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Some Properties of Tri-b Open Sets in Tri Topological Space

Ranu Sharma[1,*], Bhagyashri A. Deole[1] and Smita Verma[1]

1 Department of Applied Mathematics and Computational Science, SGSITS, Indore, Madhya Pradesh, India.

Abstract: The main aim of this paper is to study tri b-open sets in tri topological spaces along with their several properties and characterization. We study tri b-continuous, tri-b separation and obtain some of their basic properties.

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1. Introduction

The idea of b-open sets in a topological space was given by Andrijvic [1] in 1996. In 1961 Kelly [4] introduced the concept of bitopological space. Al-Hawary [3] defined the notion of b-open set and b-continuity in bitopological space and established several fundamental properties. Abo Khadra and Nasef [2] discussed b-open set in bitopological spaces. Tri topological space is a generalization of bitopological space. The study of tri-topological space was first initiated by Martin Kovar [6]. Palaniammal [7] and Hameed [5] studied separation axioms in tri-topological spaces and gives the definition of 123 open set in tri topological spaces. Tapi [9] introduced semi open and pre open set in tri topological space. Priyadharsini [8] introduced tri-b open sets in tri topological spaces. The purpose of the present paper is to study b-open sets in tri topological space and their fundamental properties in tri topological space. In this paper, we are using the name tri-open set in place of 123 open sets.

2. Preliminaries

Definition 2.1 ([7]). *Let X be a nonempty set and T_1, T_2 and T_3 are three topologies on X. The set X together with three topologies is called a tri topological space and is denoted by (X, T_1, T_2, T_3).*

Definition 2.2 ([5]). *A subset A of a topological space X is called 123 open set if $A \in T_1 \cup T_2 \cup T_3$ and complement of 123 open set is 123 closed set.*

Definition 2.3 ([8]). *Let (X, T_1, T_2, T_3) be a tri topological space, a subset A of a space X is said to be tri-b open set if $S \subset tri-cl(tri-intS) \cup tri-int(tri-clS)$.*

* *E-mail: ranusamarth@gmail.com*

Definition 2.4 ([8]). *Let (X, T_1, T_2, T_3) be a tri topological space and let $A \subset X$. The intersection of all tri-b closed sets containing A is called the tri-b closure of A and denoted by $tri - b - clA$. Tri-b-intA is the union of all tri-b open sets contained in A, and $tri - b - clA$ is the intersection of all tri-b closed sets containing A.*

Definition 2.5 ([8]). *Let X and Y be two tri topological spaces. A function $f : (X, T_1, T_2, T_3) \to (Y, \sigma_1, \sigma_2, \sigma_3)$ is said to be tri-b continuous at a point $b \in X$ if for every tri-b open set V containin $gf(b)$, $\exists$ a tri-b open set F containing b, such that $f(F) \subset V$.*

Definition 2.6 ([9]). *Let (X, P_1, P_2, P_3) be a tri topological space then a subset A of X is said to be tri semi-open set if $A \subseteq tri - cl\,(tri - int\,(A))$.*

Definition 2.7 ([9]). *Let (X, P_1, P_2, P_3) be a tri topological space then a subset A of X is said to be tri pre open set if $A \subseteq tri - int\,(tri - cl\,(A))$.*

3. Tri-b Open Sets in Tri Topological Space

Theorem 3.1. *Let (X, T_1, T_2, T_3) be tri topological space, and $F \subseteq X$. Then*

(a). $(tri - bcl(F))^c = (tri - bcl(F^c))$.

(b). $(tri - bint(F))^c = (tri - bcl(F^c))$.

Proof. Let $F \subseteq X$ where, $(X, T_1, T_2, T_3)$is a tri topological space.

(a). Now,

$$\begin{aligned}(tri - bcl(F)) &= \cap\{A : F \subset A\, and\, \text{A is tri-b closed set}\}\\ (tri - bcl(F))^c &= [\cap\{A : F \subset A\, and\, \text{A is tri-b closed set}\}]^c\\ &= \cup\{A^c : A^c \subset F^c\, and\, A^c\ \text{ is tri-b open set}\}\\ &= tri - bint(F^c)\end{aligned}$$

(b). Similarly, $(tri - bint(F))^c = (tri - bcl(F^c))$. □

Proposition 3.2. *Let G be a subset of a tri topological space X. Then*

(1). $pscl(F) = F \cup tri - cl(tri - int(tri - cl(F)))$

$psint(F) = F \cap tri - int(tri - cl(tri - int(F)))$

$pscl(p\sin t(F)) = tri - cl(tri - int(F))$

$p\sin t(pscl(F)) = tri - int(tri - cl(F))$.

