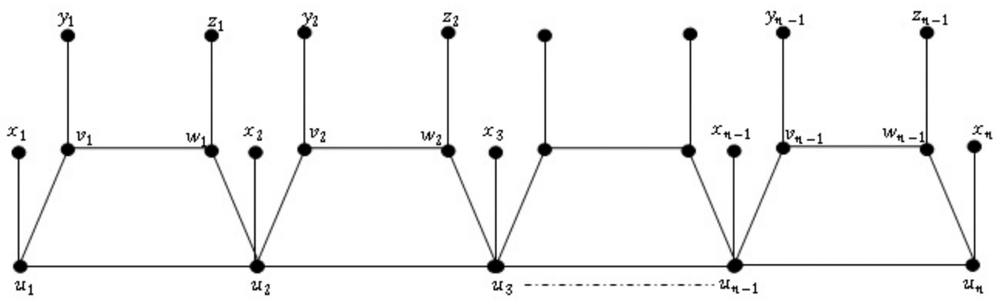


Figure 1. **500-harmonic mean labeling of $T_6 \odot K_1$.**

Theorem 2.3. *$Q_n \odot K_1, (n \geq 2)$ is k - harmonic mean graph for all $k \geq 1$.*

Proof. Let $V(Q_n \odot K_1) = \{u_i, x_i; 1 \leq i \leq n, v_i, w_i, y_i, z_i; 1 \leq i \leq n-1\}$; $E(Q_n \odot K_1) = \{u_iu_{i+1}, u_iv_i, w_iz_i, v_iy_i, u_{i+1}w_i; 1 \leq i \leq n-1; u_ix_i, 1 \leq i \leq n\}$ be denoted as in the following figure



First we label the vertices as follows. Define a function $f : V(Q_n \odot K_1) \rightarrow \{k, k+1, k+2, ..., k+q\}$ by

$$f(u_1) = k + 2$$
$$f(u_i) = k + 7i - 6,\ \ for\ \ 2 \leq i \leq n$$
$$f(v_1) = k + 1$$
$$f(v_i) = k + 7i - 5,\ \ for\ \ 2 \leq i \leq n - 1$$
$$f(x_1) = k + 3$$

$$f(x_i) = k + 7i - 8, \ for \ \ 2 \le i \le n$$

$$f(y_1) = k$$

$$f(y_i) = k + 7i - 4, \ for \ \ 2 \le i \le n-1$$

$$f(w_i) = k + 7i - 3, \ for \ \ 1 \le i \le n-1$$

$$f(z_i) = k + 7i - 2, \ for \ \ 1 \le i \le n-1$$

Then the induced edge labels are

$$f^*(u_1u_2) = \begin{cases} k+4 \, if \, k \le 5 \\ k+5 \, if \, k \ge 6 \end{cases}$$

$$f^*(u_iu_{i+1}) = k + 7i - 3, \qquad for \ \ 2 \le i \le n-1$$

$$f^*(u_1x_1) = k + 3$$

$$f^*(u_ix_i) = k + 7i - 7, \qquad for \ \ 2 \le i \le n$$

$$f^*(u_iv_i) = k + 7i - 6, \qquad for \ \ 1 \le i \le n-1$$

$$f^*(v_1y_1) = k$$

$$f^*(v_iy_i) = k + 7i - 5, \qquad for \ \ 2 \le i \le n-1$$

$$f^*(v_1w_1) = k + 2$$

$$f^*(v_iw_i) = k + 7i - 4, \qquad for \ \ 2 \le i \le n-1$$

The above defined function f provides k-harmonic mean labeling of the graph. Hence $Q_n \odot K_1, (n \ge 2)$ is k -harmonic mean graph for all $k \ge 1$. □

