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Arithmetic Operations of Hexagonal Intuitionistic Fuzzy Number Using Extension Principle

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Abstract: This paper presents an analysis of the basic definitions of the theory of intuitionistic fuzzy sets. Some arithmetic operations using membership and non-membership functions on Hexagonal intuitionistic fuzzy numbers are revealed and the ways to approve the properties using (α, β) cut intervals of intuitionistic fuzzy arithmetic are proposed. The concept is illustrated with numerical example.

Keywords: Extension principle, Hexagonal intuitionistic fuzzy number, Intuitionistic fuzzy sets, (α, β) cut intervals.

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1. Introduction

Fuzzy set theory lets us quite intuitively represent imprecise or vague information. Fuzzy numbers, which form a particular subclass of fuzzy sets of the real line, play a significant role in many important theoretical and/or practical considerations. In fuzzy sets the degree of acceptance is considered only but in IFS it is characterized by a membership function and a non-membership function so that the sum of both values is less than one [1,2]. Later Cheng [4] used interval of confidence for analyzing the fuzzy system reliability. This is because we often describe our knowledge about objects through numbers that are uncertain. To deal such fuzziness in decision making, Bellmann and Zadeh [3] and Zadeh [12] introduced the concept of fuzziness. The basic arithmetic operation on generalized intuitionstic triangular fuzzy numbers and notation of (α, β) cut sets was defined by Seikh [11]. A new operation on hexagonal has been introduced with its basic membership function followed by the properties of its arithmetic operation of fuzzy numbers [5,10]. Fuzzy set theory permits the gradual assessment of the membership and non membership of elements in a set which is described in the interval [0, 1]. Its application can be implemented in a wide range of domains where sequence is incomplete and vague. In this paper a new arithmetic operations using membership functions of Hexagonal Intuitionistic Fuzzy numbers has been introduced.

2. Definition

Definition 2.1. *An Intuitionistic Fuzzy Set (IFS) $\tilde{A}^I$ in X is defined as an object of the form $\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), v_{\tilde{A}^I}(x)\rangle : x \in X\}$ where the functions $\mu_{\tilde{A}^I} : X \to [0,1]$ and $v_{\tilde{A}^I} : X \to [0,1]$ define the degree of the membership and the degree of non membership of the element $x \in X$ in $\tilde{A}^I$, $0 \le \mu_{\tilde{A}^I}(x) + v_{\tilde{A}^I}(x) \le 1$.*

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Definition 2.2. *An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is*

(1). an intuitionistic fuzzy subset of the real line,

(2). convex for the membership function $\mu_{\tilde{A}^I}(x)$ i.e., $\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$, for every $x_1, x_2 \in R$, $\lambda \in [0,1]$.

(3). concave for the membership function $v_{\tilde{A}^I}(x)$, that is, $v_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq \max(v_{\tilde{A}^I}(x_1), v_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in R$, $\lambda \in [0,1]$.

(4). normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0) = 1, v_{\tilde{A}^I}(x_0) = 0$.

Definition 2.3. *A hexagonal intuitionistic fuzzy numbers $\tilde{A}^I_H = (a_1, a_2, a_3, a_4, a_5, a_6 : a'_{1'}, a'_2, a_3, a_4, a'_5, a'_6)$, where $a_1, a_2, a_3, a_4, a_5, a_6$ and $a'_{1'}, a'_2, a_3, a_4, a'_5, a'_6$ are real numbers such that $a'_1 \leq a_1 \leq a'_2 \leq a_2, a_3 \leq a_4 \leq a_5 \leq a'_5 \leq a_6 \leq a'_6$ and its membership and non-membership functions are given below*

