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On πg^*s-Irresolute Functions in Topological Spaces

P. Sathishmohan[1], V. Rajendran[1] and P. Jeevitha[1,*]

1 Department of Mathematics, Kongunadu Arts and Science College, Coimbatore, Tamil Nadu, India.

Abstract: The purpose of this paper is to defined and studied the notion of π generalized star semi-closed sets in topological spaces and some of their basic properties are investigated. This new class of sets lies between the class of πg-closed sets and the class of πgs-closed sets. Further the notion of πg^*s-open sets, πg^*s-continuous functions and πg^*s-irresolute functions are defined and the composition of two πg^*s-irresolute functions are discussed. Several examples are provided to illustrate the behaviour of new sets and functions.

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1. Introduction

The study of g-closed sets in a topological space was initiated by Levine [11]. Veerakumar [23] introduced the notion of g*-closed sets. Pushpalatha. P and Anitha. K [20] introduced the concept of g*s-closed sets. Zaitsav [24] introduced and investigated the concepts of π-closed sets. J. Dontchev, T. Noiri [7] obtained the concept of πg-closed sets. Aslim, Guler and Noiri [3] introduced πgs-closed sets. In this paper we study the properties of π generalized star semi-closed sets(briefly πg^*s-closed sets). Moreover in this paper, we defined πg^*s-open sets and obtained some of its properties.

2. Preliminaries

Let us recall the following definitions which we shall require in sequal.

Definition 2.1. *A subset A of a topological space (X, τ) is called*

(1). a pre-open set [14] if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.

(2). a semi-open set [10] if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.

(3). an α-open set[16] if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$.

(4). a semi-pre open set (= β-open) [2] if $A \subseteq cl(int(cl(A)))$ and a semi-pre closed set (= β-closed) if $int(cl(int(A))) \subseteq A$.

(5). a regular open set if $A = int(cl(A))$ [22] and a regular closed set if $A = cl(int(A))$.

* *E-mail: teddyjeevi@gmail.com*

(6). a π-closed set [24] if A is the union of regular closed sets.

The intersection of all semi-closed (resp. pre-closed, semi-pre-closed, regular-closed and α-closed) sets containing a subset A of (X, τ) is called the semi-closure (resp. pre-closure, semi pre-closure, regular-closure and α-closure) of A and is denoted by $scl(A)$ (respectively $pcl(A)$, $spcl(A)$, $rcl(A)$ and $\alpha cl(A)$).

Definition 2.2. *A subset A of a topological space (X, τ) is called*

(1). a generalized semi-closed set (briefly gs-closed) [1] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(2). a semi generalized closed set (briefly sg-closed) [4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X, τ).

(3). a regular generalized closed set (briefly rg-closed) [19] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in (X, τ).

(4). a generalized pre closed set (briefly gp-closed) [13] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(5). a generalized semi-preclosed set (briefly gsp-closed) [6] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(6). an α generalized closed set (briefly αg-closed) [12] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).

(7). a π generalized closed set (briefly πg-closed) [7] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).

(8). a π generalized α closed set (briefly $\pi g\alpha$-closed) [9] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).

(9). a π generalized regular closed set (briefly πgr-closed) [8] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).

(10). a π generalized pre-closed set (briefly πgp-closed) [18] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).

(11). a π generalized semi-closed set (briefly πgs-closed) [3] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).

(12). a π generalized β closed set (briefly $\pi g\beta$-closed) [21] if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is π-open in (X, τ).

(13). a regular weekly generalized closed set (briefly rwg-closed) [15] if $cl(Int((A)) \subseteq U$ whenever $A \subseteq U$ and U is r-open in (X, τ).

Definition 2.3. *A function $f : X \to Y$ from a topological space X into a topological space Y is called*

(1). r-continuous [22] if the inverse image of a closed set in Y is r-closed set in X.

(2). gs-continuous [5] if the inverse image of a closed set in Y is gs-closed set in X.

(3). gp-continuous [17] if the inverse image of a closed set in Y is gp-closed set in X.

(4). π-continuous [24] if the inverse image of a closed set in Y is π-closed set in X.

(5). πg-continuous [7] if the inverse image of a closed set in Y is πg-closed set in X.

(6). πgs-continuous [3] if the inverse image of a closed set in Y is πgs-closed set in X.

(7). πgr-continuous [8] if the inverse image of a closed set in Y is πgr-closed set in X.

(8). $\pi g\beta$-continuous [21] if the inverse image of a closed set in Y is $\pi g\beta$-closed set in X.

3. πg^*s-Closed Sets

In this section, we defined a new class of sets called πg^*s-closed sets, πg^*s-open sets and study some of its properties.