(2). $scl(F) = F \cup tri - int(tri - cl(F))$

$sint(F) = F \cap tri - cl(tri - int(F)))$.

(3). $pcl(F) = F \cup tri - cl(tri - int(F))$

$pint(F) = F \cap (tri - int(tri - cl(F)))$.

(4). $spcl(F) = F \cup tri - int(tri - cl(tri - int(F)))$

$spint(F) = F \cap tri - cl(tri - int(tri - cl(F)))$.

Theorem 3.3. *In a tri topological space (X, T_1, T_2, T_3), for a subset F of X, the following are equivalent:*

(1). F is tri-b open.

(2). $F = pintF \cup \sin tF$

(3). $F \subseteq pcl(pintF)$.

Proof. Let $F \subseteq X$, where (X, T_1, T_2, T_3) is a tri topological space.

(1) ⇒ (2) Suppose that F is a tri-b open set. So, $F \subset (tri - cl(tri - intF)) \cup (tri - int(tri - clF))$. Now,

$$
\begin{aligned}
pintF \cup \sin tF &= \{F \cap tri - int(tri - clF)\} \cup \{F \cap tri - cl(tri - intF)\}, \quad \text{(By Proposition 3.2 (2) \& (3))}\\
&= F \cap \{tri - int(tri - clF)\} \cup \{tri - cl(tri - intF)\}\\
&= F
\end{aligned}
$$

Therefore $F = pintF \cup \sin tF$.

(2) ⇒ (3) Let $F = pintF \cup sintF$ (by Proposition 3.2 (2) & (3)), we have

$$
\begin{aligned}
F &= pintF \cup (F \cap tri - cl(tri - intF)\\
&\subseteq pintF \cup (F \cap tri - cl(tri - intF)\\
&= pcl\,(pintF)\,,\\
i.e., \quad F &\subseteq pcl\,(pintF)\,.
\end{aligned}
$$

(3) ⇒ (1) Let $F \subseteq pcl(pintF)$. Then

$$
\begin{aligned}
F &\subseteq pintS \cup tri - cl(tri - intF) && \text{(by Proposition 3.2 (2))}\\
i.e., \quad F &\subseteq F \cap tri - int(tri - clF)) \cup (F \cap tri - cl(tri - intF)) && \text{(by Proposition 3.2 (3))}\\
&= \{F \cup tri - cl(tri - intF)\} \cap \{tri - int(tri - clF) \cup tri - cl(tri - intF)\}
\end{aligned}
$$

i.e., F is a tri-b open set. □

Note 3.4.

(a). Every tri-b open set can be represented as a union of tri pre-open set and a tri semi open set (by Theorem 3.3(b)).

(b). If F be a tri-b open set such that $tri - intF = \phi$ then $sintF = F \cap cl(intS)$ provides that $tri - intF = \phi$. Consequently, we have $F = pintF \cup \sin tS = pintS$ i.e S is a tri pre open set.

Theorem 3.5. *If (X, T_1, T_2, T_3) be a tri topological space, then*

(a). The intersection of a tri-α open set and a tri-b open set is a tri-b open set.

(b). tri-α and tri topological spaces have the same class of tri-b open set.

Proof. Let (X, T_1, T_2, T_3) be a tri topological space.

(a). Let F be a tri-αopen set and G be a tri-b open set. Now,

$$\begin{aligned} S &= F \cap G \\ &= tri-\alpha F \cap tri-bintG \\ &\subseteq tri-bF \cap tri-bintG \\ &= tri-bint(F \cap G) \\ &= tri-bint(S) \\ i.e., \; S &\subseteq tri-bint(S) \end{aligned}$$

But $tri-bint(S) \subseteq S$. Hence $S = tri-bint(S)$ i.e., $S = F \cap G$ is a tri-b open set. □

Theorem 3.6. *Let (X, T_1, T_2, T_3) be a tri topological space and S be a subset of X, then*

(a). $tri-bclS = sclS \cap pclS$.

(b). $tri-bintS = \sin tS \cup pintS$.

Proof. Let (X, T_1, T_2, T_3) be a tri topological space and$S \subseteq X$.