Example 2.4. *550 harmonic mean labeling of $Q_6 \odot K_1$ is as follows:*

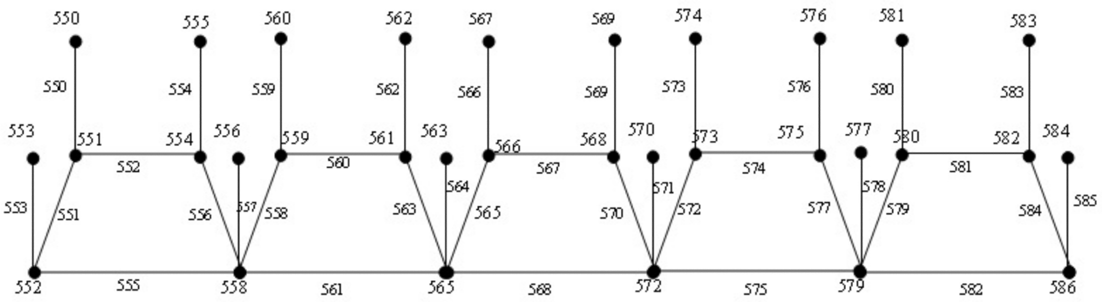
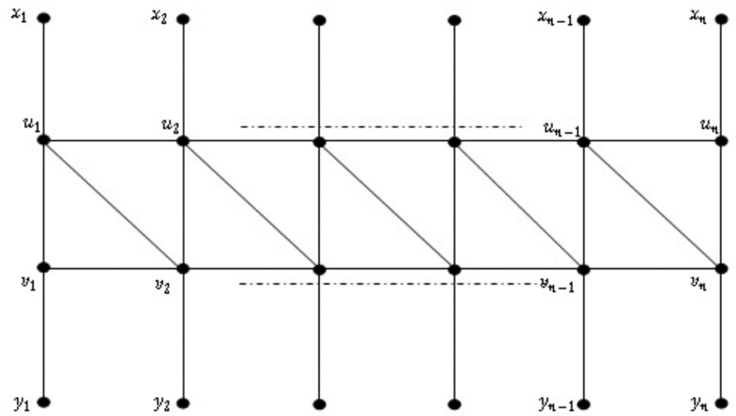


Figure 2. **550 harmonic mean labeling of $Q_6 \odot K_1$.**

Theorem 2.5. *$TL_n \odot K_1, (n \ge 2)$ is k - harmonic mean graph for all $k \ge 1$.*

Proof. Let $V(TL_n \odot K_1) = \{u_i, v_i, x_i, y_i; 1 \le i \le n\}$; $E(TL_n \odot K_1) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1}; 1 \le i \le n-1, u_iv_i, x_iu_i, y_iv_i; 1 \le i \le n\}$ be denoted as in the following figure



First we label the vertices as follows. Define a function $f : V(TL_n \odot K_1) \to \{k, k+1, k+2, ..., k+q\}$ by

$$f(u_1) = \begin{cases} k+3 \; if \; k \leq 2 \\ k+2 \; if \; k \geq 3 \end{cases}$$

$$f(u_i) = k + 6i - 3, \qquad for \;\; 2 \leq i \leq n$$

$$f(v_1) = k + 1$$

$$f(v_i) = k + 6i - 6, \qquad for \;\; 2 \leq i \leq n$$

$$f(x_1) = \begin{cases} k+2 \; ifk \leq 2 \\ k+3 \; ifk \geq 3 \end{cases}$$

$$f(x_i) = k + 6i - 4, \qquad for \;\; 2 \leq i \leq n$$

$$f(y_1) = k$$

$$f(y_i) = k + 6i - 7, \qquad for \;\; 2 \leq i \leq n$$

Then the induced edge labels are

$$f^*(u_1u_2) = k + 5$$

$$f^*(u_iu_{i+1}) = k + 6i, \qquad for \;\; 2 \leq i \leq n-1$$

$$f^*(v_iv_{i+1}) = k + 6i - 3, \;\; for \;\; 1 \leq i \leq n$$

$$f^*(u_iv_{i+1}) = k + 6i - 2, \;\; for \;\; 1 \leq i \leq n-1$$

$$f^*(u_iv_i) = k + 6i - 5, \;\; for \;\; 1 \leq i \leq n$$

$$f^*(u_ix_i) = k + 6i - 4, \;\; for \;\; 1 \leq i \leq n$$

$$f^*(v_1y_1) = k$$

$$f^*(v_1y_2) = k + 6$$

$$f^*(v_iy_i) = k + 6i - 7, \;\; for \;\; 3 \leq i \leq n$$

The above defined function f provides k-harmonic mean labeling of the graph. Hence $TL_n \odot K_1, (n \geq 2)$ is k - harmonic mean graph for all $k \geq 1$. □

