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Almost β-Regular Spaces and Almost $rg\beta$-Closed Functions

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Abstract: The aim of this paper is to introduce and study a new class of spaces, namely almost β-regular spaces which are generalizations of the concepts of almost regular spaces and β-regular spaces and to obtain some characterizations of almost β-regular spaces. Further, by using $rg\beta$-closed sets, we define almost $rg\beta$-closed functions and to obtain preservation theorems of almost β-regular spaces. The relationships among p-regular, β-regular, almost regular, almost p-regular, almost β-regular spaces are investigated. The main result of this paper is that almost β-regularity is preserved under M-β-open β-$rg\beta$-closed surjective R-maps.

MSC: 54D10, 54D15, 54A05, 54C08.

Keywords: β-open and regular open sets, almost regular and almost β-regular spaces, β-$g\beta$-closed, β-$rg\beta$-closed, almost β-closed, almost $rg\beta$-closed and almost continuous functions.

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1. Introduction

In 1969, Singal and Arya [20] introduced a weak form of regular spaces called almost regular spaces and obtained their characterizations. In 1970, Levine [6] introduced and studied generalized closed sets in general topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. In 1983, El-Deeb [5] introduced the notion of p-regular spaces by using p-open sets and obtained their characterizations and preservation theorems. In 1985, Mahmoud [7] introduced the concept of β-regular spaces. In 1990, Malghan and Navalagi [10] defined and studied almost p-regular spaces which were generalizations of both almost regular spaces and p-regular spaces. In 1993, palaniappan and Rao [16] introduced and studied regular generalized closed sets. In 1995, Dontchev [4] defined and investigated generalized β-closed sets. In 1998, Noiri [14] introduced rgp-closed sets and used these sets to obtain further characterizations of almost regular spaces and almost p-regular spaces. In 2005, Tahiliani [22] introduced generalized β-closed functions and obtained some new characterizations of β-regular spaces. Recently, by using β-closed sets, M. C. Sharma and Hamant Kumar [19] introduced almost β-closed functions.

In this paper, we introduce and study a new class of spaces, namely almost β-regular spaces and to obtain some characterizations of almost β-regular spaces. Further, by using $rg\beta$-closed sets, we define β-$rg\beta$-closed functions and to obtain preservation theorems of almost β-regular spaces. The relationships among p-regular, β-regular, almost regular, almost p-regular, almost β-regular spaces are investigated. The main result of this paper is that almost β-regularity is preserved under M-β-open β-$rg\beta$-closed surjective R-maps.

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2. Preliminaries

Throughout this paper, spaces (X,τ), (Y,σ), and (Z,γ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively. A is said to be α-open [12] if $A \subset Int(Cl(Int(A)))$, p-open [11] if $A \subset Int(Cl(A))$, β-open [1] if $A \subset Cl(Int(Cl(A)))$. The complement of a α-open (respectively p-open, β-open) set is said to be α-closed (respectively p-closed, β-closed). The intersection of all α-closed (respectively p-closed, β-closed) sets containing A is called α-closure (respectively p-closure, β-closure) of A, and is denoted by $\alpha Cl(A)$ (respectively $pCl(A)$, $\beta Cl(A)$). The β-Interior [2] of A, denoted by $\beta Int(A)$, is defined as union of all β-open sets contained in A. The family of regularly open (respectively regularly closed, β-open, β-closed) sets of a space X is denoted by $RO(X)$ (respectively $RC(X)$, $\beta O(X)$, $\beta C(X)$). It is well known that $sCl(A) = A \cup Int(Cl(A))$, $sInt(A) = A \cap Cl(Int(A))$, $pCl = A \cup Cl(Int(A))$, $pInt(A) = A \cap Int(Cl(A))$, $\alpha Cl(A) = A \cup Cl(Int(Cl(A)))$, $\alpha Int(A) = A \cap Int(Cl(Int(A)))$, $\beta Cl(A) = A \cup Int(Cl(Int(A)))$ and $\beta Int(A) = A \cap Cl(Int(Cl(A)))$.