$$\mu_{\tilde{A}^I_H}(x) = \begin{cases} 0, & for \quad x \leq a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right), & for\ a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_3)}{a_3-a_2}\right), & for\ a_2 \leq x \leq a_3 \\ 1, & for\ a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\left(\frac{x-a_4}{a_5-a_4}\right), & for\ a_4 \leq x \leq a_5 \\ \frac{1}{2}\left(\frac{(a_6-x}{a_6-a_5}\right), & for\ a_5 \leq x \leq a_6 \\ 0, & otherwise \end{cases} \qquad \nu_{\tilde{A}^I_H}(x) = \begin{cases} 1 - \frac{1}{2}\left(\frac{x-a'_1}{a'_2-a'_1}\right), & for\ a'_1 \leq x \leq a'_2 \\ \frac{1}{2}\left(\frac{a_3-x)}{a_3-a'_2}\right), & for\ a'_2 \leq x \leq a_3 \\ 0, & for\ a_3 \leq x \leq a_4 \\ \frac{1}{2}\left(\frac{x-a_4}{a'_5-a_4}\right), & for\ a_4 \leq x \leq a'_5 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{(x-a'_5}{a'_6-a'_5}\right), & for\ a'_5 \leq x \leq a'_6 \\ 1, & otherwise \end{cases}$$

3. Degree of Membership Function of Alpha Cut

The classical set $\tilde{A}_\alpha$ called alpha cut set of elements whose degree of membership is the set of elements in $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6 : a'_1, a'_2, a_3, a_4, a'_5, a'_6)$ is not less than α. It is defined as $A_\alpha = \left\{x \in X / \mu_{\tilde{A}_H}(x) \geq \alpha, \nu_{\tilde{A}_H}(x) \geq \alpha\right\}$

$$\mu_{\tilde{A}_H} = \begin{cases} P_1(\alpha), P_2(\alpha), & for \alpha \in [0, 0.5) \\ Q_1(\alpha), Q_2(\alpha), & for \alpha \in [0.5, 1] \end{cases} \qquad \nu_{\tilde{A}_H} = \begin{cases} R'_1(\alpha), R'_2(\alpha), & for \alpha \in [0, 0.5), \\ S'_1(\alpha), S'_2(\alpha), & for \alpha \in [0.5, 1] \end{cases}$$

The $\alpha-$cut intervals are as follows

$$\begin{aligned} P_1(\alpha) &= a_1 + \alpha(a_3 - a_1), & P_2(\alpha) &= a_6 + \alpha(a_4 - a_6) \\ Q_1(\alpha) &= 2\alpha(a_3 - a_2) - 2a_3, & Q_2(\alpha) &= -2\alpha(a_5 - a_4) + 2a_5 - a_4 \\ R'_1(\alpha) &= a'_1 + \alpha(a_3 - a'_1), & R'_2(\alpha) &= a'_6 + \alpha(a_4 - a'_6) \\ S'_1(\alpha) &= 2\alpha(a_3 - a'_2) - 2a_3, & S'_2(\alpha) &= -2\alpha(a'_5 - a_4) + 2a'_5 - a_4 \end{aligned}$$

Theorem 3.1. *If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6 : a'_1, a'_2, a_3, a_4, a'_5, a'_6)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6 : b'_1, b'_2, b_3, b_4, b'_5, b'_6)$ are two hexagonal intustinuistic fuzzy numbers then $\tilde{D}_H = \tilde{A}_H + \tilde{B}_H$ is also a fuzzy number.*

Proof. The membership function and the non-membership $\mu_{\tilde{A}_H}(x), \nu_{\tilde{A}_H}(x)$ functions can be transformed in to $\tilde{D}_H(\alpha) = (x + y)$. The membership function of $\tilde{A}_H(x)$ is

$$x \in \begin{cases} P_1(\alpha) = a_1 + \alpha(a_3 - a_1), & P_2(\alpha) = a_6 + \alpha(a_4 - a_6) \\ Q_1(\alpha) = 2\alpha(a_3 - a_2) - 2a_3, & Q_2(\alpha) = -2\alpha(a_5 - a_4) + 2a_5 - a_4 \end{cases}$$

The non-membership function of $\tilde{A}_H(x)$ is

$$x' \in \begin{cases} R_1'(\alpha) = a_1' + \alpha(a_3 - a_1') & R_2'(\alpha) = a_6' + \alpha(a_4 - a_6') \\ S_1'(\alpha) = 2\alpha(a_3 - a_2') - 2a_3 & S_2'(\alpha) = -2\alpha(a_5' - a_4) + 2a_5' - a_4 \end{cases}$$