Example 2.6. *300 harmonic mean labeling of $TL_8 \odot K_1$ is as follows:*

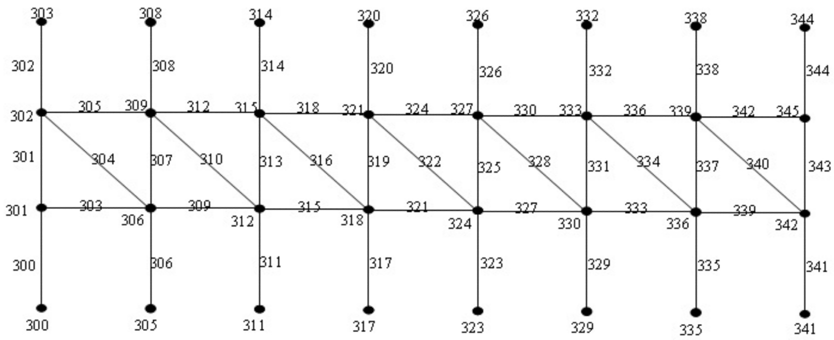
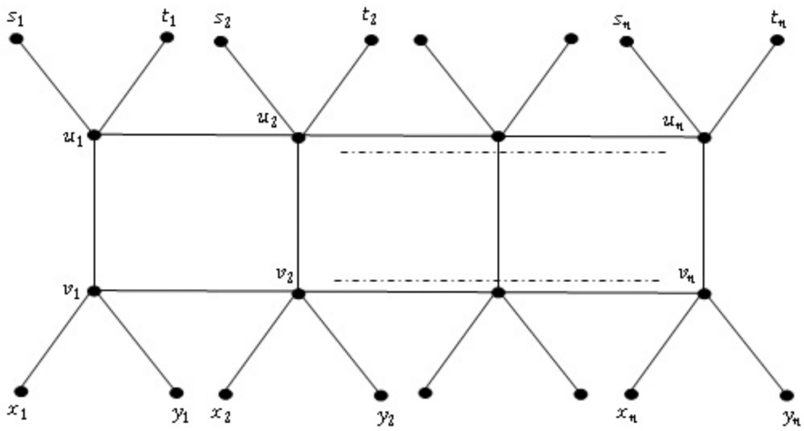


Figure 3. **300 harmonic mean labeling of $TL_8 \odot K_1$.**

Theorem 2.7. *$L_n \odot \overline{k_2} (n \geq 2)$ is k - harmonic mean graph for all $k \geq 1$.*

Proof. Let $V(L_n \odot \overline{k_2}) = \{u_i, v_i, s_i, t_i, x_i, y_i; 1 \leq i \leq n\}$; $E(L_n \odot \overline{k_2}) = \{u_iu_{i+1}, v_iv_{i+1}; 1 \leq i \leq n - 1, s_iu_i, t_iu_i, u_iv_i, v_ix_i, v_iy_i; 1 \leq i \leq n\}$ be denoted as in the following figure



First we label the vertices as follows. Define a function $f : V(L_n \odot \overline{k_2}) \to \{k, k+1, k+2, ..., k+q\}$ by

$$f(u_i) = k + 7i - 6, \qquad for \;\; 1 \leq i \leq n$$

$$f(v_1) = k + 4$$

$$f(v_2) = k + 9$$

$$f(v_3) = k + 18$$

$$f(v_i) = k + 7i - 5, \qquad for \;\; 4 \leq i \leq n$$

$$f(x_i) = k + 7i - 4, \qquad for \;\; 1 \leq i \leq n$$

$$f(y_1) = k + 5$$

$$f(y_2) = k + 16$$

$$f(y_i) = k + 7i - 2, \qquad for \;\; 3 \leq i \leq n$$

$$f(s_1) = k$$

$$f(s_i) = k + 7i - 8, \qquad for \;\; 2 \leq i \leq n$$

$$f(t_1) = k + 2$$

$$f(t_i) = k + 7i - 7, \;\; for \;\; 2 \leq i \leq n$$

Then the induced edge labels are

$$f^*(u_1u_2) = \begin{cases} k+3 \; if \; k \le 3 \\ k+4 \; if \; k \ge 4 \end{cases}$$

