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An Inventory Model for Deteriorating Products with Demand Appraise by Promotional Effort

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Abstract: In Modern era when trends in technology, fashion and consumer change rapidly; the major problem faced by the retailer is how they can manage the deteriorating stock at time of product's self-life. In general they use some promotions efforts like price discount to prop up demand. We have tried to deal with such problem in this paper. We study the inventory problem of non-instantaneous single period deteriorating items with two parameter weibull distribution. We are considering two types of demand in this paper. First, it is function of time and then contain price factor to grown promotional effort. The holding cost is considered as constant. An approach is proposed to maximize total profit and optimal order quantity. Numerical examples illustrate the theoretical results and sensitivity analysis done as well.

Keywords: Deterministic inventory model, weibull distribution, promotional efforts, demand function.

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1. Introduction

Inventory control is an essential part of a well doing business, without a proper inventory management a business may run out of stock of essential items or it may run out at an important time that will result into loss of income and goodwill. Deterioration is a natural phenomenon which cannot be ignored in real life. In genuine situation, the life cycle of seasonal products, fruits, electric components, volatile liquids, food, etc. are small and finite and usually can suffer deterioration. Thus, the item may not serve the purpose after a period of time and will have to be rejected as it cannot be used to satisfy the future demand of consumers. The model for Deteriorate item was first developed by Ghare and Schrader [10], where they assumed the deterioration rate as constant after that Covert and Philip [13] extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. This topic has consequently been investigated by many researchers. Sharma and Sharma [1] have developed a model with weibull deterioration and power pattern demand rate with time dependent holding cost with shortage. An inventory model for deteriorating items with time proportional demand was investigated by Dave and Patel [15]. Mehta and shah [9] has suggested an inventory model for deteriorating items with exponentially increasing demand and shortages under inflation and time discounting.

The basic inventory model has first introduced by Harish [3] and [4]. He presented an economic order quantity model which shows that how much a product should be ordered and when orders should take place so that the inventory costs could be minimized. After some time Wilson [12] has extend Harish's model. Deng [11] has improved Inventory Models with Ramp Type Demand and weibull Deterioration. An inventory model for a deteriorating item with, a quadratic time-varying demand and shortages proposed by Ghosh and Chaudhuri [14]. The price dependent model with shortage was developed by

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Sharma [2]. An inventory model for deteriorating items with stock-dependent demand was developed by Hou [7]. He has shown that the total cost function is convex. With the convexity, a simple solution algorithm is presented to determine the optimal order quantity and the optimal interval of the total cost function. Wang [6] used the Verhulst's population growth model to construct a demand function toward EOQ model. Nagre [8] discussed a non-instanious deterioration model with promotional efforts. He first modified price dependent demand function to include demand accrued from promotional effort and developed a model to maximize total profit.

In this paper we extend the work of Wang [6] as follow: (i) the time and price dependent demand function is modified to include demand accrued from promotional effort and developed a modified demand function (ii) the deterioration is considered and it is two parameter weibull distribution deterioration (iii) an example is given to compare Wang at el results (iv) sensitivity analysis is done as well.

1.1. Assumptions

- Demand is price dependent.
- We consider two period in this paper, first period consider no deterioration and in second period (Promotional Period) the rate of deterioration is two parameter weibull distribution (see Dang [11], Sharma [2], Ghosh and Chaurdhry [14]).
- Shortage is not occurring.
- The promotional effort cost is an increasing function of promotional effort(see Tsao, Sheen [16], Wang [6], Nagre [8]).
- Holding cost is constant.
- There is no repair or replenishment of deteriorated items.

1.2. Notations

- $I_1(t)$ = the inventory level that changes with time t during non-deteriorating period.
- $I_2(t)$ = the inventory level that changes with time t during deteriorating period.
- T = Length of selling Period.
- t_p = Non deteriorating period of product and peak demand time .
- a = Price incurred demand coefficient.
- $\emptyset(t)$ = Deterioration rate = $\alpha\beta t^{\beta-1}$.
- q = Order quantity (decision variable).
- P_0 = regular selling price (decision variable).
- P_d = Discount selling Price ($P_0 < P_d$)
- h_1 = Holding cost per unit time in non-deteriorating period.
- h_2 = Holding cost per unit time in deteriorating period.
- D_1 = demand in period ($0 < t < t_d$).
- D_2 = Demand in period ($t_d < t < T$).

