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On m-δ-I-Open Sets and m-δ-I-Continuity

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Abstract: In this paper, we introduce and study the notions of m-δ-I-open sets and m-δ-I-continuity and their related notions in ideal minimal spaces. Also we investigate the decomposition of m-semi-I-continuous functions and m-α-I-continuous functions using m-δ-I-open sets.

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1. Introduction

In 2001, Popa and Noiri introduced the notions of minimal structure and m-continuous function as a function defined between a minimal structure and a topological space [3]. A minimal structure m on a nonempty set X is a collection of subsets of X such that $\emptyset \in m$ and $X \in m$ [3]. By (X, m), we denote a nonempty set X with minimal structure m on X. The members of the minimal structure m are called m-open sets and the complement of m-open set is said to be m-closed [3]. The closure of A and the interior of A are denoted by $Cl(A)$ and $Int(A)$, respectively. An ideal is defined as a nonempty collection I of subsets of X satisfying the following two conditions: (i) If $A \in I$ and $B \subset A$, then $B \in I$; (ii) If $A \in I$ and $B \in I$, then $A \cup B \in I$ [1]. For a subset $A \subset X$, $A^*_m(m, I)=\{x \in X : U \cap A \notin I$ for each neighborhood U of $x\}$ is called the local minimal function of A with respect to I and m [2]. We simply write A^*_m instead of $A^*_m(I, m)$ in case there is no chance for confusion. For every ideal topological space (X, I, m), there exists a minimal structure $m^*(m, I)$ called the $*$-minimal, finner than m. Additionally $mCl^*(A) = A \cup A^*_m$ for every $A \subset X$.

2. Preliminaries

Definition 2.1. *A subset A of a minimal space (X, m) is said to be αm-open[7] (respectively m-preopen [6], m-semiopen [5], βm-open [9], m-b-open [10]) if $A \subset mInt(mCl(mInt(A)))$ (respectively $A \subset mInt(mCl(A))$, $A \subset mCl(mInt(A))$, $A \subset mCl(mInt(mCl(A)))$, $A \subset mInt(mCl(A)) \cup mCl(mInt(A))$).*

Definition 2.2. *Let (X, m) be a minimal space. For a subset A of X, the m-closure of A and m-interior of A are defined in [11] as follows:*

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(a). $mCl(A) = \cap\{F : A \subset F, X - F \in m\}$,

(b). $mInt(A) = \cup\{U : U \subset A, U \in m\}$.

Definition 2.3. *A function* f:$(X, m) \to (Y, \tau)$ *is said to be* αm*-continuous [7] (respectively* m*-semicontinuous [5],* m*-precontinous [6],* βm*-continuous, [9]* m*-b-continuous [10]) if the inverse image of every open set of* Y *is* αm*-open (respectively* m*-semiopen,* m*-preopen,* βm*-open,* m*-b-open) in* (X, m).

Definition 2.4 ([13])**.** *A subset* A *of an ideal minimal space* (X, m, I) *is said to be* m*-semi-*I*-open (resp.* m*-pre-*I*-open,* m*-*α*-*I*-open,* m*-*β*-*I*-open, strongly* m*-*β*-*I*-open,* m*-*δ*-*I*-open) if* $A \subset mCl^*(mInt(A))$ *(respectively* $A \subset mInt(mCl^*(A))$, $A \subset mInt(mCl^*(mInt(A)))$, $A \subset mCl(mInt(mCl^*(A)))$, $A \subset mCl^*(mInt(mCl^*(A)))$, $mInt(mCl^*(A)) \subset mCl^*(mInt(A))$*).*

Remark 2.5. *Let* (X, τ) *be a topological space. The families* $\alpha IO(X, m)$, $SIO(X, m)$, $PIO(X, m)$ *and* $\beta IO(X, m)$ *are all minimal structures on* X.

Definition 2.6 ([13])**.** *A function* f:$(X, m, I) \to (Y, \tau)$ *is said to be* m*-pre-*I*-continuous (respectively* m*-semi-*I*-continuous,* m*-*α*-*I*-continuous,* m*-*δ*-*I*-continuous) if the inverse image of every open set of* (Y, τ) *is* m*-pre-*I*-open (respectively* m*-semi-*I*-open,* m*-*α*-*I*-open,* m*-*δ*-*I*-open) in* (X, m, I).