$$f^*(u_2u_3) = k + 11$$

$$f^*(u_3u_4) = k + 18$$

$$f^*(u_iu_{i+1}) = k + 7i - 2, \qquad for \;\; 4 \le i \le n-1$$

$$f^*(v_iv_{i+1}) = k + 7i - 1, \qquad for \;\; 1 \le i \le n-1$$

$$f^*(v_1x_1) = \begin{cases} k+4 \; if \; k \le 3 \\ k+3 \; if \; k \ge 4 \end{cases}$$

$$f^*(v_ix_i) = k + 7i - 4, \qquad for \;\; 2 \le i \le n$$

$$f^*(v_1y_1) = k + 5$$

$$f^*(v_2y_2) = k + 12$$

$$f^*(v_3y_3) = k + 19$$

$$f^*(v_iy_i) = k + 7i - 3, \qquad for \;\; 4 \le i \le n$$

$$f^*(u_iv_i) = k + 7i - 5, \qquad for \;\; 1 \le i \le n$$

$$f^*(u_is_i) = k + 7i - 7, \qquad for \;\; 1 \le i \le n$$

$$f^*(u_it_i) = k + 7i - 6, \qquad for \;\; 1 \le i \le n$$

The above defined function f provides k-harmonic mean labeling of the graph. Hence $L_n \odot \overline{k_2}, (n \ge 2)$ is k - harmonic mean graph for all $k \ge 1$. $\square$

Example 2.8. *800 harmonic mean labeling of* $L_6 \odot \overline{k_2}$ *is as follows:*

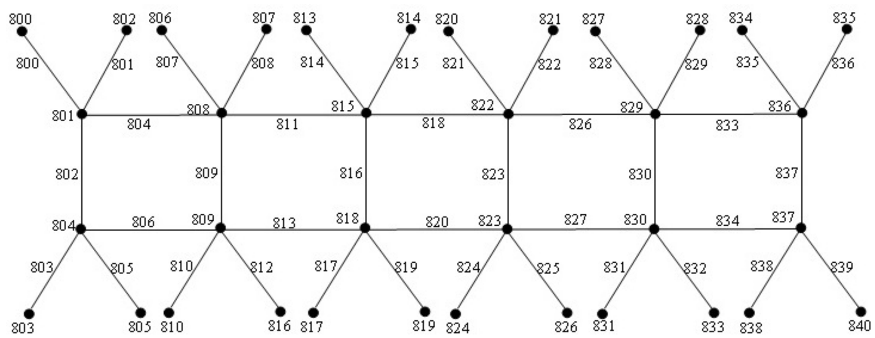
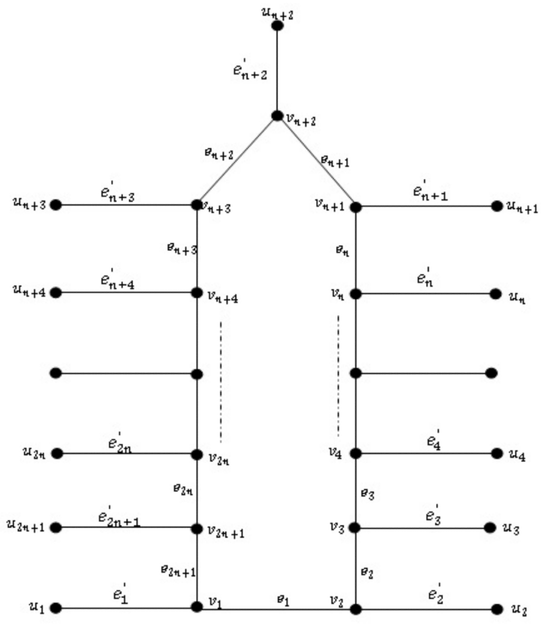


Figure 4. **800 harmonic mean labeling of** $L_6 \odot \overline{k_2}$.