Definition 3.1. *A subset A of a topological space (X, τ) is called π generalized star semi-closed set(briefly πg^*s-closed set) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is πg-open in (X, τ).*

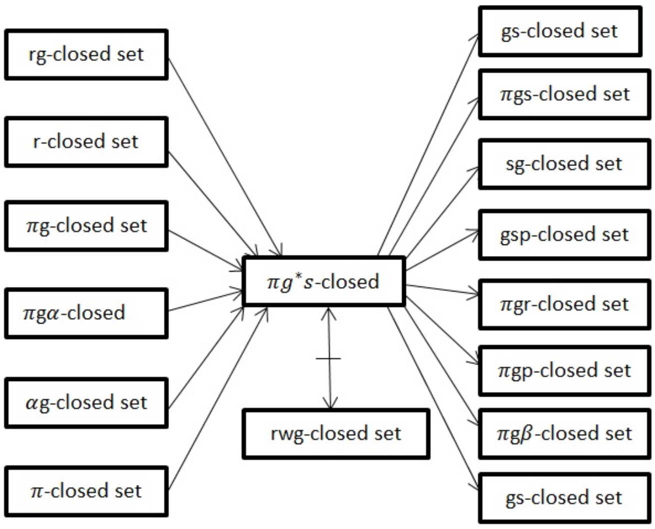
Theorem 3.2. *Every r-closed set is πg^*s-closed.*

Proof. Let A be a r-closed set in X. Let U be a πg-open set such that $A \subseteq U$. Since A is r-closed we have $rcl(A) = A \subseteq U$. But, scl(A)$\subseteq rcl(A) \subseteq U$. Therefore $scl(A) \subseteq U$. Hence A is a πg^*s-closed set in X. □

Remark 3.3. *The converse of the above theorem need not be true as seen in the following example.*

Example 3.4. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{b\}, X\}$. Then r-closed set= $\{\phi, X\}$ and πg^*s-closed set= $\{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. Let $A = \{a\}$. Then the subset A is πg^*s-closed but not a r-closed set.*

Remark 3.5. *The following diagram shows the relationship of πg^*s-closed set with other known existing sets.*



$A \to B$ represents A implies B but not conversely.

Example 3.6. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{c\}, \{a, b\}, X\}$. Then πg^*s-closed set= $\{\phi, \{c\}, \{a, b\}, X\}$, gs-closed, sg-closed, gp-closed, gsp-closed, πgr-closed, πgp-closed, πgs-closed and $\pi g\beta$-closed set= $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $A = \{a\}$. Then the subset A is gs-closed, sg-closed, gp-closed, gsp-closed, πgr-closed, πgp-closed, πgs-closed and $\pi g\beta$-closed set but not πg^*s-closed set.*

Example 3.7. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then πg^*s-closed set= $\{\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$, π-closed, rg-closed, αg-closed, πg-closed, $\pi g\alpha$-closed set= $\{\phi, \{c\}, \{b, c\}, \{a, c\}, X\}$ and rwg= $\{\phi, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $A = \{a\}$. Then the subset A is πg^*s-closed but not π-closed, rg-closed, αg-closed, πg-closed, $\pi g\alpha$-closed and rwg-closed set.*

Theorem 3.8. *Union of two πg^*s-closed subset is πg^*s-closed.*

Proof. Let A and B be any two πg^*s-closed sets in X. Such that $A \subseteq U$ and $B \subseteq U$ where U is πg-open in X and so $A \cup B \subseteq U$. Since A and B are πg^*s-closed. $A \subseteq scl(A)$ and $B \subseteq scl(A)$ and hence $A \cup B \subseteq scl(A) \cup scl(B) \subseteq scl(A \cup B)$. Thus, $A \cup B$ is πg^*s-closed set in (X, τ). □

Example 3.9. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$. Then πg^*s-closed= $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Let $A = \{a\}$ is πg^*s-closed set and $B = \{b\}$ is πg^*s-closed set, then $A \cup B = \{a\} \cup \{b\} = \{a, b\}$ is also πg^*s-closed set.*

Theorem 3.10. *Intersection of two πg^*s-closed subset is πg^*s-closed.*

Proof. Let A and B be any two πg^*s-closed sets in X. Such that $A \subseteq U$ and $B \subseteq U$ where U is πg-open in X and so $A \cap B \subseteq U$. Since A and B are πg^*s-closed. $A \subseteq scl(A)$ and $B \subseteq scl(A)$ and hence $A \cap B \subseteq scl(A) \cap scl(B) \subseteq scl(A \cap B)$. Thus, $A \cap B$ is πg^*s-closed set in (X, τ). □

Example 3.11. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. Then πg^*s-closed= $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$. Let $A = \{a, b\}$ is and $B = \{b, c\}$ then $A \cap B = \{b\}$ is also πg^*s closed set.*

Theorem 3.12. *A subset A of X is πg^*s-closed if and only if $scl(A) - A$ contains no non-empty closed set in X.*

