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On $\mathcal{NI}_{\widehat{g}}$-Closed Sets in Nano Ideal Topological Spaces

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Abstract: This paper focuses on $\mathcal{NI}_{\widehat{g}}$-Closed sets (nano $\mathcal{I}_{\widehat{g}}$-closed sets) and $\mathcal{NI}_{\widehat{g}}$-open sets (nano $\mathcal{I}_{\widehat{g}}$-closed sets) in nano ideal topological spaces and certain properties of these are investigated. We also investigate the concept of nano $\mathcal{I}_{\widehat{g}}$-closed sets and discussed their relationships with other forms of nano ideal sets. Further, we have given an appropriate examples to understand the abstract concepts clearly.

MSC: 54A05.

Keywords: $\mathcal{N}g$-closed sets, $\mathcal{N}\widehat{g}$-closed sets, $\mathcal{NI}_g$-closed sets, $\mathcal{NI}_{\widehat{g}}$-closed sets and $\mathcal{NI}_{\widehat{g}}$-open sets.

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1. Introduction

In 1970, Levine [11] introduced the concept of generalized closed sets in topological spaces. This concept was found to be useful to develop many results in general topology. In 1991, Balachandran et.al [2] introduced and investigated the notion of generalized continuous functions in topological spaces. In 2003, Veerakumar [13] introduced $\widehat{g}$-closed sets in topological spaces. The concept of ideal topological space was introduced by kuratowski [6]. Also he defined the local functions in ideal topological spaces. Further, Jankovic and Hamlett [5] investigated further properties of ideal topological spaces. In 2011, Ravi [1] introduced $\mathcal{I}_{\widehat{g}}$-closed sets in ideal topological spaces. The notion of nano topology was introduced by Lellis Thivagar [8, 9] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analyzed the nano forms of weakly open sets such as nano α-open sets, nano semi-open sets and nano pre-open sets. Bhuvaneswari [3], introduced and studied the concept of Nano generalized-closed sets. Lellis Thivagar et.al [10] defined nano ideal topological spaces. The structure of this manuscript is as follows. In section 2, we recall some fundamental definitions and results which are useful to prove our main results. In section 3, we define and study the notion of $\mathcal{NI}_{\widehat{g}}$-closed sets and $\mathcal{NI}_{\widehat{g}}$-open sets in nano ideal topological spaces. We also discuss the concept of $\mathcal{NI}_{\widehat{g}}$-closed sets and discussed the relationships between the other existing nano ideal sets.

2. Preliminaries

Definition 2.1 ([8])**.** *Let U be a non-empty finite set of objects called the universe R be an equivalence relation on U named as the indiscerniblity relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

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(1). The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}\}$, where $R(x)$ denotes the equivalence class determined by x.

(2). The Upper approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}\}$

(3). The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X with respect to R and it is denoted by$B_R(X)$. That is, $B_R(X) = U_R(X)$-$L_R(X)$

Definition 2.2 ([8]). *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:*

(1). U and $\phi \in \tau_R(X)$

(2). The union of elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

(3). The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call $\{U,\tau_R(X)\}$ is called the nano topological space.

Definition 2.3 ([4]). *An ideal $\mathcal{I}$ on a topological space is a non-empty collection of subsets of X which satisfies*

(1). $A \in \mathcal{I}$ and $B \subseteq A \Rightarrow B \in \mathcal{I}$.

(2). $A \in \mathcal{I}$ and $B \in \mathcal{I} \Rightarrow A \cup B \in \mathcal{I}$.

Definition 2.4. *A nano topological space $\{U, \tau_R(X)\}$ with an ideal $\mathcal{I}$ on U is called a nano ideal topological space or nano ideal space and denoted as $\{U, \tau_R(X), \mathcal{I}\}$.*

Definition 2.5. *Let $\{U, \tau_R(X), \mathcal{I}\}$ be a nano ideal topological space. A set operator $(A)^{*\mathcal{N}} : P(U) \to P(U)$ is called the nano local function of $\mathcal{I}$ on U with respect to $\mathcal{I}$ on $\tau_R(X)$ is defined as $(A)^{*\mathcal{N}} = \{x \in U : U \cap A \notin \mathcal{I}$; for every $U \in \tau_R(X)\}$ and is denoted by $(A)^{*\mathcal{N}}$, where nano closure operator is defined as $\mathcal{N}cl^*(A) = A \cup (A)^{*\mathcal{N}}$.*

Result 2.6 ([10]). *Let $\{U, \tau_R(X), \mathcal{I}\}$ be a nano ideal topological space and let A and B be subsets of U, then*

(1). $(\phi)^{*\mathcal{N}} = \phi$.