Let the membership function of $\tilde{B}_H(x)$ is y The non-membership function of $\tilde{B}_H(x)$ is y'

$$y' \in \begin{cases} T_1'(\alpha) = a_1' + \alpha(a_3 - a_1') & T_2'(\alpha) = a_6' + \alpha(a_4 - a_6') \\ U_1'(\alpha) = 2\alpha(a_3 - a_2') - 2a_3 & U_2'(\alpha) = -2\alpha(a_5' - a_4) + 2a_5' - a_4 \end{cases}$$

$$v \in (x+y) = \begin{cases} 2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6 + & \\ 2\alpha(b_2 - b_1) + b_1, -2\alpha(b_6 - b_5) + b_6 & for\ \alpha \in [0.05) \\ 2\alpha(a_3 - a_2) - a_3 + 2a_2, -2\alpha(a_5 - a_4) + 2a_5 - a_4, & \\ 2\alpha(b_3 - b_2) - b_3 + 2b_2, -2\alpha(b_5 - b_4) + 2b_5 - b_4 & for\ \alpha \in (0.5, 1] \end{cases}$$

$$v' \in (x'+y') = \begin{cases} 2\alpha(a_2' - a_1') + a_1', -2\alpha(a_6' - a_5) + a_6' + & \\ 2\alpha(b_2' - b_1') + b_1', -2\alpha(b_6' - b_5') + b_6' & for\ \alpha \in [0.05) \\ 2\alpha(a_3 - a_2') - a_3 + 2a_2', -2\alpha(a_5' - a_4) + 2a_5' - a_4, & \\ 2\alpha(b_3 - b_2') - b_3 + 2b_2', -2\alpha(b_5' - b_4) + 2b_5' - b_4 & for\ \alpha \in (0.5, 1] \end{cases}$$

The membership function $\tilde{D}_H = \tilde{A}_H + \tilde{B}_H$ is

$$z = (x+y) \in \begin{cases} 0, & for \quad z \le (a_1 + b_1) \\ \frac{z-(a_1+b_1)}{2[(a_2+b_2)-(a_1+b_1)]}, & for\ a_1 + b_1 \le z \le a_2 + b_2 \\ \frac{z-2(a_2+b_2)+(a_3+b_3)}{2[(a_3+b_3)-(a_2+b_2)]}, & for\ a_2 + b_2 \le z \le \ a_3 + b_3 \\ 1, & for\ a_3 + b_3 \le z \le a_4 + b_4 \\ \frac{z+(a_4+b_4)-2(a_5+b_5)}{-2[(a_5+b_5)-(a_4+b_4)]}, & for\ a_4 + b_4 \le z \le a_5 + b_5 \\ \frac{z-(a_6+b_6)}{-2[(a_6+b_6)-(a_5+b_5)]}, & for\ a_5 + b_5 \le z \le a_6 + b_6 \\ 0, & for\ z \ge a_6 + b_6 \end{cases} \tag{1}$$

And the non membership function

$$z' = (x'+y') \in \begin{cases} 1 + \frac{(a_1'+b_1')-z}{2[(a_2'+(b_2')-(a_1'+b_1')]}, & for\ a_1' + b_1' \le z \le a_2' + b_2' \\ \frac{(a_3+b_3)-z}{2[(a_3+b_3)-(a_2'+b_2')]}, & for\ a_2' + b_2' \le z \le \ a_3 + b_3 \\ 0, & for\ a_3 + b_3 \le z \le a_4 + b_4 \\ \frac{z-(a_4+b_4)}{2[(a_5'+b_5')-(a_4+b_4)]}, & for\ a_4 + b_4 \le z \le a_5' + b_5' \\ \frac{1}{2} + \frac{z-(a_5'+b_5')}{2[(a_6'+b_6')-(a_5'+b_5')]}, & for\ a_5' + b_5' \le z \le a'_6 + b_6' \\ 1, & for\ z \ge \ a_6' + b_6' \end{cases} \tag{2}$$