- D_p = Price gained demand.
- C_0 = Ordering Cost.
- C_Q = Purchase Cost.
- ρ = Promotional effort demand multiplier.

2. Basic Demand Model

The demand function developed by Wang [6] is shown in equation (1) and (2) and demand pattern shown in Figure 1.

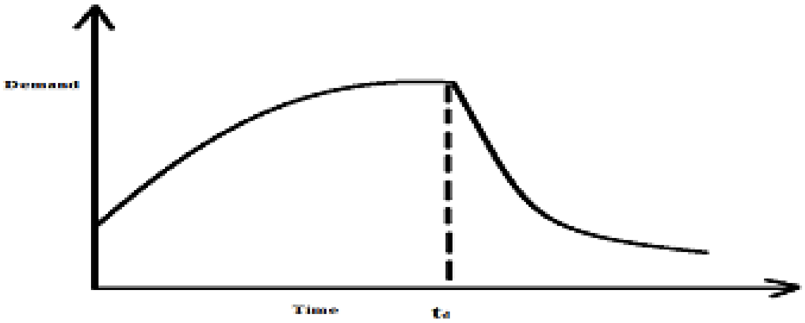

Figure 1.

The demand rate for two intervals in basic demand model is

$$\frac{dD}{dt} = \begin{cases} \lambda D_1\left(m - D_1\left(t\right)\right) & 0 < t \le t_p \\ \lambda D_2\left(-D_1\left(t\right)\right) & t_p < t \le T \end{cases} \tag{1}$$

λ is constant and $D_1\left(t_p\right) = D_2(t_p)$. Solution of equation (1) is

$$D\left(t\right) = \begin{cases} D_1 = \frac{m}{(1+ke^{-m\lambda t})} & 0 < t \le t_p \\ D_2 = \frac{1}{\lambda(t-t_p)+z} & t_p < t \le T \end{cases} \tag{2}$$

Where $k = \frac{m}{D_0} - 1$ and $z = \frac{1+ke^{-m\lambda t}}{m\lambda}$.

2.1. Modified Demand Model

Wang et al improved demand D_2 to add price demand as

$$D_p\left(t\right) = a\left(P_0 - P_d\right); \quad (a > 0) \tag{3}$$

We further modified it to include demand accrued from promotional effort using demand multiplier $(\rho \ge 1)$. Then modified demand become $\rho\left(D_2 + D_p\right)$.

Modified demand will be

$$D\left(t\right) = \begin{cases} \frac{m}{(1+ke^{-m\lambda t})} & 0 < t \le t_p \\ \rho\left\{\frac{1}{\lambda(t-t_p)+z} + a(P_0 - P_d)\right\} & t_p < t \le T \end{cases} \tag{4}$$

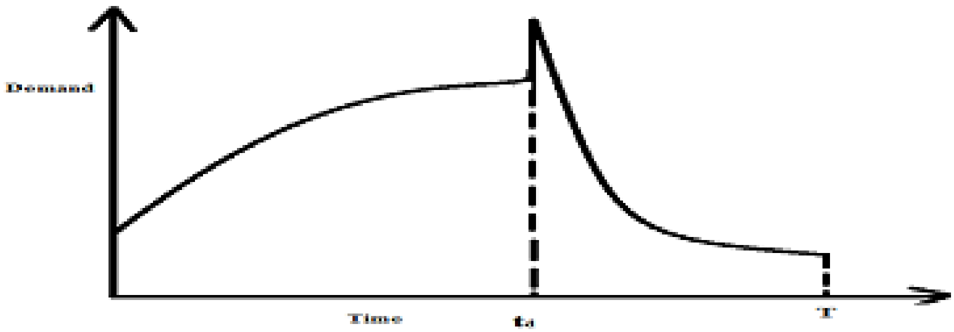

Figure 2.