Proof. Let A be a πg^*s-closed set. Suppose F is a non-empty closed set such that $F \subseteq scl(A) - A$. Then $F \subseteq scl(A) \cap A^c$, since $scl(A) - A = scl(A) \cap A^c$. Therefore $F \subseteq scl(A)$ and $F \subseteq A^c$. Since F^c is open, it is πg-open. Now, by the definition of πg*-closed set, $scl(A) \subseteq F^c$, That is $F \subseteq [scl(A)]^c$. Hence $F \subseteq scl(A) \cap [scl(A)]^c = \phi$. That is $F = \phi$, which is a contradiction. Thus, $scl(A) - A$ contains no non-empty closed set in X. Conversely, assume that $scl(A) - A$ contains no non-empty closed set. Let $A \subseteq U$, where U is πg-open. Suppose that $scl(A)$ is not contained in U, then $scl(A) \cap U^c$ is a non-empty closed subset of $scl(A) - A$, which is a contradiction. Therefore $scl(A) \subseteq U$ and hence A is πg^*s-closed. □

Theorem 3.13. *For any element $x \in X$. the set $X/\{x\}$ is πg^*s-closed set or πg-open.*

Proof. Suppose $X/\{x\}$ is not πg-open, then X is the only π g-open set containing $X/\{x\}$. This implies $sclX/\{x\} \subseteq X$. Hence $X/\{x\}$ is πg^*s-closed or πg-open in X. □

Theorem 3.14. *If A is an πg^*s-closed subset of X such that $A \subseteq B \subseteq scl(A)$, then B is an πg^*s-closed set in X.*

Proof. Let A be a πg^*s-closed set of X such that $A \subseteq B \subseteq scl(A)$. Let U be a πg-open set of X such that $B \subseteq U$, then $A \subseteq U$. Since A is πg^*s-closed. We have $scl(A) \subseteq U$. Now $scl(B) \subseteq scl(scl(A)) = scl(A) \subseteq U$. Therefore B is an πg^*s-closed set in X. □

Definition 3.15. *A subset A of (X, τ) is called πg^*s-open set if and only if A^c is πg^*s-closed in (X, τ).*

Theorem 3.16. *A set A is πg^*s-open if and only if $F \subseteq Int(A)$, where F is πg-closed and $F \subseteq A$.*

Proof. If $F \subseteq Int(A)$ where F is πg-closed and $F \subseteq A$. Let $A^c \subseteq G$ where $G = F^c$ is πg-open. Then $G^c \subseteq A$ and $G^c \subseteq Int(A)$. Then we have A^c is πg^*s-closed. Hence A is πg^*s-open. Conversely If A is πg^* s-open, $F \subseteq A$ and F is πg-closed. Then F^c is πg-open and $A^c \subseteq F$. Therefore $cl(A^c) \subseteq (F^c)$. But $cl(A^c) = (Int(A))^c$. Hence $F \subseteq Int(A)$. □

Theorem 3.17. *If A is a subset of a topological space X is s-open and πgs-closed then A is πg^*s-open set.*

Proof. Suppose a subset A of X is both s-open and πgs-closed. Now $A \supseteq scl(A) \supseteq [scl(A)]^c$. Therefore $A \subseteq sInt(A)$. Thus A is open in X. □

Theorem 3.18. *If a subset A of a topological space X is πg^*s-closed then it is πgs-closed.*

Proof. Suppose A is πg^*s-closed in X. Let U be an open set containing A in X. Then $U \subseteq scl(A)$. Now $U \supseteq scl(A)$. Thus A is πgs-closed in X. □

Theorem 3.19. *If A is both rwg-open and πg-closed in X, Then it is πg^*s-closed in X.*

Proof. Let A be an rwg-open and πg-closed set in X. Let $A \subset U$ and let U be πg-open in X. Now $A \subset A$. By hypothesis $scl(A)$. That is $scl(A) \subset U$. Thus A is πg^*s-closed in X. □

4. πg^*s-Continuous Functions

In this chapter, we introduce a function called πg^*s-continuous function and obtain some of its basic results.

Definition 4.1. *If $f : X \to Y$ is called πg^*s-continuous, if $f^{-1}(A)$ is πg^*s-closed set in X for every closed set A in Y.*

Theorem 4.2.

(i). If $f : X \to Y$ is πg^*s-continuous, then f is r-continuous.

(ii). If $f : X \to Y$ is πg^*s-continuous, then f is π-continuous.

Proof.

(i). Let F be a closed set in Y. Since f is πg^*s-continuous, then $f^{-1}(F)$ is πg^*s-closed in X. Since every πg^*s-closed set is r-closed, then $f^{-1}(F)$ is r-closed in X. Hence f is r-continuous.