Hence addition rule is proved. Hence we have $\tilde{D}_H = \tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6 : a_1' + b_1' : a_2' + b_2', a_3 + b_3, a_4 + b_4, a_5' + b_5', a_6' + b_6')$ is also a HIFN. □

Example 3.2 (Addition of two HIFN by (α, β)-cut method)**.** *Let us consider two HIFN* $\tilde{A}_H(x) = (7, 9, 11, 13, 16, 20; 5, 7, 11, 13, 19, 23)$*;* $\tilde{B}_H(y) = (6, 8, 11, 14, 19, 25; 4, 7, 11, 14, 21, 27)$*. Using* (1) *and* (2) *we get the addition of two HIFN approximation as*

$$(\tilde{A}_H + \tilde{B}_H)(x+y) = (13, 17, 22, 27, 35, 45; 9, 14, 22, 27, 40, 50)$$

Now when we consider the addition using its membership function and the non membership functions we arrive at the following results.

$$\mu_{\tilde{H}}(z) \in x+y = \begin{cases} \frac{x-13}{8}, & for\ 13 \leq x \leq 17 \\ \frac{x-12}{10}, & for\ 17 \leq x \leq 22 \\ 1, & for\ 22 \leq x \leq 27 \\ \frac{x-43}{-16}, & for\ 27 \leq x \leq 35 \\ \frac{x-45}{-20}, & for\ 35 \leq x \leq 35 \\ 0, & for\ x \geq 45 \end{cases} \qquad \nu_{\tilde{H}}(z) \in x+y = \begin{cases} \frac{y-12}{10}, & for\ 9 \leq y \leq 14 \\ \frac{22-y}{16}, & for\ 14 \leq y \leq 22 \\ 0 & \\ \frac{y-27}{10}, & for\ 22 \leq y \leq 27 \\ \frac{y-30}{20}, & for\ 40 \leq y \leq 50 \\ 1, & for\ y \geq 50 \end{cases}$$

Theorem 3.3. *If $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6 : a_1', a_2', a_3, a_4, a_5', a_6')$; $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6 : b_1', b_2', b_3, b_4, b_5', b_6')$ are two hexagonal intustinustic fuzzy numbers then $\tilde{D}_H = \tilde{A}_H \otimes \tilde{B}_H$ is approximated HIFN*

$$\tilde{A}_H \otimes \tilde{B}_H = (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6 : a_1'b_1', a_2'b_2', a_3b_3, a_4b_4, a_5'b_5', a_6'b_6')$$

Proof. Now with the conversion of $z = (x+y)$ we can find the membership and non-membership function of acceptance IFS $\tilde{D}_H = \tilde{A}_H \otimes \tilde{B}_H$ byα-cut method

$$z = (x \times y) = \begin{cases} (2\alpha(a_2 - a_1) + a_1, -2\alpha(a_6 - a_5) + a_6)\otimes & \\ (2\alpha(b_2 - b_1) + b_1, -2\alpha(b_6 - b_5) + b_6), & for \quad \alpha \in [0.05) \\ (2\alpha(a_3 - a_2) - a_3 + 2a_2, -2\alpha(a_5 - a_4) + 2a_5 - a_4)\otimes & \\ (2\alpha(b_3 - b_2) - b_3 + 2b_2, -2\alpha(b_5 - b_4) + 2b_5 - b_4), & for \quad \alpha \in (0.5, 1] \end{cases}$$

So the membership and the non- membership (rejection) function $\tilde{D}_H = \tilde{A}_H \otimes \tilde{B}_H$