Definition 2.1. *A subset A of a space (X,τ) is said to be*

(1). generalized-closed (briefly g-closed) [6] if $Cl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

(2). regular generalized-closed (briefly rg-closed) [16] if $Cl(A) \subset U$ whenever $A \subset U$ and $U \in RO(X)$.

(3). α-generalized closed (briefly αg-closed) [9]) if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

(4). regular generalized α-closed (briefly $rg\alpha$-closed) [15]) if $\alpha Cl(A) \subset U$ whenever $A \subset U$ and $U \in RO(X)$.

(5). generalized pre-closed (briefly gp-closed) [13] if $pCl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

(6). regular generalized pre-closed (briefly rgp-closed) [14] if $pCl(A) \subset U$ whenever $A \subset U$ and $U \in RO(X)$.

(7). generalized β-closed (briefly $g\beta$-closed) [4] if $\beta Cl(A) \subset U$ whenever $A \subset U$ and $U \in \tau$.

(8). regular generalized β-closed (briefly $rg\beta$-closed) [17] if $\beta Cl(A) \subset U$ whenever $A \subset U$ and $U \in RO(X)$.

(9). g-open (respectively rg-open, αg-open, $rg\alpha$-open, gp-open, rgp-open, $g\beta$-open, $rg\beta$-open) if the complement of A is g-closed (respectively rg-closed, αg-closed, $rg\alpha$-closed, gp-closed, rgp-closed, $g\beta$-closed, $rg\beta$-closed).

Remark 2.2. *We have the following implications for the properties of subsets:*

$$\begin{array}{ccccc}
\text{closed} & \Rightarrow & g\text{-closed} & \Rightarrow & rg\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\alpha\text{-closed} & \Rightarrow & \alpha g\text{-closed} & \Rightarrow & rg\alpha\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
p\text{-closed} & \Rightarrow & gp\text{-closed} & \Rightarrow & rgp\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow \\
\beta\text{-closed} & \Rightarrow & g\beta\text{-closed} & \Rightarrow & rg\beta\text{-closed}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

Example 2.3. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, \{a\}, X\}$. Then the subset $A = \{b\}$ is g-closed as well as αg-cloed but not closed.*

Example 2.4. *Let $X = \{a,b,c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a,b\}, X\}$. Then the subset $A = \{a\}$ is $rg\beta$-closed not g-closed.*

Example 2.5. *Let $X = \{a, b, c, \}$ and $\tau = \{\phi, \{a\}, X\}$. Then the subset $A = \{a, b\}$ is g-closed as well as rg-closed.*

Example 2.6. *Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, \{c\}, \{d\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$. Then the subset $A = \{a\}$ is α-closed as well as p-closed but not closed.*

Example 2.7. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then the subset $A = \{a, c\}$ is g-closed as well rg-closed. But it is not p-closed.*

Example 2.8. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then the subset $A = \{b\}$ is $rg\beta$-closed not rg-closed.*

Lemma 2.9 ([22]). *Let A be a subset of a space X and $x \in X$. The following properties hold for $\beta Cl(A)$:*

(1). $x \in \beta C1(A)$ if and only if $A \cap U \neq \phi$ for every $U \in \beta O(X)$ containing x.

(2). A is β-closed if and only if $A = \beta Cl(A)$.

(3). $\beta Cl(A) \subset \beta Cl(B)$ if $A \subset B$.

(4). $\beta Cl(\beta Cl(A)) = \beta Cl(A)$.

(5). $\beta Cl(A)$ is β-closed.

Lemma 2.10 ([22]). *A subset A of a space X is $g\beta$-open in X if and only if $F \subset \beta Int(A)$ whenever $F \subset A$ and F is closed in X.*

Lemma 2.11 ([17]). *A subset A of a space X is $rg\beta$-open in X if and only if $F \subset \beta Int(A)$ whenever $\subset A$ and $F \in RC(X)$.*

3. Some Generalizations of Almost Closed Functions

Definition 3.1. *A function $f : X \to Y$ is said to be*

(a). strongly β-closed (respectively M-β-closed, β-$g\beta$-closed [22], β-$rg\beta$-closed) if f(F) is closed (respectively β-closed, $g\beta$-closed, $rg\beta$-closed) in Y for every β-closed set F of X.