(2). $A \subset B \to (A)^{*\mathcal{N}} \subset (B)^{*\mathcal{N}}$.

(3). For another $J \supseteq \mathcal{I}$ on U, $(A)^{*\mathcal{N}}(J) \subset (A)^{*\mathcal{N}}(\mathcal{I})$.

(4). $(A)^{*\mathcal{N}} \subset \mathcal{N}cl^*(A)$.

(5). $(A)^{*\mathcal{N}}$ is a nano closed set.

(6). $((A)^{*\mathcal{N}})^{*\mathcal{N}} \subset (A)^{*\mathcal{N}}$.

(7). $(A)^{*\mathcal{N}} \cup (B)^{*\mathcal{N}} = (A \cup B)^{*\mathcal{N}}$.

(8). $(A \cap B)^{*\mathcal{N}} = (A)^{*\mathcal{N}} \cap (B)^{*\mathcal{N}}$.

(9). For every nano open set V, $V \cap (V \cap A)^{*\mathcal{N}} \subset (V \cap A)^{*\mathcal{N}}$.

(10). For $\mathcal{I} \in \mathcal{I}$, $(A \cup \mathcal{I})^{*\mathcal{N}} = (A)^{*\mathcal{N}} = (A - \mathcal{I})^{*\mathcal{N}}$.

Result 2.7 ([10]). *Let $\{U, \tau_R(X), \mathcal{I}\}$ be a nano ideal topological space and A be a subset of U, If $A \subset (A)^{*\mathcal{N}}$, then $(A)^{*\mathcal{N}} = \mathcal{N}cl(A)^{*\mathcal{N}} = \mathcal{N}cl(A) = \mathcal{N}cl^*(A)$.*

Definition 2.8. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be*

(1). Nano semi-closed [8], if $\mathcal{N}cl(\mathcal{N}int(A)) \subseteq A$.

(2). $\mathcal{N}g$-closed [3], if $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open.

(3). $\mathcal{N}\widehat{g}$-closed [7], if $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open.

Definition 2.9. *A subset A of a nano ideal space. Let $(U, \tau_R(X), \mathcal{I})$ is said to be*

(1). $*^{\mathcal{N}}$-closed [12], if $(A)^{*\mathcal{N}} \subseteq A$.

(2). $*^{\mathcal{N}}$-dense [12], if $A \subseteq (A)^{*\mathcal{N}}$.

(3). $\mathcal{NI}_g$-closed [12], if $(A)^{*\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano open.

3. $\mathcal{NI}_{\widehat{g}}$-Closed Sets

In this section we define and study the notion of $\mathcal{NI}_{\widehat{g}}$-closed sets and $\mathcal{NI}_{\widehat{g}}$-open sets in nano ideal topological spaces.

Definition 3.1. *A subset A of a nano ideal space $(U, \tau_R(X), \mathcal{I})$ is said to be $\mathcal{NI}_{\widehat{g}}$-closed if $(A)^{*\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open.*

Corollary 3.2. *A subset A of a nano ideal space $(U, \tau_R(X), \mathcal{I})$ is said to be $\mathcal{NI}_{\widehat{g}}$- open if $U - A$ is $\mathcal{NI}_{\widehat{g}-}$closed.*

Theorem 3.3. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space, then every $\mathcal{NI}_{\widehat{g}}$-closed set is $\mathcal{NI}_g$-closed but not conversely.*

Example 3.4. *Let $U = \{a, b, c, d\}$, $U \backslash R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Let the nano ideal space $\tau_R(X) = \{U, \phi, \{a, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{a\}\}$. Then $\mathcal{NI}_{\widehat{g}}$ closed sets are $\{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\mathcal{NI}_g$-closed sets $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. It is clear that $\{b\}$ is $\mathcal{NI}_g$-closed but it is not in $\mathcal{NI}_{\widehat{g}}$-closed.*

Theorem 3.5. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space and $A \subseteq U$, then the following are equivalent.*

(1). A is $\mathcal{NI}_{\widehat{g}}$-closed.