(a). Since $tri-bclS$ is a tri-b closed set. Hence, $tri-int(tri-cl(tri-bclS)) \cap tri-cl(tri-int(tri-bclS)) \subseteq tri-bclS$. Again,

$$\begin{aligned} tri-int(tri-clS)) \cap tri-cl(tri-intS) &\subseteq tri-int(tri-cl(tri-bclS)) \cap tri-cl(tri-int(tri-bclS)) \\ tri-int(tri-clS) \cap tri-cl(tri-intS) &\subseteq tri-bclS \\ S \cup tri-int(tri-clS) \cap tri-cl(tri-intS) &\subseteq S \cup tri-bclS \\ sclS \cap pclS &\subseteq tri-bclS \qquad \text{(i)} \end{aligned}$$

Next,

$$\begin{aligned} tri-bclS \subseteq sclS \;\; & \;\; tri-bclS \subseteq pclS \\ tri-bclS \subseteq sclS \cap pclS & \qquad \text{(ii)} \end{aligned}$$

From (i) and (ii), it follows that $tri-bclS = sclS \cap pclS$.

(b). Since $tri-bintS$ is a $tri-b$ open set, we have

$$tri-cl(tri-int(tri-bintS)) \cup tri-int(tri-cl(tri-bintS)) \supseteq tri-bintS$$

Again,

$$\begin{aligned} tri-cl(tri-int(tri-bintS)) \cup tri-int(tri-cl(tri-bintS)) &\subseteq tri-cl(tri-intS) \\ tri-bintS &\subseteq (tri-cl(tri-intS)) \cup tri-int(tri-clS) \\ S \cap tri-bintS &\subseteq (S \cap tri-cl(tri-intS)) \cup \{S \cap tri-int(tri-clS)\} \\ tri-bintS &\subseteq sintS \cup pintS \qquad \text{(i)} \end{aligned}$$

Next,

$$sintS \subseteq tri - bintS \ \ \& \ \ pintS \subseteq tri - bintS$$

$$sintS \cup pintS \subseteq tri - bintS \tag{ii}$$

From (i) and (ii), it follows that $tri - bintS = sintS \cup pintS$. □

Theorem 3.7. *If F be a subset of a tri topological space (X, T_1, T_2, T_3), then $tri - bint\,(tri - bcl\ F) = tri - bcl\,(tri - bintF)$.*

Proof. Let (X, T_1, T_2, T_3) be a tri topological space. Now,

$$\begin{aligned} tri - bint\,(tri - bcl\ F) &= sint(tri - bclF) \cup pint\,(tri - bclF) \\ &= tri - bcl\,(sint\ F) \cup pint\,(tri - bclS) \qquad \text{(i)} \\ tri - bcl\,(tri - bint\ F) &= tri - bcl(\sin tS \cup pintS) \\ &= tri - bcl(\sin tS) \cup tri - bcl(pintS) \\ &= scl(\sin tS) \cup pint(pclS) \qquad \text{(ii)} \end{aligned}$$

Hence from (i) and (ii), $tri - bint\,(tri - bcl\ F) = tri - bcl\,(tri - bintF)$. □

Theorem 3.8. *A subset B of a tri topological space (X, T_1, T_2, T_3) is tri-b open if and only if every closed set F containing B, there exists the union of maximal tri open set M contained in $tri - cl\,(B)$ and the minimal tri closed set N containing $tri\,int\,(B)$ such that $B \subseteq M \cup N \subseteq F$.*

Proof. Let A be a tri-b open set in a tri topological space (X, T_1, T_2, T_3). Then

$$B \subseteq tri - cl(tri - int(B)) \cup tri - int(tri - cl(B)) \tag{1}$$

Let $B \subseteq F$ and F is tri-closed so that$tri - cl\,(B) \subseteq F$. Let $M = tri - int\,(tri - cl\,(B))$, then M is the maximal open set contained in $tri - cl\,(B)$. Let $N = tri - cl\,(tri - int\,(B))$, then N is the minimal closed set containing $tri - int\,(B)$. Again, $B \subseteq tri - cl\,(B) \subseteq F$ and $tri - int\,(tri - cl\,(B)) \subseteq tri - cl\,(B)$.