Theorem 2.9. $C_{2n+1} \odot K_1, (n \ge 1)$ *is k - harmonic mean graph for all* $k \ge 1$.

Proof. Let $V(C_{2n+1} \odot K_1) = \{v_i, u_i; 1 \le i \le 2n+1\}$; $E(C_{2n+1} \odot K_1) = \{e_i = (v_iv_{i+1}); 1 \le i \le 2n\} \cup e_{2n+1} = (v_{2n+1}, v_1) \cup \{e'_i = (v_i, u_i); 1 \le i \le 2n+1\}$ be denoted as in the following figure



First we label the vertices as follows. Define a function $f : V(C_{2n+1} \odot K_1) \to \{k, k+1, k+2, ..., k+q\}$ by

$$f(v_1) = k+1$$
$$f(v_i) = k+4i-6, \quad for \;\; 2 \le i \le n+2$$
$$f(v_{n+i}) = k+4n-4i+9, \quad for \;\; 3 \le i \le n+1$$
$$f(u_1) = k$$
$$f(u_i) = k+4i-5, \quad for \;\; 2 \le i \le n+1$$
$$f(u_{n+2}) = k+4n+1$$
$$f(u_{n+i}) = k+4n-4i+8, \quad for \;\; 3 \le i \le n+1$$

Then the induced edge labels are

$$f^*(e_1) = k+1$$
$$f^*(e_i) = k+4i-4, \quad for \;\; 2 \le i \le n+1$$
$$f^*(e_{n+i}) = k+4n-4i+7, \quad for \;\; 2 \le i \le n$$
$$f^*(u_iv_i) = k+5i-4, \quad for \;\; 1 \le i \le n-1$$
$$f^*(u_{i+1}v_i) = k+5i-1, \quad for \;\; 1 \le i \le n-1$$
$$f^*({e_1}') = k$$
$$f^*({e_2}') = \begin{cases} k+3 \; if \; k=1 \\ k+2 \; if \; k \ge 2 \end{cases}$$
$$f^*({e_i}') = k+4i-6, \quad for \;\; 3 \le i \le n+1$$
$$f^*({e_{n+i}}') = k+4n-4i+9, \quad for \;\; 2 \le i \le 2n+1$$

The above defined function f provides k-harmonic mean labeling of the graph. Hence $C_{2n+1} \odot K_1, (n \ge 1)$ is k - harmonic mean graph for all $k \ge 1$. □

Example 2.10. *200 harmonic mean labeling of $C_{19} \odot K_1$ is as follows:*

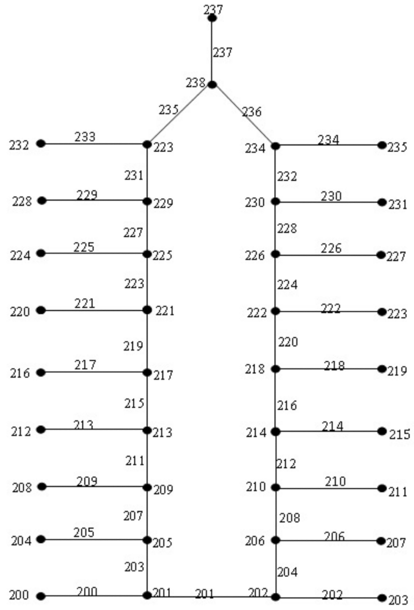


Figure 5. **200 harmonic mean labeling of $C_{19} \odot K_1$.**

Theorem 2.11. *$C_{2n} \odot K_1, (n \geq 2)$ is k - harmonic mean graph for all $k \geq 1$.*

Proof. Let $V(C_{2n} \odot K_1) = \{v_i, u_i; 1 \leq i \leq 2n\}$; $E(C_{2n} \odot K_1) = \{e_i = (v_i v_{i+1}); 1 \leq i \leq 2n-1\} \cup e_{2n} = (v_{2n}, v_1) \cup \{e_i{}' = (v_i, u_i); 1 \leq i \leq 2n\}$ be denoted as in the following figure