(2). $\mathcal{N}cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open in U

(3). For all $x \in \mathcal{N}cl^*(A)$, $\mathcal{N}scl(\{x\}) \cap A \neq \phi$.

(4). $\mathcal{N}cl^*(A) - A$ contains no nonempty nano semi-closed set.

Proof. (1) $\Rightarrow$ (2) If A is $\mathcal{NI}_{\widehat{g}}$-closed, then $(A)^{*\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open in U and so $\mathcal{N}cl^*(A) = A \cup (A)^{*\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open in U. This proves (2).
(2) $\Rightarrow$ (3) Suppose $x \in \mathcal{N}cl^*(A)$. If $\mathcal{N}scl(\{x\}) \cap A = \phi$, then $A \subseteq U - \mathcal{N}scl(\{x\})$. By (2), $\mathcal{N}cl^*(A) \subseteq U - \mathcal{N}scl(\{x\})$, a contradiction, since $x \in \mathcal{N}cl^*(A)$.
(3) $\Rightarrow$ (4) Suppose $F \subseteq \mathcal{N}cl^*(A) - A$, F is nano semi-closed and $x \in F$. Since $F \subseteq U - A$ and F is nano semi-closed,

then $A \subseteq U - F$ and F is nano semi-closed, $\mathcal{N}scl(\{x\}) \cap A = \phi$. Since $x \in \mathcal{N}cl^*(A)$ by (3), $\mathcal{N}scl(\{x\}) \cap A \neq \phi$. Therefore $\mathcal{N}cl^*(A) - A$ contains no nonempty nano semi-closed set.

(4) $\Rightarrow$ (1) Let $A \subseteq G$ where G is nano semi-open set. Therefore $U-G \subseteq U-A$ and so $\mathcal{N}cl^*(A)\cap(U-G) \subseteq \mathcal{N}cl^*(A)\cap(U-A) = \mathcal{N}cl^*(A) - A$. Therefore $\mathcal{N}cl^*(A) \cap (U - G) \subseteq \mathcal{N}cl^*(A) - A$. Since $\mathcal{N}cl^*(A)$ is always nano closed set, so $\mathcal{N}cl^*(A) \cap (U - G)$ is a nano semi-closed set contained in $\mathcal{N}cl^*(A) - A$. Therefore $(A)^{*\mathcal{N}} \cap (U - G) = \phi$ and hence $(A)^{*\mathcal{N}} \subseteq G$. Therefore A is $\mathcal{NI}_{\widehat{g}}$-closed. □

Theorem 3.6. *Every $*^{\mathcal{N}}$ closed set is $\mathcal{NI}_{\widehat{g}}$-closed but not conversely.*

Proof. Let A be a $*^{\mathcal{N}}$-closed, then $(A)^{*\mathcal{N}} \subseteq A$. Let $A \subset G$ where G is nano semi-open. Hence $(A)^{*\mathcal{N}} \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open. Therefore A is $\mathcal{NI}_{\widehat{g}}$-closed. □

Example 3.7. *Let $U = \{a, b, c, d\}$, $U \backslash R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Let the nano ideal space $\tau_R(X)=\{U, \phi, \{a\}, \{a, b, d\}, \{b, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{a\}, \{a, b, d\}\}$. Then $\mathcal{NI}_{\widehat{g}}$-closed sets are $\{U, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ and $*^{\mathcal{N}}$-closed sets are $\{U, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}\}$. It is clear that $\{b, c\}$ is $\mathcal{NI}_{\widehat{g}}$-closed set but it is not in $*^{\mathcal{N}}$-closed.*

Theorem 3.8. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. For every $A \in \mathcal{I}$, A is $\mathcal{NI}_{\widehat{g}}$-closed.*

Proof. Let $A \subseteq G$ where G is nano semi-open set. Since $(A)^{*\mathcal{N}} = \phi$ for every $A \in \mathcal{I}$, then $\mathcal{N}cl^*(A) = A \cup (A)^{*\mathcal{N}} = A \subseteq G$. Therefore, by Theorem 3.5 (4), A is $\mathcal{NI}_{\widehat{g}}$-closed. □

Theorem 3.9. *If $(U, \tau_R(X), \mathcal{I})$ is a nano ideal space, then $(A)^{*\mathcal{N}}$ is always $\mathcal{NI}_{\widehat{g}}$-closed for every subset A of U.*