$$\Rightarrow tri - int\,(tri - cl\ (\mathrm{B})) \subseteq F \tag{2}$$

Next $tri - int\,(B) \subseteq B \Rightarrow tri - cl\,(tri - int\,(B)) \subseteq tri - cl\,(B)$ and $tri - cl\,(B) \subseteq F$.

$$\Rightarrow tri - cl\,(tri - intB) \subseteq F \tag{3}$$

From (2) and (3), we have

$$tri - int\,(tri - cl\,(B)) \cup tri - cl\,(tri - int\,(B)) \subseteq F \tag{4}$$

Combining (1) and (4), we have $B \subseteq tri - cl\,(tri - int\,(B)) \cup tri - int\,(tri - cl\,(B)) \subseteq F$ or $B \subseteq M \cup N \subseteq F$.

Conversely, assume that the condition holds good i.e., $B \subseteq M \cup N \subset F$, where B is a subset in a tri topological space, F is closed and M is the maximal tri open set contained in a $tri - cl(B)$, N is the minimal closed set containing $tri - int(B)$. Therefore, $M = tri - int\,(tri - cl\,(B))$ and $N = tri - cl\,(tri - int\,(B))$. Thus, the above condition reduces to $B \subseteq tri - cl\,(tri - int\,(B)) \cup tri - int\,(tri - cl\,(B)) \subseteq F$. This means that B is a tri-b open set. □

Theorem 3.9. *A subset B in a tri topological space* (X, T_1, T_2, T_3) *is tri-b open if and only if there exists a tri pre-open set U in* (X, T_1, T_2, T_3) *such that* $U \subseteq B \subseteq pcl\,(U)$.

Proof. Let $B \subset X$. Then by Theorem 3.8,

$$B \subseteq pcl\,(pint\ B) \tag{5}$$

Now, as usual $pintB \subseteq B$ and $U = pint\,(B) =$ a tri pre-open set. Hence, from (5) it follows that $U \subseteq B \subseteq pcl\,(U)$. Conversely, for a set B there exists tri pre-open set U such that

$$U \subseteq B \subseteq pcl\,(U) \tag{6}$$

Since, $pint\,(B)$ is the maximal tri pre-open set contained in B. Hence

$$U \subseteq pint\,(B) \subseteq B \tag{7}$$

Now,

$$pcl\,(U) \subseteq pcl\ \ (pintB) \quad \text{[from (7)]} \tag{8}$$

Combining (6) and (7) we get, $B \subseteq pcl\,(pint\ B)$, which means that B is a tri-b open set. □

Corollary 3.10. *A subset B in a topological space* (X, T_1, T_2, T_3) *is tri-b open if and only if it contains tri pre-open set but not its tri pre-closure.*

Theorem 3.11. *If H is a tri- b open set in a space* (X, T_1, T_2, T_3)*then* $H \subseteq tri - cl\,(tri - intH) \cup tri - int(tri - clH)$ *is a tri pre-open set.*

Proof. Let H be a tri b-open set in a space(X, T_1, T_2, T_3), then $H \subseteq tri - cl\,(tri - intH) \cup tri - int\,(tri - cl\ H)$. Since, $tri - int\ B \subseteq B$ for all $B = X$, hence substituting $tri - cl\,(tri - int\ H)$ for B, we have,

$$tri - int\,(tri - cl\,(tri - intH)) \subseteq tri - cl\,(tri - int\ H)\,.$$

This means that $tri - int\,(tri - cl\,(tri - int\ H))$ is a tri semi-closed set. And in term it is tri b-closed. Now, $S = H - tri - int(tri - cl\,(tri - int\ H))$ is tri- b open. Also, $tri - int\ S = \varphi$. Using Note 3,.4 (b), the above two facts provide that S is a tri pre-open set. □

4. Tri-b Continuous Function in Tri Topological Space

Definition 4.1. *A function* $f : (X, T_1, T_2, T_3) \to (Y, \sigma_1, \sigma_2, \sigma_3)$ *is said to be tri b-closed (respectively tri-b open) if for every tri-b closed (respectively tri-b open) subset B of X,* $f\,(B)$ *is tri b-closed (respectively tri b-open) in Y.*

Definition 4.2. *Let* (X, T_1, T_2, T_3) *and* $(Y, \sigma_1, \sigma_2, \sigma_3)$ *be two tri-topological spaces. A function* $f : X \to Y$ *is called tri-b open map if* $f(G)$ *tri-b open in* Y *for every tri-b open set* G *in* X.