(b). β-closed [1] (briefly $g\beta$-closed [22], $rg\beta$-closed) if f(F) is β-closed (respectively $g\beta$-closed, $rg\beta$-closed) in Y for every closed set F of X.

(c). almost closed [21] (briefly almost β-closed [19], almost $g\beta$-closed, almost $rg\beta$-closed) if f(F) is closed (respectively β-closed, $g\beta$-closed, $rg\beta$-closed) in Y for every $A \in RC(X)$.

From the definitions stated above, we have the following diagram:

$$\begin{array}{ccccccc}
\text{strongly } \beta\text{-closed} & \Rightarrow & M\text{-}\beta\text{-closed} & \Rightarrow & \beta\text{-}g\beta\text{-closed} & \Rightarrow & \beta\text{-}rg\beta\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
\text{closed} & \Rightarrow & \beta\text{-closed} & \Rightarrow & g\beta\text{-closed} & \Rightarrow & rg\beta\text{-closed} \\
\Downarrow & & \Downarrow & & \Downarrow & & \Downarrow \\
\text{almost closed} & \Rightarrow & \text{almost } \beta\text{-closed} & \Rightarrow & \text{almost } g\beta\text{-closed} & \Rightarrow & \text{almost } rg\beta\text{-closed}
\end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

Example 3.2. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is closed as well as $rg\beta$-closed.*

Example 3.3. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is almost closed as well as almost $rg\beta$-closed.*

Example 3.4. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, c\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is M-β-closed as well as β-closed but not closed.*

Example 3.5. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, X\}$. Then the identity function $f : (X, \tau) \to (X, \sigma)$ is β-closed as well as almost β-closed but not almost closed.*

Theorem 3.6. *A function $f : Xt \to Y$ is almost β-closed (respectively almost $g\beta$-closed, almost $rg\beta$-closed) if and only if for each subset B of Y and each $U \in RO(X)$ containing $f^{-1}(B)$, there exists a β-open (respectively $g\beta$-open, $rg\beta$-open) set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.*

Proof. We prove only the first case; the proof of the others being entirely analogous.
Necessity: Suppose that f is almost β-closed. Let B be any subset of Y and $U \in RO(X)$ containing $f^{-1}(B)$. Put $V = Y - f(X - U)$ then V is a β-open set of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.
Sufficiency: Let F be any regular closed set of X. Then $f^{-1}(Y - f(F)) \subset X - F$ and $X - F \in RO(X)$. There exists a β-open set V of Y such that $Y - f(F) \subset V$ and $f^{-1}(V) \subset X - F$. Therefore, we have $f(F) = Y - V$ and $f(F)$ is β-closed in Y. This shows that f is almost β-closed. □

Theorem 3.7. *A function $f : X \to Y$ is β-$rg\beta$-closed (respectively $rg\beta$-closed) if and only if for each subset B of Y and each β-open (respectively open) set U of X containing $f^{-1}(B)$, there exists an $rg\beta$-open set V of Y such that $B \subset V$ and $f^{-1}(V) \subset U$.*

Proof. The proof is similar to that of Theorem 3.6 and is thus omitted. □

4. Almost β-regular Spaces

Definition 4.1. *A space X is said to be*

(a). β-regular [7] (respectively p-regular [5]) if for each closed set A and each point $x \in X - A$, there exist disjoint β-open (respectively p-open) sets U, V such that $x \in U$, $A \subset V$, and $U \cap V = \phi$.

(b). almost regular [20] if for each $A \in RC(X)$ and each point $x \in X - A$, there exist disjoint open sets U, V such that $x \in U$, $A \subset V$, and $U \cap V = \phi$.