Proof. Let $(A)^{*\mathcal{N}} \subseteq G$ where G is nano semi-open. Since $((A)^{*\mathcal{N}})^{*\mathcal{N}} \subseteq (A)^{*\mathcal{N}}$, we have $((A)^{*\mathcal{N}})^{*\mathcal{N}} \subseteq G$ whenever $(A)^{*\mathcal{N}} \subseteq G$ and G is nano semi-open. Hence $(A)^{*\mathcal{N}}$ is $\mathcal{NI}_{\widehat{g}}$-closed. □

Corollary 3.10. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and A be a $\mathcal{NI}_{\widehat{g}}$- closed set. Then the following are equivalent.*

(1). A is a $*^{\mathcal{N}}$-closed set.

(2). $\mathcal{N}cl^*(A) - A$ is a nano semi-closed set.

Proof. (1) $\Rightarrow$ (2) If A is $*^{\mathcal{N}}$ - closed, then $(A)^{*\mathcal{N}} \subseteq A$ and so $\mathcal{N}cl^*(A) - A = (A \cup (A)^{*\mathcal{N}}) - A = \phi$. Hence $\mathcal{N}cl^*(A) - A$ is a nano semi - closed set.

(2) $\Rightarrow$ (1) If $\mathcal{N}cl^*(A) - A$ is a nano semi - closed set, since A is $\mathcal{NI}_{\widehat{g}}$ - closed set, by Theorem 3.5(4), $\mathcal{N}cl^*(A) - A = \phi$ and so A is $*^{\mathcal{N}}$ - closed. □

Theorem 3.11. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. Then every $\mathcal{N}\widehat{g}$-closed set is a $\mathcal{NI}_{\widehat{g}}$-closed set but not conversely.*

Proof. Let A be a $\mathcal{N}\widehat{g}$-closed set. Then $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open. We have $\mathcal{N}cl^*(A) \subseteq \mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open. Hence A is $\mathcal{NI}_{\widehat{g}}$ -closed. □

Example 3.12. *Let $U = \{a, b, c, d\}, U \backslash R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Let the nano ideal space $\tau_R(X) = \{U, \phi, \{a, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{a\}\}$. Then $\mathcal{NI}_{\widehat{g}}$- closed sets are $\{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\mathcal{N}\widehat{g}$-closed sets are $\{U, \phi, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$. It is clear that $\{a\}$ is $\mathcal{NI}_{\widehat{g}}$-closed set but it is not in $\mathcal{N}\widehat{g}$-closed.*

Theorem 3.13. *If $(U, \tau_R(X), \mathcal{I})$ is a nano ideal space and A is a $*^{\mathcal{N}}$-dense in itself, $\mathcal{NI}_{\widehat{g}}$-closed subset of U then A is $\mathcal{N}\widehat{g}$-closed.*

Proof. Suppose A is a $*^{\mathcal{N}}$-dense in itself, $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed subset of U. Let $A \subseteq G$ where G is nano semi-open. Then by Theorem 3.5 (2), $\mathcal{N}cl^*(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open. Since A is $*^{\mathcal{N}}$-dense in itself, by Result 2.7, $\mathcal{N}cl(A) = \mathcal{N}cl^*(A)$. Therefore $\mathcal{N}cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi-open. Hence A is $\mathcal{N}\widehat{g}$-closed. □

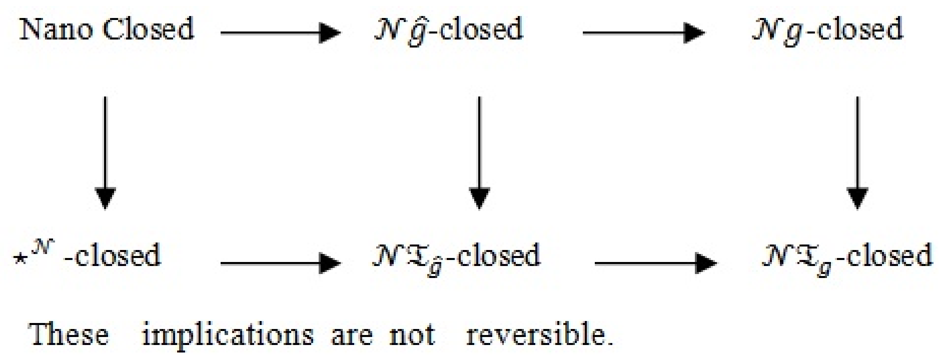
Corollary 3.14. *If $(U, \tau_R(X), \mathcal{I})$ is any nano ideal space where $\mathcal{I} = \{\phi\}$ then A is $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed if and only if A is $\mathcal{N}\widehat{g}$-closed.*