Definition 4.3. *Let* (X, T_1, T_2, T_3) *and* $(Y, \sigma_1, \sigma_2, \sigma_3)$ *be two tri topological spaces. Let* $f : X \to Y$ *be a mapping. f is called tri- b closed map if* $f(F)$ *is tri-b closed in* Y *for every tri-b closed set* F *in* X.

5. Tri-gb Open Sets in Tri Topological Space

Definition 5.1. *A subset F of a tri topological space (X, T_1, T_2, T_3) is said to be tri-gb-closed if $tri - bcl(F) \subset V$ whenever $F \subset V$ and V is tri open set.*

Remark 5.2.

(a). The complement of tri-gb closed is tri-gb open.

(b). The intersection of all tri closed sets of V containing a subset B of V is called tri-gb-closure of B and is denoted by $tri - gb - cl(B)$ and the union of all tri-gb open sets contained in B denoted by $tri - gbint(B)$ is called tri-gb-interior of B.

Theorem 5.3. *Every tri closed subset of a tri topological space V is tri-b closed.*

Proof. Let $B \subset V$ is a tri closed set, since $B^0 \subset triclB^0$, hence $triintB^0 \subset triint(triclB^0)$, hence $triintB \subset B$ for any subset B, hence $B^0 \subset triint(triclB^0)$ and $B^0 \subset triint(triclB^0) \cup tricl(triintB^0)$ hence B^0 is tri-b open set, hence B is tri-b open set. □

Theorem 5.4. *Every tri-b closed subset of a tri topological space V is tri-gb closed.*

Proof. Let $B \subset V$ is a tri-b closed set, and let $B \subset F$, where F is tri-b open, since B is tri-b closed set, hence $tri - int(tri - clB) \cap tri - cl(tri - intB) \subset B$, $tri - int(tri - clB) \cap tri - cl(tri - intB) \subset B$, $tri - int(tri - clB) \cap tri - cl(tri - intB) \subset F$ because $tri - cl - bB$ is the smallest $tri - b - cl$ set containing B,

$$tricl(B) = B \cup triint(tricl(B)) \cap tricl(triint(B)) \subset B \subset B \cup V \subset V,$$

i.e., B is tri b-closed. □

6. Tri-b Separation Axioms in Tri Topological Space

Definition 6.1. *A tri topological space X is said to be $tri - b - T_0$ space if and only if to given any pair of distinct points x_1, y_1 in V, there exists a tri-b open set containing one of the points but not the other*

Example 6.2. *Let $X = \{a, b, c\}$, $T_1 = \{X, \phi\}$, $T_2 = \{X, \phi, \{a\}\}$, $T_3 = \{X, \phi, \{b, c\}\}$ Tri open sets in tri topological spaces are union of all tri topologies. Then tri open sets of $X = \{X, \phi, \{a\}, \{b, c\}\}$ Tri-b open set of X is denoted by $tri - BO(X) = \{X, \phi, \{a, b\}, \{c\}, \{a, c\}, \{b, c\}\}$. So (X, T_1, T_2, T_3) is $tri - b - T_0$ space.*

Theorem 6.3. *If $\{x_1\}$ is tri-b-open for some $x_1 \in V$ then $x_1 \in tri - b - cl\{y_1\}$, for all $y_1 \neq x_1$.*

Proof. Let $\{x_1\}$ be a tri-b-open for some $x_1 \in V$, then $V - \{x_1\}$ is tri-b closed. If $x_1 \in tri - b(\{y_1\})$, for some $y_1 \neq x_1$, then y_1, x_1 both are in all the tri-gb-closed sets containing y_1, so $x_1 \in V - \{x_1\}$ which is contradiction, hence $x_1 \in tri - cl - b(\{y_1\})$. □

Theorem 6.4. *In any tri topological space (X, T_1, T_2, T_3), any distinct points have distinct tri-b-closure.*

Proof. Let $x_1, y_1 \in X$ with $x_1 \neq y_1$, and let $B = \{x_1\}^c$ hence $tri - b - cl(B) = B$ or U. Now if then $tri - b - cl(B) = B$, then B is tri-b-closed so $X - A = \{x_1\}$ is tri-b open and not containing y_1. So by Theorem 6.3 $x_1 \notin tri - b - cl(y_1)$ and $y_1 \in tri - b - cl(y_1)$ which implies that $tri - b - cl(y_1)$ and $tri - b - cl(x_1)$ are distinct. If $tri - b - cl(B) = X$ then A is tri-b open, hence $\{x_1\}$ is tri-b closed, which mean that $tri - b - cl(\{x_1\}) = \{x_1\}$ which is not equal to $tri - b - cl(\{y_1\})$. □