2.2. Mathematical formulation of the inventory Model

Let $I_1(t)$ and $I_2(t)$ be the net on hand stock level at time t ($0 < t < t_d$ and $t_d < t < T$ respectively). Then the governing differential equations are

$$\begin{aligned} \frac{dI_1}{dt} &= -D_1(t); \quad 0 < t \le t_p \\ \frac{dI_2}{dt} + \emptyset(t) I_2 &= -D_2(t); \quad t_p < t \le T \end{aligned} \tag{5}$$

With boundary conditions $I_1(0) = q$ and $I_2(T) = 0$. Solution of above differential equations will be

$$\begin{aligned} &I_1 = q - mt + \frac{1}{\lambda}\ln(k+1) \\ &I_2 = \frac{\rho}{\lambda} ln\left(\frac{z - t_p}{t + z - t_p}\right) + \frac{\rho\alpha}{\lambda}(\Delta_T - \Delta_t) + \frac{\alpha\alpha\rho}{\beta+1}(P_0 - P_d)\left(T^{\beta+1} - t^{\beta+1}\right) - \frac{\rho\alpha}{\lambda} t^{\beta} \ln\left(\frac{z - t_p}{t + z - t_p}\right) \end{aligned} \tag{6}$$

For the order quantity q: $I_1(t_p) = I_2(t_p)$ i.e.,

$$q = mt_p - \frac{1}{\lambda}\ln(k+1) + \frac{\rho}{\lambda} ln\left(\frac{z - t_p}{z}\right) + \frac{\rho\alpha}{\lambda}\Delta_T + \frac{\alpha\alpha\rho}{\beta+1}(P_0 - P_d)\left(T^{\beta+1} - t^{\beta+1}\right) - \frac{\rho\alpha}{\lambda} t^{\beta} \ln\left(\frac{z - t_p}{z}\right) \tag{7}$$

Suitable units for second interval (Na): $N_a = I_2(t_p) - N_d$. Where $N_d = \int_{t_p}^{T} \emptyset(t) I_2 dt$ i.e.

$$N_a = ln\left(\frac{z - t_p}{z}\right)\left(\frac{\rho}{\lambda} - \frac{\rho\alpha}{\lambda} {t_p}^{\beta} + \frac{{t_p}^{\beta}}{\beta}\right) + \Delta_T\left(\frac{\rho\alpha}{\lambda} - \frac{1}{\beta}\right) + \frac{\alpha\alpha\rho}{\beta+1}(P_0 - P_d)\left(T^{\beta+1} - t_p^{\beta+1}\right) - \frac{T^{\beta}}{\beta} ln\left(\frac{z - t_p}{T + z - t_p}\right) \tag{8}$$

Now, we derive the Expression for total relevant Profit which is covered of the Ordering cost, Purchase Cost, Sale revenue, Promotional cost, Holding Cos and Price Labelling Charge Cost are derived as follows

(1). Ordering Cost

$$Ordering\ Cost = C_0 \tag{9}$$

(2). Purchase Cost

$$Purchase\ Cost = C_q \tag{10}$$

(3). Sales Revenue (R)

$$R = P_0\left(mt_p - \frac{1}{\lambda}\ln(k+1)\right) + N_a P_d \tag{11}$$

(4). Promotional Cost (C_P). By Tsao and Sheen [16]

$$C_p = K(\rho-1)^2 \int_{t_d}^{T} (D_2 + D_p)\, dt$$
$$C_p = K(\rho-1)^2 \frac{1}{\lambda} \ln\left(\frac{T-t_p+z}{z}\right) + a\,(P_0 - P_d)\,(T - t_d) \tag{12}$$

(5). Price Labelling Changing Cost

$$Price\ Labelling\ Changing\ Cost = C_1 \tag{13}$$

(6). Holding Cost (H)

$$H = h_1 \int_0^{t_p} I_1 dt + h_2 \int_{t_p}^{T} I_2 dt$$
$$H = h_1 t_p \left[q - \frac{m t_p}{2} + \frac{1}{\lambda}\ln(k+1)\right] + \frac{h_2\rho}{\lambda}\left[\ln(T+z-t_p)\left(t_p - z - T + \frac{\alpha T^{\beta+1}}{\beta+1}\right)\right.$$
$$+ \ln(z-t_p)\left(T - t_p - \frac{\alpha}{\beta+1}\left(T^{\beta+1} - t_p^{\beta+1}\right)\right) + \ln z\left(z - \frac{\alpha t^{\beta+1}}{\beta+1}\right) + \alpha\Delta_T (T - t_p)$$
$$\left. -\alpha \int_{t_p}^{T} \Delta_t dt + \frac{\alpha\alpha}{\beta+1}(P_0 - P_d)\left(T^{\beta+1}(T-t_p) - \frac{T^{\beta+2} - t_p^{\beta+2}}{\beta+2}\right)\right] \tag{14}$$