Lemma 2.7 ([8])**.** *Let* (X, m) *be a minimal space and* A, B *subsets of* X*. Then* $x \in mCl(A)$ *if and only if* $U \cap A \neq \emptyset$ *for every* $U \in m$ *containing* x*. And satisfying the following properties:*

(a). $mCl(mCl(A)) = mCl(A)$.

(b). $mInt(mInt(a)) = mCl(A)$.

(c). $mInt(X - A) = X - mCl(A)$.

(d). $mCl(X - A) = X - mInt(A)$.

(e). If $A \subset B$*, then* $mCl(A) \subset mCl(B)$.

(f). $mCl(A \cap B) \subset mCl(A) \cup mcl(B)$.

(g). $A \subset mCl(A)$ *and* $mInt(A) \subset A$.

Lemma 2.8 ([4])**.** *Let* (X, m) *be a minimal space and* A *a subset of* X*. Then* $x \in mCl(A)$ *if and only if* $U \cap A \neq \emptyset$ *for each* $U \in m$ *containing* x.

3. m-δ-I-Open Set

Definition 3.1. *A subset* A *of minimal space* (X, m, I) *is said to be* m*-*δ*-*I*-open set if* $mInt(mCl^*(A)) \subset mCl^*(mInt(A))$*. The Complement of a* m*-*δ*-*I*-open set is called a* m*-*δ*-*I*-closed set.*

The family of all m-δ-I-open sets of (X, m, I) is denoted by $m\delta IO(X, m)$.

Proposition 3.2. *Let* (X, m, I) *be a minimal space and* A *be a subset of* X*. Then* A *is* m*-semi-*I*-open if and only if it is both* m*-*δ*-*I*-open and strongly* m*-*β*-*I*-open.*

Proof. Let A be a m-semi-I-open set, then we have $A \subset mCl^*(mInt(A)) \subset mCl^*(mInt(mCl^*(A)))$. This shows that A is a strongly m-β-I-open set. Moreover, $mInt(mCl^*(A)) \subset mCl^*(A) \subset mCl^*(mCl^*(mInt(A))) = mCl^*(mInt(A))$. Therefore, A is a m-δ-I-open set. Let A be a m-δ-I-open set and strongly m-β-I-open set, then we have $mInt(mCl^*(A)) \subset mCl^*(mInt(A))$. Thus we obtain that $mCl^*(mInt(mCl^*(A))) \subset mCl^*(mInt(A))$. Since A is a strongly m-β-I-open set, we have $A \subset mCl^*(mInt(mCl^*(A))) \subset mCl^*(mInt(A))$ and $A \subset mCl^*(mInt(A))$. Hence A is a m-semi-I-open set. □

Proposition 3.3. *Let (X, m, I) be a minimal space and A be a subset of X. Then A is an m-α-I-open set if and only if it is both m-δ-I-open set and m-pre-I-open set.*

Proof. Let A be an m-α-I-open set. Clearly every m-α-I-open set is m-pre-I-open set. Since A is an m-α-I-open set, we have $A \subset mInt(mCl^*(mInt(A))) \subset mInt(mCl^*(A))$. Hence A is a m-pre-I-open set. Let A be a m-δ-I-open set and m-pre-I-open set. Then we have $mInt(mCl^*(A)) \subset mCl^*(mInt(A))$ and hence $mInt(mCl^*(A)) \subset mInt(mCl^*(mInt(A)))$. Since A is m-pre-I-open, we have $A \subset mInt(mCl^*(A))$. Therefore we obtain that $A \subset mInt(mCl^*(mInt(A)))$ and hence A is an m-α-I-open set. □

Remark 3.4. *The notion of m-α-I-openness is different from that of strongly m-β-I-openness and m-pre-I-openness.*

Example 3.5. *Let $X = \{a, b, c, d\}$, $m = \{\emptyset, \{a\}, \{a, b\}, \{b, c\}, \{c, d\}, X\}$ and $I = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Then*

(a). $A = \{b, d\}$ is m-δ-I-open but not strongly m-β-I-open.