Proof. From the fact that for $\mathcal{I} = \{\phi\}$, $(A)^{*\mathcal{N}} = \mathcal{N}cl(A) \supseteq A$. Therefore A is $*^{\mathcal{N}}$-dense in itself. Since A is $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed, by Theorem 3.13, A is $\mathcal{N}\widehat{g}$-closed. Conversely, by Theorem 3.11, every $\mathcal{N}\widehat{g}$-closed set is $\mathcal{N}\mathcal{I}_{\widehat{g}}$ -closed set. □

Example 3.15. *Let $U = \{a, b, c, d\}$, $U \backslash R = \{\{a\}, \{d\}, \{b, c\}\}$ and $X = \{a, d\}$. Then in the nano ideal space $\tau_R(X) = \{U, \phi, \{a, d\}\}$ with a nano ideal $\mathcal{I} = \{\phi, \{a\}\}$. Then $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed sets are $\{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ and $\mathcal{N}g$-closed sets are $\{U, \phi, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \{b, c, d\}\}$. It is clear that $\{b\}$ is $\mathcal{N}g$-closed set but it is not in $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed and $\{a\}$ is a $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed set but it is not in $\mathcal{N}g$-closed.*

Remark 3.16. *From the above Example, $\mathcal{N}g$-closed sets and $\mathcal{N}\mathcal{I}_{\widehat{g}}$ -closed sets are independent of each other.*

Remark 3.17. *The following figure shows that the relationship of $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed sets with some of the existing sets, which we have discussed in this section*



Theorem 3.18. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. Then A is $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed if and only if $A = F - N$ where F is $*^{\mathcal{N}}$-closed and N contains no nonempty nano semi-closed set.*

Proof. If A is $\mathcal{N}\mathcal{I}_{\widehat{g}}$-closed, then by Theorem 3.5(4) and Result 2.7, $N = (A)^{*\mathcal{N}} - A$ contains no nonempty nano semi-closed set. If $F = \mathcal{N}cl^*(A)$, then F is $*^{\mathcal{N}}$-closed such that $F - N = (A \cup (A)^{*\mathcal{N}}) - ((A)^{*\mathcal{N}} - A) = (A \cup (A)^{*\mathcal{N}}) \cap ((A)^{*\mathcal{N}} \cap A^c)^c = (A \cup (A)^{*\mathcal{N}}) \cap (((A)^{*\mathcal{N}})^c \cup A) = (A \cup (A)^{*\mathcal{N}}) \cap (A \cup ((A)^{*\mathcal{N}})^c) = A \cup ((A)^{*\mathcal{N}} \cap ((A)^{*\mathcal{N}})^c) = A$.
Conversely, suppose $A = F - N$ where F is $*^{\mathcal{N}}$-closed and N contains no nonempty nano semi-closed set. Let G be a nano semi-open set such that $A \subseteq G$. Then $F - N \subseteq G \Rightarrow F \cap (U - G) \subseteq N$. Now $A \subseteq F$ and $(F)^{*\mathcal{N}} \subseteq F$ then $(A)^{*\mathcal{N}} \subseteq (F)^{*\mathcal{N}}$ and so $(A)^{*\mathcal{N}} \cap (U - G) \subseteq (F)^{*\mathcal{N}} \cap (U - G) \subseteq F \cap (U - G) \subseteq N$. By hypothesis, since $(A)^{*\mathcal{N}} \cap (U - G)$ is nano semi-closed, $(A)^{*\mathcal{N}} \cap (U - G) = \phi$ and so $(A)^{*\mathcal{N}} \subseteq G$. Hence A is $\mathcal{N}\mathcal{I}_{\widehat{g}}$ -closed. □

Theorem 3.19. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. If $A \subseteq B \subseteq (A)^{*\mathcal{N}}$ then $(A)^{*\mathcal{N}} = (B)^{*\mathcal{N}}$ and B is $*^{\mathcal{N}}$-dense in itself.*