Theorem 6.5. *In any tri topological space X, if distinct points have distinct tri-b closure then U is* $tri-b-T_0$ *space.*

Proof. Let $x_1, y_1 \in V$ with $x_1 = y_1$, with $tri-cl-b(\{y_1\})$ is not equal to $tri-b-cl(\{x_1\})$, hence there exists $z \in V$ such that $z \in tri-b-cl\{x_1\}$ but $z \notin tri-b-cl\{y_1\}$ or $z \in tri-b-cl\{x_1\}$ but now without loss of generality, let $z \in tri-b-cl(\{x_1\})$. But $z \notin tri-b-cl(\{x_1\})$, if $x \in tri-b-cl(\{y_1\})$, then $tri-b-cl(\{x_1\})$ is contained in $tri-b-cl(\{y_1\})$, hence $z \in tri-cl(\{y_1\})$, which is a contradiction, this mean that $x \notin tri-b-cl\{y_1\})$ hence $z \notin tri-b-cl\{y_1\}x_1 \in tri-cl-b\{y_1\}^c$, hence X is tri-b -$T_0$ space. □

Definition 6.6. *A tri topological space X is said to be tri-b-*T_1 *space if and only if to given any pair of distinct point* x_1 *and* y_1 *of X there exist two tri-b-open sets U, V such that* $x_1 \in U_1$, $y_1 \notin U_1$ *and* $y_1 \in V_1$, $x_1 \notin V_1$.

Theorem 6.7. *Every tri-*T_1 *space is a tri-b-*T_0 *space.*

Definition 6.8. *A tri topological space X is said to be tri-b-*T_2 *space if and only if for* $x_1, y_1 \in X$, $x_1 \neq y_1$ *there exist two disjoint tri-b open sets* U_1, V_1 *in X such that* $x_1 \in U_1$, $y_1 \in V_1$.

Theorem 6.9. *Every tri-b-*T_2 *space is tri-b-*T_1 *space.*

Proof. Let X is tri-b T_2 space and let x_1, y_1 in X with $x_1 \neq y_1$, so by hypothesis there exist two disjoint tri-b open, say U_1, V_1 such that $x_1 \in U_1$ and $y_1 \in V_1$ but $U_1 \cap V_1 = \phi$ hence $x_1 \notin V_1$ and $y_1 \notin U_1$ i.e., X is tri-b-T_1 space. □

Definition 6.10. *A tri topological space X is said to be tri-regular space if and only if for each tri-b closed set G and each point* $x_1 \notin G$, *there exist disjoint tri-b open sets* U_1 *and* V_1 *such that* $x \in U_1$ *and* $G \in V_1$.

Theorem 6.11. *Every tri-b-*T_3 *space is a tri-b-*T_2 *space.*

Proof. Let (X, T_1, T_2, T_3) be a tri-bT_3 space and letx_1, y_1 be two distinct points of X. Now by definition, X is also a tri-b-T_1 space and so $\{x_1\}$ is a tri- closed set. Also $y_1 \notin \{x_1\}$. Since (X, T_1, T_2, T_3) is a tri-b regular space, there exist tri-b open sets G_1 and H_1 such that $\{x_1\} \subset G_1, \{y_1\} \subset H_1$ and $G_1 \cap H_1 = \phi$. Also $\{x_1\} \subset G_1 \Rightarrow x_1 \in G_1$. Thus x_1, y_1 belong respectively to disjoint tri-b open sets G_1 and H_1. According (X, T_1, T_2, T_3) is a $tri-b-T_2$ space.
Since $B \subset tri-b-clB$, hence $tri-bintB \subset tri-bint(tri-bclB)$, since $tri-bintB \subset B$ for any subset B, hence $B \subset tri-b-int(tri-b-clB)$ and $B \subset tri-bint(tri-bclB) \cup tri-bcl(tri-bintB)$, hence B is tri b open set. □

7. Conclusion

In this paper the idea of tri-b separation and tri-gb open set in tri topological spaces were introduced and tri-b continuity were studied, Also properties of tri-b open set and tri-b closed sets were studied.

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