Then Total relevant profit $TP = Sale\ revenue(R) - Ordering\ cost\ (C_0)\ Purchase\ Cost(C_Q) - Promotional\ cost\ (C_P) - Holding\ Cost\ (H) - Price\ Labelling\ Charge\ Cost(C_L)$. Hence Total Profit will be

$$TP = P_0\left(m t_p - \frac{1}{\lambda}\ln(k+1)\right) + P_d\left[ln\left(\frac{z-t_p}{z}\right)\left(\frac{\rho}{\lambda} - \frac{\rho\alpha}{\lambda}{t_p}^{\beta} + \frac{{t_p}^{\beta}}{\beta}\right) + \Delta_T\left(\frac{\rho\alpha}{\lambda} - \frac{1}{\beta}\right)\right.$$
$$\left. + \frac{\alpha\alpha\rho}{\beta+1}(P_0 - P_d)\left(T^{\beta+1} - t_p^{\beta+1}\right) - \frac{T^{\beta}}{\beta} ln\left(\frac{z-t_p}{T+z-t_p}\right)\right] - C_0 - C_q - C_1$$
$$- \left[K(\rho-1)^2\frac{1}{\lambda}\ln\left(\frac{T-t_p+z}{z}\right) + \alpha(P_0 - P_d)(T - t_d)\right] - \left[h_1 t_p\left[q - \frac{m t_p}{2} + \frac{1}{\lambda}\ln(k+1)\right]\right.$$
$$+ \frac{h_2\rho}{\lambda}\left[\ln(T+z-t_p)\left(t_p - z - T + \frac{\alpha T^{\beta+1}}{\beta+1}\right) + \ln(z-t_p)\left(T - t_p - \frac{\alpha}{\beta+1}\left(T^{\beta+1} - t_p^{\beta+1}\right)\right)\right.$$
$$\left.\left. + \ln z\left(z - \frac{\alpha t^{\beta+1}}{\beta+1}\right) + \alpha\Delta_T(T-t_p) - \alpha\int_{t_p}^{T}\Delta_t dt + \frac{\alpha\alpha}{\beta+1}(P_0 - P_d)\left(T^{\beta+1}(T-t_p) - \frac{T^{\beta+2} - t_p^{\beta+2}}{\beta+2}\right)\right]\right] \tag{15}$$

For optimality test

Theorem 2.1. *The total profit is strictly concave in P_d*

$$\frac{dTP}{dP_d} = ln\left(\frac{z-t_p}{z}\right)\left(\frac{\rho}{\lambda} - \frac{\rho\alpha}{\lambda}{t_p}^{\beta} + \frac{{t_p}^{\beta}}{\beta}\right) + \Delta_T\left(\frac{\rho\lambda}{\lambda} - \frac{1}{\beta}\right) + \frac{\alpha\alpha\rho}{\beta+1}(P_0 - 2P_d)\left(T^{\beta+1} - t_p^{\beta+1}\right)$$
$$- \frac{T^{\beta}}{\beta} ln\left(\frac{z-t_p}{T+z-t_p}\right) - \frac{h_2\rho}{\lambda}\frac{\alpha\alpha}{\beta+1}(P_0 - P_d)\left(T^{\beta+1}(T-t_p) - \frac{T^{\beta+2} - t_p^{\beta+2}}{\beta+2}\right) \tag{16}$$
$$\frac{d^2TP}{dP_d} = -2\frac{\alpha\alpha\rho}{\beta+1}\left(T^{\beta+1} - t_p^{\beta+1}\right) \tag{17}$$

Hence $\frac{d^2TP}{dP_d}$ is always negative in P_d so The total profit is strictly concave in P_d.