Proof. Since $A \subseteq B$, then $(A)^{*\mathcal{N}} \subseteq (B)^{*\mathcal{N}}$ and since $B \subseteq (A)^{*\mathcal{N}}$, then $(B)^{*\mathcal{N}} \subseteq ((A)^{*\mathcal{N}})^{*\mathcal{N}} \subseteq (A)^{*\mathcal{N}}$. Therefore $(A)^{*\mathcal{N}} = (B)^{*\mathcal{N}}$ and $B \subseteq (A)^{*\mathcal{N}} \subseteq (B)^{*\mathcal{N}}$. □

Theorem 3.20. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. If A and B are subsets of U such that $A \subseteq B \subseteq \mathcal{N}cl^*(A)$ and A is $\mathcal{N}\mathcal{I}_{\widehat{g}}$ -closed then B is $\mathcal{N}\mathcal{I}_{\widehat{g}}$ -closed.*

Proof. Since A is $\mathcal{NI}_{\widehat{g}}$ -closed then by Theorem 3.5 (4), $\mathcal{N}cl^*(A) - A$ contains no nonempty nano semi-closed set. Since $\mathcal{N}cl^*(B) - B \subseteq \mathcal{N}cl^*(A) - A$ and so $\mathcal{N}cl^*(B) - B$ contains no nonempty nano semi-closed set. Hence B is $\mathcal{NI}_{\widehat{g}}$ -closed. □

Corollary 3.21. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. If A and B are subsets of U such that $A \subseteq B \subseteq (A)^{*\mathcal{N}}$ and A is $\mathcal{NI}_{\widehat{g}}$ -closed then A and B are $\mathcal{N}\widehat{g}$-closed sets.*

Proof. Let A and B be subsets of U such that $A \subseteq B \subseteq (A)^{*\mathcal{N}} \Rightarrow A \subseteq B \subseteq (A)^{*\mathcal{N}} \subseteq \mathcal{N}cl^*(A)$ and A is $\mathcal{NI}_{\widehat{g}}$-closed. By the above Theorem, B is $\mathcal{NI}_{\widehat{g}}$-closed. Since $A \subseteq B \subseteq (A)^{*\mathcal{N}}$, then $(A)^{*\mathcal{N}} = (B)^{*\mathcal{N}}$ and so A and B are $*^{\mathcal{N}}$-dense in itself. By Theorem 3.13, A and B are $\mathcal{N}\widehat{g}$-closed. □

Theorem 3.22. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. Then A is $\mathcal{NI}_{\widehat{g}}$-open if and only if $F \subseteq \mathcal{N}int^*(A)$ whenever F is nano semi-closed and $F \subseteq A$.*

Proof. Suppose A is $\mathcal{NI}_{\widehat{g}}$-open. If F is nano semi-closed and $F \subseteq A$, then $U - A \subseteq U - F$ and so $\mathcal{N}cl^*(U - A) \subseteq U - F$ by Theorem 3.5(2). Therefore $F \subseteq U - \mathcal{N}cl^*(U - A) = \mathcal{N}int^*(A)$. Hence $F \subseteq \mathcal{N}int^*(A)$.
Conversely, suppose the condition holds. Let G be a nano semi-open set such that $U - A \subseteq G$. Then $U - G \subseteq A$ and so $U - G \subseteq \mathcal{N}int^*(A)$. Therefore $\mathcal{N}cl^*(U - A) \subseteq G$. By Theorem 3.5 (2), $U - A$ is $\mathcal{NI}_{\widehat{g}}$-closed. Hence A is $\mathcal{NI}_{\widehat{g}}$-open. □

Corollary 3.23. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. If A is $\mathcal{NI}_{\widehat{g}}$-open then $F \subseteq \mathcal{N}int^*(A)$ whenever F is closed and $F \subseteq A$.*

Theorem 3.24. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. If A is $\mathcal{NI}_{\widehat{g}}$-open and $\mathcal{N}int^*(A) \subseteq B \subseteq A$, then B is $\mathcal{NI}_{\widehat{g}}$-open.*

Proof. Since A is $\mathcal{NI}_{\widehat{g}}$-open, then $U - A$ is $\mathcal{NI}_{\widehat{g}}$-closed. By Theorem 3.5(4), $\mathcal{N}cl^*(U - A) - (U - A)$ contains nononempty nano semi-closed set. Since $\mathcal{N}int^*(A) \subseteq \mathcal{N}int^*(B)$ which implies that $\mathcal{N}cl^*(U - B) \subseteq \mathcal{N}cl^*(U - A)$ and so $\mathcal{N}cl^*(U - B) - (U - B) \subseteq \mathcal{N}cl^*(U - A) - (U - A)$. Hence B is $\mathcal{NI}_{\widehat{g}}$-open. □

Theorem 3.25. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space and $A \subseteq U$. Then following are equivalent.*

(1). A is $\mathcal{NI}_{\widehat{g}}$ -closed.