$$\mu_{\tilde{D}_H}(z) = \begin{cases} \frac{-B_1 + \sqrt{B_1^2 - 4A_1(a_1b_1 - z)}}{2A_1}, & for\ a_1b_1 \leq z \leq a_2b_2 \\ \frac{-B_2 + \sqrt{B_2^2 - 4A_2(4a_2b_2 - 2\ (a_2b_3 + a_3b_2) + a_3b_3 - z)}}{2A_2}, & for\ a_2b_2 \leq z \leq a_3b_3 \\ 1, & for\ a_3b_3 \leq z \leq a_4b_4 \\ \frac{B_3 - \sqrt{B_3^2 - 4A_3(4a_5b_5 - 2\ (a_5b_4 + a_4b_5) + a_4b_4 - z)}}{2A_3}, & for\ a_4b_4 \leq z \leq a_5b_5 \\ \frac{B_4 - \sqrt{B_4^2 - 4A_4(a_6b_6 - z)}}{2A_4}, & for\ a_5b_5 \leq z \leq a_6b_6 \end{cases} \tag{3}$$

where $A_i = 4(a_{i+1} - a_i)(b_{i+1} - b_i)$, $i = 1, 2, 3, 4$; $B_1 = 2[b_1(a_2 - a_1) + a_1(b_2 - b_1)]$; $B_j = 2[(a_{j+1} - a_j)(jb_j - b_{j+1}) + (b_{j+1} - b_j)(ja_j - (j-1)a_{j+1})]$, $j = 2, 3, 4$.

$$\nu_{\tilde{D}_H}(z) = \begin{cases} 1 - \frac{-B_1 + \sqrt{B_1^2 - 4A_1(a_1'b_1' - z)}}{2A_1}, & for\ a_1'b_1' \leq z \leq a_2'b_2' \\ 1 - \frac{-B_2 + \sqrt{B_2^2 - 4A_2(4a_2'b_2' - 2\ (a_2'b_3 + a_3b_2') + a_3b_3 - z)}}{2A_2}, & for\ a_2'b_2' \leq z \leq a_3b_3 \\ 0, & for\ a_3b_3 \leq z \leq a_4b_4 \\ 1 - \frac{B_3 - \sqrt{B_3^2 - 4A_3(4a_5'b_5' - 2\ (a_5'b_4 + a_4b_5') + a_4b_4 - z)}}{2A_4}, & for\ a_4b_4 \leq z \leq a_5'b_5' \\ 1 - \frac{B_4 - \sqrt{B_4^2 - 4A_4(a_6'b_6' - z)}}{2A_3}, & for\ a_5'b_5' \leq z \leq a_6'b_6' \end{cases} \tag{4}$$

where

$$A_1 = (a_3 - a_1')(b_3 - b_1'), \qquad B_1 = (a_1'(b_3 - b_1') + b_1'(a_3 - a_1')$$
$$A_2 = 4(a_3 - a')(b_3 - b_1'), \qquad B_2 = 4[b_3(a_3 - a_2') + a_3(b_3 - b_2')]$$

$$A_3 = 4(a_5' - a_4')(b_5 - b_4'), \qquad B_3 = 2[(a_5' - a_4)(2b_5' - b_4) + (b_5' - b_4)(2a_5' - a_4)$$
$$A_4 = (a_4 - a_6')(b_4 - b_6'), \qquad B_4 = a_6'(b_4 - b_6') + b_6'(a_4 - a_6')]$$

So $\tilde{D}_H = \tilde{A}_H \otimes \tilde{B}_H$ represented by (3) and (4) is a Hexagonal IFN. □

Example 3.4 (Multiplication of two HIFN by (α,β)-cut method)**.** *Let us consider two HIFN* $\tilde{A}_H(x) = (7,9,11,13,16,20;5,7,11,13,19,23)$ *and* $\tilde{B}_H(y) = (6,8,11,14,19,25;4,7,11,14,21,27)$ *Using* (3) *and* (4) *the multiplication of these two HIFN is defined by*

$$(\tilde{A}_H \times \tilde{B}_H)(x \times y) = (48,72,121,182,304,500 : 20,49,121,182,399,621)$$