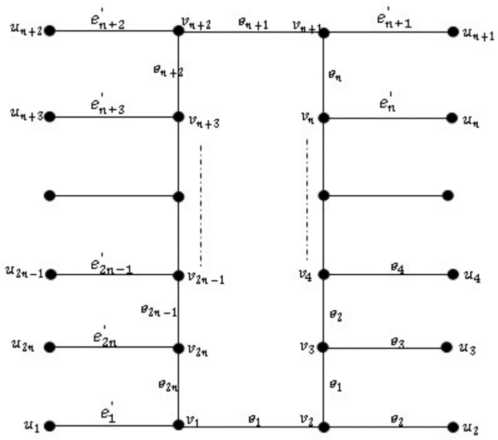


Figure 6. **200 harmonic mean labeling of $C_{19} \odot K_1$.**

First we label the vertices as follows. Define a function $f : V(C_{2n} \odot K_1) \to \{k, k+1, k+2, ..., k+q\}$ by

$$f(v_1) = k + 1$$

$$f(v_i) = k + 4i - 6, \qquad for \;\; 2 \leq i \leq n+1$$

$$f(v_{n+i}) = k + 4n - 4i + 5, \;\; for \;\; 2 \leq i \leq n$$

$$f(u_1) = k$$

$$f(u_i) = k + 4i - 5, \qquad for \;\; 2 \leq i \leq n+1$$

$$f(u_{n+2}) = k + 4n + 1$$
$$f(u_{n+i}) = k + 4n - 4i + 4, \;\; for \;\; 3 \leq i \leq n$$

Then the induced edge labels are

$$f^*(e_1) = k + 1$$
$$f^*(e_i) = k + 4i - 4, \qquad for \;\; 2 \leq i \leq n$$
$$f^*(e_{n+1}) = k + 4n - 3$$
$$f^*(e_{n+i}) = k + 4n - 4i + 3, \quad for \;\; 2 \leq i \leq n - 1$$
$$f^*(e_1{}') = k$$
$$f^*(e_2{}') = \begin{cases} k + 3 \; if \; k = 1 \\ k + 2 \; if \; k \geq 2 \end{cases}$$
$$f^*(e_i{}') = k + 4i - 6, \qquad for \;\; 3 \leq i \leq n$$
$$f^*(e_{n+1}{}') = k + 4n - 1$$
$$f^*(e_{n+2}{}') = k + 4n - 2$$
$$f^*(e_{n+i}{}') = k + 4n - 4i + 5, \quad for \;\; 3 \leq i \leq n - 1$$
$$f^*(e_2{}') = \begin{cases} k + 2 \; if \; k = 1 \\ k + 3 \; if \; k \geq 2 \end{cases}$$

The above defined function f provides k-harmonic mean labeling of the graph. Hence $C_{2n} \odot K_1, (n \geq 2)$ is k - harmonic mean graph for all $k \geq 1$. □

Example 2.12. *100 harmonic mean labeling of $C_{16} \odot K_1$ is as follows:*

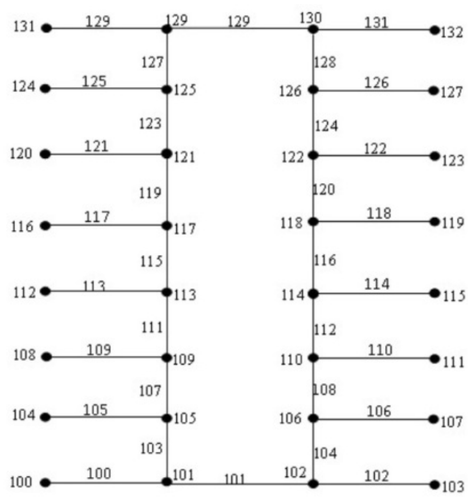


Figure 7. **100 harmonic mean labeling of $C_{16} \odot K_1$.**

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