(c). almost β-regular (respectively almost p-regular [10]) if for each $A \in RC(X)$ and each point $x \in X - A$, there exist disjoint β-open (respectively p-open) sets U, V such that $x \in U$, $A \subset V$, and $U \cap V = \phi$.

By the definitions stated above, we have the following diagram:

$$\begin{array}{ccccc} regularity & \Rightarrow & p-regularity & \Rightarrow & \beta-regularity \\ \Downarrow & & \Downarrow & & \Downarrow \\ almost\ regularity & \Rightarrow & almost\ p-regularity & \Rightarrow & almost\ \beta-regularity \end{array}$$

Where none of the implications is reversible as can be seen from the following examples:

Example 4.2. *Let* $X = \{a, b, c\}$ *and* $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. *Consider the closed set* $\{b, c\}$ *and a point 'a' such that* $a \notin \{b, c\}$. *Then* $\{b, c\}$ *and* $\{a\}$ *are open sets such that* $\{b, c\} \subset \{b, c\}$, $a \in \{a\}$ *and* $\{b, c\} \cap \{a\} = \phi$. *Similarly for the closed set* $\{a\}$ *and a point 'c' such that* $c \notin \{a\}$. *Then there exist open sets* $\{a\}$ *and* $\{b, c\}$ *such that* $\{a\} \subset \{a\}$, $c \notin \{b, c\}$ *and* $\{a\} \cap \{b, c\} = \phi$. *It follows that X is regular space.*

Example 4.3. *Let* $X = \{a, b, c\}$ *and* $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. *Then the space X is* β*-regular but not p-regular, since for the closed set* $\{b, c\}$ *and the point* $a \notin \{b, c\}$, *there do not exist disjoint p-open sets containing them.*

Example 4.4. *Let* $X = \{a, b, c\}$ *and* $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. *Then the space X is almost* β*-regular but it is not almost p-regular, since for the regular closed set* $\{a, c\}$ *and the point* $b \notin \{a, c\}$, *there do not exist disjoint p-open sets containing them.*

Example 4.5. *Let* $X = \{a, b, c\}$ *and* $\tau = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. *Then the space X is almost regular.*

Theorem 4.6. *The following properties are equivalent for a space X:*

(a). X is almost β*-regular;*

(b). for each $F \in RC(X)$ *and each point* $x \in X - F$, *there exists a* β*-open set U and an* $rg\beta$*-open set V such that* $x \in U$, $F \subset V$, *and* $U \cap V = \phi$.

(c). for each $F \in RC(X)$, $F = \cap\{\beta Cl(V) : F \subset V \text{ and } V \text{ is } rg\beta - open \text{ in } X\}$,

(d). for each $F \in RC(X)$ *and each nonempty subset A of X such that* $A \cap F = \phi$, *there exist* $U \in \beta O(X)$ *and an* $rg\beta$*-open set V such that* $A \cap U \neq \phi$, $F \subset V$ *and* $U \cap V = \phi$.

Proof. $(a) \Rightarrow (b)$ The proof is obvious from the fact that β-openness implies $g\beta$-openness and $g\beta$-openness implies $rg\beta$-openness.

$(b) \Rightarrow (c)$ For any $F \in RC(X)$, we always have $F \subset \cap\{\beta Cl(V) : F \subset V \text{ and } V \text{ is } rg\beta - open \text{ in } X\}$. Suppose that $x \in X - F$. By assumption, there exist a β-open set U and an $rg\beta$-open set V such that $x \in U$, $F \subset V$, and $U \cap V = \phi$. By Lemma 2.9, we have $x \in X - \beta Cl(V)$ and hence $x \in X - \cap\{\beta Cl(V) : F \subset V \text{ and } V \text{ is } rg\beta - open \text{ in } X\}$. Therefore, we obtain $F \supset \cap\{\beta Cl(V) : F \subset V \text{ and } V \text{ is } rg\beta - open \text{ in } X\}$ and hence $F = \cap\{\beta Cl(V) : F \subset V \text{ and } V \text{ is } rg\beta - open \text{ in } X\}$.