$$P_d^* = \frac{P_0}{2} + \frac{\beta+1}{2\alpha\alpha\rho\left(T^{\beta+1} - t_p^{\beta+1}\right)}\left[\ln\left(\frac{z-t_p}{z}\right)\left(\frac{\rho}{\lambda} - \frac{\rho\alpha}{\lambda}{t_p}^{\beta} + \frac{{t_p}^{\beta}}{\beta}\right) + \Delta_T\left(\frac{\rho\alpha}{\lambda} - \frac{1}{\beta}\right) - \frac{T^{\beta}}{\beta}\ln\left(\frac{z-t_p}{T+z-t_p}\right)\right.$$
$$\left. - \frac{h_2\rho}{\lambda}\frac{\alpha\alpha}{\beta+1}\left(T^{\beta+1}(T-t_p) - \frac{T^{\beta+2} - t_p^{\beta+2}}{\beta+2}\right)\right] \tag{18}$$

2.3. Numerical Example

In this section, we provide a numerical example to illustrate several distinct theoretical results as well as to gain some managerial insights.

Example 2.2. *The proposed model is an extension of the model presented by Wang [6] so Consider the same input parameters from the paper of Wang [6], but assuming that there are not Promotional effort and deterioration* $\theta \approx 0$ *(taking* $a\alpha \ll 0$*) Hence, we have* $C_q = 30$; $h_1 = 0.05$; $h_2 = 0$; $\alpha = 50$; $C_0 = 1000$; $C_1 = 200$; $m = 1000$; $D_1 = 90$; $\lambda = 0.01$; $T = 3t_p = 2$; $\rho = 1$; $\alpha = 0.00001$; $\beta = 1$. *The optimal solution obtained using equations* (15) *to* (18) *is* $P^* = 68.908$, $Q^* = 3544.5$; *and* $TP^* = 188601$.

Example 2.3. *To know the effect of promotional efforts under deterioration we consider* $C_q = 30$; $h_1 = 0.05$; $h_2 = 2$; $\alpha = 50$; $C_0 = 1000$; $C_1 = 200$; $m = 1000$; $D_1 = 90$; $\lambda = 0.01$; $T = 3t_p = 2$; $\rho = 1.1$; $\alpha = 0.00001$; $\beta = 8$. *Then* $P^* = 59.88$; $q^* = 2129$; $TP^* = 492940$. *This result show that the total profit obtain in this model is more than the total profit obtain by Wang [6] after taking promotional results.*

3. Sensitivity Analysis

We present in this section a sensitivity analysis of the optimal inventory strategy, studying whether this strategy is affected by changes in the input parameters. We analyse the behaviour of the total cost TP^*, Optimal order quantity q^* and P_d against changes in the parameters α, β, ρ, and a of the inventory system.

Parameters		P_d*	q^*	TP
α	0.000010	44.88	2111	463100
	0.000011	46.88	2117	462970
	0.000012	48.55	2118	462870
	0.000013	49.96	2122	462800
	0.000014	51.17	2125	462740
β	6	982	1923	302220
	7	305.54	1972	303110
	8	44.88	2111	463100
ρ	1	44.88	2111	463100
	1.1	59.88	2129	492940
	1.2	72.37	2146	523310
	1.3	82.94	2164	554070
	1.4	92.01	2182	585730
a	35	35.29	2101	462800
	40	39.29	2104	462870
	45	42.40	2108	462970
	50	44.88	2111	463100
	55	46.92	2115	463240

Table 1.

The sensitivity analysis shows that when the parameter a increase optimal order quantity q^* increase and total profit TP^* decrease but in case of distribution parameter β increase q^* and TP^* both increases. While increase of Promotional effort demand multiplier (ρ), q^* and TP^* both increases. But sensitivity of ρ is more than of α, β and a.

4. Conclusion

This paper presents an inventory model that includes some realistic features. First, an item is deteriorated over time and follows a weibull distribution that makes a broader application scope. Second, when the demand is failed down then some promotional effort like price discount always works in reality. These assumptions are consistent with economic senses. The numerical example shown in this paper is a comparative the Wang [6] results. However, this paper is open for that assumption. This inventory model can further be elaborate by taking some features such as probabilistic demand rate, shortages with full or partial backlogging, quantity discounts, multiple products, and partial credit trade policy. Also, another possible extension could be to investigate the effect of the repair or replenishment of the deteriorated items.

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