With membership and non-membership function we get the result as follows

$$\mu_{\tilde{H}}(z) \in x + y = \begin{cases} \frac{x-42}{60}, & for\ 42 \le x \le 72 \\ \frac{x-72}{98}, & for\ 72 \le x \le 121 \\ 1, & for\ 121 \le z \le 182 \\ \frac{426-x}{244}, & for\ 182 \le x \le 304 \\ \frac{500-x}{392}, & for\ 304 \le x \le 500 \\ 0, & for\ x \ge 500 \end{cases} \qquad \nu_{\tilde{H}}(z) \in x + y = \begin{cases} \frac{78-y}{58}, & for\ 20 \le y \le 49 \\ \frac{121-y}{144}, & for\ 49 \le y \le 121 \\ 0 & \\ \frac{y-182}{434}, & for\ 121 \le y \le 399 \\ \frac{y-177}{444}, & for\ 399 \le y \le 621 \\ 1, & for\ y \ge 621 \end{cases}$$

4. Extension Principle for Intuitionistic Fuzzy Sets

Let $f : X \to Y$ be a mapping from a set X to a set Y. Then the extension principle allows us to define the IFS $\tilde{B}^I$ in Y induced by the IFS $\tilde{A}^I$ in X,through f as follows $\tilde{B}^I = \{< y, \mu_{\tilde{B}^I}(y), \nu_{\tilde{B}^I}(y) >: y = f(x), x \in X\}$ with

$$\mu_{\tilde{B}^I}(y) = \begin{cases} \sup\limits_{y=f(x)} \mu_{\tilde{A}^I}(x) & : \ f^{-1}(y) \neq \phi \\ 0 & : \ f^{-1}(y) = \phi \end{cases} \quad and \quad \nu_{\tilde{B}^I}(y) = \begin{cases} \inf\limits_{y=f(x)} \nu_{\tilde{A}^I}(x) & : \ f^{-1}(y) \neq \phi \\ 0 & : \ f^{-1}(y) = \phi \end{cases}$$

Where $f^{-1}(y)$ is the inverse image of y.

4.1. Cartesian product of Intuitionistic Fuzzy sets

Let $\tilde{A}_1^I, \tilde{A}_2^I, \tilde{A}_3^I, \ldots, \tilde{A}_n^I$ be IFSs in $X_1 \times X_2 \times X_3 \times \cdots \times X_n$ with the corresponding membership function $\mu_{\tilde{A}_1^I}(y), \mu_{\tilde{A}_2^I}(y), \ldots, \mu_{\tilde{A}_n^I}(y)$ and the non membership function $\nu_{\tilde{A}_1^I}(y), \nu_{\tilde{A}_2^I}(y), \ldots, \nu_{\tilde{A}_n^I}(y)$ respectively. Then the Cartesian product of IFSs $\tilde{A}_1^I, \tilde{A}_2^I, \tilde{A}_3^I, \ldots, \tilde{A}_n^I$ denoted by $\tilde{A}_1^I \times \tilde{A}_2^I \times \tilde{A}_3^I \times \cdots \times \tilde{A}_n^I$ is defined as IFS in $X_1 \times X_2 \times X_3 \times \cdots \times X_n$ whose membership and non-membership functions are expressed by $\mu_{\tilde{A}_1^I \times \tilde{A}_2^I \times \cdots \times \tilde{A}_n^I}(x_1 \ldots x_n) = \min[\mu_{\tilde{A}_1^I}(x) \ldots \mu_{\tilde{A}_n^I}(x)]$ and

$$\nu_{\tilde{A}_1^I \times \tilde{A}_2^I \times \cdots \times \tilde{A}_n^I}(x_1 \ldots x_n) = \max[\nu_{\tilde{A}_1^I}(x) \ldots \nu_{\tilde{A}_n^I}(x)]$$

4.2. Arithmetic Operations of intuitionistic Fuzzy Number based on Extension Principle

The arithmetic operation $(*)$ of two IFNs is a mapping of an input vector $X = [x_1, x_2]^T$. Define in the Cartesian product space $R \times R$ onto an output y defined in the real space R. If $\tilde{A}_1^I$ and $\tilde{A}_2^I$ are IFNs then their outcome of arithmetic operation is also a IFN determined with the formula

$$\left[\tilde{A}_1^I * \tilde{A}_2^I\right](y) = \left\{y, \sup_{y=x_1*x_2}\left[\min\left(\mu_{\tilde{A}_1^I}(x_1), \mu_{\tilde{A}_2^I}(x_2)\right)\right], \inf_{y=x_1*x_2}\left[\max\left(\nu_{\tilde{A}_1^I}(x_1), \nu_{\tilde{A}_2^I}(x_2)\right)\right] \forall x_1, x_2, y \in R\right\}$$