$(c) \Rightarrow (d)$ Let $F \in RC(X)$ and A be a nonempty subset of X such that $A \cap F = \phi$. Take a point $x \in A$ so that $x \in X - F$. Therefore, there exists an $rg\beta$-open set V such that $F \subset V$ and $x \in X - \beta Cl(V)$. Now, put $U = X - \beta Cl(V)$, then we have $U \in \beta O(X)$, $A \cap U \neq \phi$, and $U \cap V = \phi$.

$(d) \Rightarrow (a)$ For each $F \in RC(X)$ and each point $x \in X - F$, there exists a β-open set U and an $rg\beta$-open set V such that $x \in U$, $F \subset V$, and $U \cap V = \phi$. Since $F \in RC(X)$, we have $F \subset \beta Int(V)$, $\beta Int(V) \in \beta O(X)$, and $U \cap \beta Int(V) = \phi$. This shows that X is almost β-regular. □

Remark 4.7. *We can obtain characterizations of almost* β*-regular spaces by replacing "V is* $rg\beta$*-open" in (b), (c), and (d) of Theorem 4.6 with "V is* $g\beta$*-open". The proof is quite similar to that of Theorem 4.6.*

Theorem 4.8. *Almost* β*-regularity is regular open hereditary.*

Lemma 4.9. *If Y is an open subspace of a space X and A is a subset of Y, then* $\beta Cl_Y(A) = Y \cap \beta Cl(A)$.

Lemma 4.10. *If Y is an open subspace of a space X and* $A \in \beta O(X)$, *then* $A \cap Y \in \beta O(Y)$.

Theorem 4.11. *If X is an almost* β*-regular space and Y is an open set of X, then the subspace Y is almost* β*-regular.*

Proof. Let $G \in RO(Y)$ and $x \in G$. There exists $H \in RO(X)$ such that $G = Y \cap H$, since $G = Int_Y(Cl_Y(G)) = Int(Cl(G) \cap Y) = Int(Cl(G)) \cap Y$. Since X is almost β-regular, there exists $U \in \beta O(X)$ such that $x \in U \subset \beta Cl(U) \subset H$. We have $x \in U \cap Y \subset \beta Cl(U) \cap Y \subset H \cap Y$ and $U \cap Y \in \beta O(Y)$ by Lemma 4.10. By Lemma 4.9, we obtain $\beta Cl_Y(U \cap Y) = \beta Cl(U \cap Y) \cap Y \subset \beta Cl(U) \cap Y$ and hence $x \in U \cap Y \subset \beta Cl_Y(U \cap Y) \subset G$. Therefore, Y is almost β-regular. □

Corollary 4.12. *If X is an almost β-regular space and X_0 is a regular open set of X, then the subspace X_0 is almost β-regular.*

5. Preservation Theorems

Definition 5.1. *A function $f: X \to Y$ is said to be*

(a). almost continuous [21] if $f^{-1}(V)$ is open in X for every $V \in RO(Y)$,

(b). R-map [3] if $f^{-1}(V) \in RO(X)$ for every $V \in RO(Y)$,

(c). almost open [21] if $f(U)$ is open in Y for every $U \in RO(X)$,

(d). M-β-open if $f(U) \in \beta O(Y)$ for every $U \in \beta O(X)$,

(e). weakly open [18] if $f(U) \in Int(f(Cl(U)))$ for every open set U of X.