(2). $A \cup (U - (A)^{*N})$ is $\mathcal{NI}_{\widehat{g}}$-closed.

(3). $(A)^{*\mathcal{N}} - A$ is $\mathcal{NI}_{\widehat{g}}$ -open.

Proof. (1) $\Rightarrow$ (2) Suppose A is $\mathcal{NI}_{\widehat{g}}$-closed. If G is any nano semi-open set such that $A \cup (U - (A)^{*\mathcal{N}}) \subseteq G$, then $U - G \subseteq U - (A \cup (U - (A)^{*\mathcal{N}})) = U \cap (A((A)^{*N})^c)^c = (A)^{*\mathcal{N}} \cap A^c = (A)^{*\mathcal{N}} - A$. Since A is $\mathcal{NI}_{\widehat{g}}$-closed, by Result 2.7 and Theorem 3.5 (4), it follows that $U - G = \phi$ and so $U = G$. Therefore $A \cup (U - (A)^{*\mathcal{N}}) \subseteq G \Rightarrow A \cup (U - (A)^{*\mathcal{N}}) \subseteq U$ and so $(A \cup (U - (A)^{*\mathcal{N}}))^{*\mathcal{N}} \subseteq U^* \subseteq U = G$. Hence $A \cup (U - (A)^{*\mathcal{N}})$ is $\mathcal{NI}_{\widehat{g}}$-closed.
(2) $\Rightarrow$ (1) Suppose $A \cup (U - (A)^{*\mathcal{N}})$ is $\mathcal{NI}_{\widehat{g}}$-closed. If F is any nano semi-closed set such that $F \subseteq (A)^{*\mathcal{N}} - A$, then $F \subseteq (A)^{*\mathcal{N}}$ and $F \nsubseteq A \Rightarrow U - (A)^{*\mathcal{N}} \subseteq U - F$ and $A \subseteq U - F$. Therefore $A \cup (U - (A)^{*\mathcal{N}}) \subseteq A \cup (U - F) = U - F$ and $U - F$ is nano semi-open. Since $(A \cup (U - (A)^{*\mathcal{N}}))^{*\mathcal{N}} \subseteq U - F \Rightarrow (A)^{*\mathcal{N}} \cup (U - (A)^{*\mathcal{N}})^{*\mathcal{N}} \subseteq U - F$ and so $(A)^{*\mathcal{N}} \subseteq U - F \Rightarrow F \subseteq U - (A)^{*\mathcal{N}}$. Since $F \subseteq (A)^{*\mathcal{N}}$, it follows that $F = \phi$. Hence A is $\mathcal{NI}_{\widehat{g}}$-closed.
(2) $\Rightarrow$ (3) Since $U - ((A)^{*\mathcal{N}} - A) = U \cap ((A)^{*\mathcal{N}} \cap A^c)^c = U \cap ((A)^{*\mathcal{N}})^c \cup A) = (U \cap ((A)^{*\mathcal{N}})^c) \cup (U \cap A) = A \cup (U - (A)^{*\mathcal{N}})$. □

Theorem 3.26. *Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal space. Then every subset of U is $\mathcal{NI}_{\widehat{g}}$-closed if and only if every nano semi-open set is $*^{\mathcal{N}}$-closed.*

Proof. Suppose every subset of U is $\mathcal{NI}_{\widehat{g}}$-closed. If $G \subseteq U$ is nano semi-open then G is $\mathcal{NI}_{\widehat{g}}$-closed and so $(G)^{*\mathcal{N}} \subseteq G$. Hence G is $*^{\mathcal{N}}$-closed. Conversely, suppose that every nano semi-open set is $*^{\mathcal{N}}$-closed. If G is nano semi-open set such that $A \subseteq G \subseteq U$ then $(A)^{*\mathcal{N}} \subseteq (G)^{*\mathcal{N}} \subseteq G$ and so A is $\mathcal{NI}_{\widehat{g}}$-closed. □

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