(ii). Obvious. □

Example 4.3. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Then $\sigma = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$, $\sigma^c = \{\phi, \{a, c\}, \{c\}, \{a\}, X\}$. Let r-closed= $\{\phi, X\}$, π-closed= $\{\phi, X\}$ and πg^*s-closed= $\{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Define a map $f(a) = b$, $f(b) = a$, $f(c) = c$, then $f^{-1}(ac) = bc$, $f^{-1}(c) = c$, $f^{-1}(a) = b$ which is in πg^*s-closed set in X. But not in r-closed set and π-closed set. Therefore f is πg^*s-continuous function but not r-continuous and π-continuous function.*

Theorem 4.4. *(i). If $f : X \to Y$ is πgr-continuous, then f is πg^*s-continuous.*

(ii). If $f : X \to Y$ is gs-continuous, then f is πg^*s-continuous.

(iii). If $f : X \to Y$ is gp-continuous, then f is πg^*s-continuous.

(iv). If $f : X \to Y$ is $\pi g\beta$-continuous, then f is πg^*s-continuous.

Proof. (i). Let F be a closed set in Y. Since f is πgr-continuous, then $f^{-1}(F)$ is πgr-closed in X. Since every πgr-closed set is πg^*s-closed, then $f^{-1}(F)$ is πgr-closed in X. Hence f is πgr-continuous.
Proof of (ii),(iii),(iv) is obvious □

Example 4.5. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, \{b, c\}X\}$. Then $\sigma = \{\phi, \{c\}, X\}$, πgr, gs, gp-closed= $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$ and πg^*s-closed= $\{\phi, \{a\}, \{b, c\}, X\}$. Define a map $f(a) = a$, $f(b) = b$, $f(c) = c$, then $f^{-1}(ab) = ab$, which is in πgr, gs, gp-closed set in X. But not in πg^*s-closed set. Therefore f is πgr, gp, gs-continuous function but not πg^*s-continuous function.*

5. πg^*s-Irresolute Function

In this section we defined the concept of πg^*s-irresolute functions in topological spaces.

Definition 5.1. *If $f : X \to Y$ is called πg^*s-irresolute function, if $f^{-1}(A)$ is πg^*s-closed set in X for every πg^*s-closed set A in Y.*

Theorem 5.2. *Every πg^*s-irresolute functions is πg^*s-continuous.*

Remark 5.3. *The converse of the above theorem need not be true as seen in the following example.*

Example 5.4. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then πg^*s-closed$= \{\phi, \{c\}, X\}$, $\sigma = \{\phi, \{a\}, \{b, c\}, X\}$ and $\pi g^*s = \{\phi, \{a\}, \{b, c\}, X\}$. Define a map $f(a) = a$ and $f(b) = b$, $f(c) = c$ then $f^{-1}(a) = a$, $f^{-1}(b, c) = bc$, which is not πg^*s-irresolute, since it is πg^*s-closed set of Y but the inverse is not a πg^*s-closed set of X. But it is πg^*s-continuous.*

Theorem 5.5. *A map $f : X \to Y$ is πg^*s-irresolute function if and only if, for every πg^*s-open set A of Y, $f^{-1}(A)$ is πg^*s-open in Y.*

Proof. **Necessity:** If $f : X \to Y$ is πg^*s-irresolute, then for every πg^*s-closed B of Y, $f^{-1}(B)$ is πg^*s-closed in X. If A is any πg^*s-open subset of Y, then A^c is πg^*s-closed. Thus $f^{-1}(A^c)$ is πg^*s-closed, but $f^{-1}(A^c) = (f^{-1}(A))^c$ so that $f^{-1}(A)$ is πg^*s-open.

Sufficiency: If for all πg^*s-open subsets A of Y, $f^{-1}(A)$ is πg^*s-open in X, and if B is any πg^*s-closed subset of Y, then B ^{c} is πg^*s-open. Also $f^{-1}(B^c) = (f^{-1}(B))^c)$ is πg^*s-open. Thus $f^{-1}(B)$ is πg^*s-closed. □

Theorem 5.6. *If $f : X \to Y$ and $g : Y \to Z$ are both πg^*s-irresolute function, then $gof : X \to Z$ is πg^*s-irresolute.*

Proof. If $A \subset Z$ is πg^*s-open, then $g^{-1}(A)$ is πg^*s-open and $f^{-1}(g^{-1}(A))$ is πg^*s-open since g and f are πg^*s-irresolute. Thus $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is πg^*s-open and gof is πg^*s-irresolute. □

Theorem 5.7. *Let X, Y and Z be any topological spaces. For any πg^*s-irresolute map $f : X \to Y$ and any πg^*s-continuous map $g : Y \to Z$, the composition $gof : X \to Z$ is πg^*s-continuous.*

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