Remark 5.2. *A R-map is also said to be regular irresolute by Palaniappan and Rao [16]. In [18], it was shown that almost openness implies weak openness but the converse is false.*

Theorem 5.3. *If $f : X \to Y$ is an almost continuous and almost β-closed surjection with compact point inverses and X is regular, then Y is almost β-regular.*

Proof. Let F be a regular closed set of Y and $y \in Y - F$. We have $f^{-1}(Y) \cap f^{-1}(F) = \phi$. Since $f^{-1}(Y)$ is compact and $f^{-1}(F)$ is closed in the regular space X, there exist disjoint open sets U_0, V_0 of X such that $f^{-1}(Y) \subset U_0$ and $f^{-1}(F) \subset V_0$. Now, put $U = Int(Cl(U_0))$ and $V = Int(Cl(V_0))$, then U and V are disjoint regular open sets such that $f^{-1}(Y) \subset U$ and $f^{-1}(F) \subset V$. Since f is almost β-closed, by Theorem 3.6, there exist β-open sets G, H of Y such that $y \in G$, $F \subset H$, $f^{-1}(G) \subset U$ and $f^{-1}(H) \subset V$. Moreover, G and H are disjoint since U and V are disjoint. This shows that Y is almost β-regular. □

Corollary 5.4. *If $f : X \to Y$ is an almost continuous and β-closed surjection with compact point inverses and X is regular, then Y is almost β-regular.*

Theorem 5.5. *If $f : X \to Y$ be an M-β-open β-$rg\beta$-closed surjective R-map. If X is almost β-regular, then Y is almost β-regular.*

Proof. Let $F \in RC(Y)$ and $y \in Y - F$. Then $f^{-1}(Y)$ and $f^{-1}(F) = \phi$ are disjoint. Since f is an R-map, $f^{-1}(F)$ is regular closed in X. For each $x \in f^{-1}(Y)$, there exist disjoint β-open sets U and V of X such that $x \in U$ and $f^{-1}(F) \subset V$. Since f is M-β-open, we have $y = f(x) \in f(U)$ and $f(U) \in \beta O(Y)$. Since f is β-$rg\beta$-closed, by Theorem 3.7, there exists an $rg\beta$-open set W of Y such that $F \subset W$ and $f^{-1}(W) \subset V$. Since $f(U)$ and W are disjoint, by Theorem 4.6, we obtain that Y is almost β-regular. □

Corollary 5.6. *If $f : X \to Y$ be an almost continuous, almost open, M-β-open, M-β-closed function from an almost β-regular space X on to a space Y, then Y is almost β-regular.*

Proof. Every almost continuous almost open function is an R-map. Every M-β-closed function is β-$rg\beta$-closed and the proof follows immediately from Theorem 5.5. □

Theorem 5.7. *If $f : X \to Y$ be continuous weakly open β-$rg\beta$-closed surjection and X is almost β-regular, then Y is almost β-regular.*

Proof. First we show that f is an R-map. Let V be any regular open set of Y. Since f is continuous, $f^{-1}(V)$ is open in X and hence $f^{-1}(V) \subset Int(Cl(f^{-1}(V)))$. Since f is a weakly open continuous surjection, we have $f(Int(Cl(f^{-1}(V)))) \subset Int[f(Cl(Int(Cl(f^{-1}(V)))))] \subset Int(Cl(f(f^{-1}(V)))) \subset Int(Cl(V)) = V$. Therefore, we obtain $Int(Cl(f^{-1}(V))) \subset f^{-1}(V)$ and hence $Int(Cl(f^{-1}(V))) = f^{-1}(V)$. Thus, $f^{-1}(V)$ is regular open in X and f is an R-map. Next, we show that f is M-β-open. Let U be any β-open set in X. Then, we have $f(U) \subset f(Int(Cl(U))) \subset Int[f(Cl(Int(Cl(U))))] \subset Int[f(Cl(U))] \subset Int(Cl(f(U)))$. Therefore, $f(U)$ is β-open in Y and f is M-β-open. Theorem 5.5 completes the proof. □

6. Conclusion

In this paper, we introduce and study a new class of spaces, namely almost β-regular spaces and to obtain some characterizations of almost β-regular spaces. Further, by using β-closed sets, we define almost β-closed functions and to obtain preservation theorems of almost β-regular spaces. The relationships among p-regular, β-regular, almost regular, almost p-regular, almost β-regular spaces are investigated. The main result of this paper is that almost β-regularity is preserved under M-β-open β-$rg\beta$-closed surjective R-maps.

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