To calculate the arithmetic operations of IFNs it is sufficient to determine the membership function and non-membership function as follows

$$\mu_{\tilde{A}_1^I * \tilde{A}_2^I}(y) = \sup_{y=x_1*x_2}\left[\min\left(\mu_{\tilde{A}_1^I}(x_1), \mu_{\tilde{A}_2^I}(x_2)\right)\right] \quad and \; \nu_{\tilde{A}_1^I * \tilde{A}_2^I}(y) = \inf_{y=x_1*x_2}\left[\max\left(\nu_{\tilde{A}_1^I}(x_1), \nu_{\tilde{A}_2^I}(x_2)\right)\right]$$

4.3. Addition of Two HIFN by Intuitionistic Fuzzy Extension principle

Let $\tilde{A}_H(x) = (7, 9, 11, 13, 16, 20 : 5, 7, 11, 13, 19, 23)$; $\tilde{B}_H(y) = (6, 8, 11, 14, 19, 25 : 4, 7, 11, 14, 21, 27)$ and $\tilde{D}_H = \tilde{A}_H + \tilde{B}_H$. By extension principal $\mu_{\tilde{D}_1^I}(z) = \sup\{(\min(\mu_{\tilde{A}^I}(x), \mu_{\tilde{B}^I}(y)) : x + y = z\}$ and $\nu_{\tilde{D}_1^I}(z) = \inf\{(\max(\nu_{\tilde{A}^I}(x), \nu_{\tilde{B}^I}(y)) : x + y = z\}$

$$(\tilde{A}_H + \tilde{B}_H)(z) = (13, 17, 22, 27, 35, 45; 9, 14, 22, 27, 40, 50)$$

Let us solve the above problem using the extension principal and find the left and right divergent for both membership and non membership functions. Let us select a number $z = 14$ for membership function and $z = 10$ for the non-membership function and split it between x and y (i.e.,) $x = 7$ and $y = 7$ and $x = 5$, $y = 5$. Few values can be taken as shown in Table 1 and Table 2. Next we have to evaluate $(\min(\mu_{\tilde{A}^I}(x), \mu_{\tilde{B}^I}(y))$ and $(\max(\nu_{\tilde{A}^I}(x), \nu_{\tilde{B}^I}(y))$for both x and y.

x	$\mu_{\tilde{A}}(x)$	y	$\mu_B(y)$	$\min(\mu_{\tilde{A}}(x), \mu_B(y))$
7	0	7	.25	0
7.25	0.06	6.25	.18	.06
7.5	0.125	6.5	.125	**.125**
7.75	0.1875	6.25	.06	.06
8	.25	6	0	0

Table 1. Membership function of sum of two HIFNs

The Maximum appears at $x = 7.5$ and $y = 6.5$ so that $\mu_{\tilde{C}}(14) = .125$.

x	$\nu_{\tilde{A}}(x)$	y	$\nu_B(y)$	$\max(\nu_{\tilde{A}}(x), \nu_B(y))$
5	1	5	.8333	1
5.25	0.9375	4.75	.875	.9375
5.5	0.875	4.5	.9166	**.9166**
5.75	0.8125	4.25	.9583	.9583
6	.75	4	1	1

Table 2. Non-Membership function of sum of two HIFNs

The Minimum appears at $x = 5.5$ and $y = 4.5$ so that $\nu_{\tilde{C}}(10) = .9166$.

5. Conclusion

In this paper, we have proposed the definition of HIFN according to the intuitionistic fuzzy number approach. Arithmetic operations are proposed on HIFN and explained through theorems and the concepts are made clear using examples. Also arithmetic operations of Hexagonal intuitionistic Fuzzy Number based on Extension Principle and (α, β)-cuts method has been illustrated..

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