(b). $A = \{a, b, c\}$ is strongly m-β-I-open but not m-δ-I-open.

(c). $A = \{c\}$ is m-pre-I-open but not m-δ-I-open.

(d). $A = \{a, d\}$ is m-δ-I-open but notm-pre-I-open.

Proposition 3.6. *Let A and B be subsets of a minimal space (X, m, I). If $A \subset B \subset mCl^*(A)$ and A is a m-δ-I-open set, then B is a m-δ-I-open set.*

Proof. Suppose that $A \subset B \subset mCl^*(A)$. Since A is a m-δ-I-open set, we have $mInt(mCl^*(A)) \subset mCl^*(mInt(A))$. Since $A \subset B$, $mCl^*(mInt(A)) \subset mCl^*(mInt(B))$ and $mInt(mCl^*(A)) \subset mCl^*(mInt(B))$. Since $B \subset mCl^*(A)$, we have $mCl^*(B) \subset mCl^*(A)$ and $mInt(mCl^*(B)) \subset mInt(mCl^*(A))$. Therefore, we obtain $mInt(mCl^*(B)) \subset mCl^*(mInt(B))$. This shows that B is a m-δ-I-open set. □

Proposition 3.7. *Let A and B be subsets of a minimal space (X, m, I). If A is a m-δ-I-open set, then $A = B \cup C$, where B is an m-α-I-open set, $mInt(mCl^*(C)) = \emptyset$ and $B \cap C = \emptyset$.*

Proof. Suppose that A is a m-δ-I-open set, then we have $mInt(mCl^*(A)) \subset mCl^*(mInt(A))$ and $mInt(mCl^*(A)) \subset mInt(mCl^*(mInt(A)))$. Now we have $A = (mInt(mCl^*(A)) \cap A) \cup (A - mInt(mCl^*(A)))$. Now we set $B = mInt(mCl^*(A)) \cap A$ and $C = A - mInt(mCl^*(A))$. We first show that B is an m-α-I-open set. Now we have $mInt(mCl^*(B)) = mInt(mCl^*(mInt(mInt(A)) \cap A)) = mInt(mCl^*(mInt(mCl^*(A)) \cap mInt(A))) = mInt(mCl^*(mInt(A)))$. Since A is a m-δ-I-open set, we have $mInt(mCl^*(mInt(A))) \supset mInt(mCl^*(A)) \supset B$ and thus B is an m-α-I-open set. Next we show that $mInt(mCl^*(C)) = \emptyset$. Since $mCl^*(A) \subset mCl(A)$ for any subset A of X. Therefore, we have $mInt(mCl^*(C)) = mInt(mCl^*(A \cap (X - mInt(mCl^*(A))))) \subset mInt(mCl^*(A)) \cap mInt(mCl^*(X - mInt(mCl^*(A)))) \subset mInt(mCl^*(A)) \cap mInt(mCl(X - mInt(mCl^*(A)))) \subset mInt(mCl^*(A)) \cap (X - mInt(mCl^*(A))) = \emptyset$. It is obvious that $B \cap C = \emptyset$. □

4. On Decompositions of m-α-I-continuity and m-semi-I-continuity

Definition 4.1. *A function A function f:$(X, m, I) \to (Y, \tau)$ is said to be m-δ-I-continuous function if the inverse image of every open set of (Y, τ) is m-δ-I-open set in (X, m, I).*

Proposition 4.2. *For a function $f : (X, m, I) \to (Y, \tau)$, the following statements are equivalent:*

(a). f is a m-semi-I-continuous.

(b). f is strongly m-β-I-continuous and m-δ-I-continuous.

Proof. The proof is obvious from Proposition 3.1. □

Proposition 4.3. *For a function $f : (X, m, I) \to (Y, \tau)$, the following statements are equivalent:*

(a). f is a m-α-I-continuous.

(b). f is m-pre-I-continuous and m-semi-I-continuous.

(c). f is m-pre-I-continuous and m-δ-I-continuous.

Proof. The proof is obvious from Proposition 3.1 and Proposition 3.2. □

Proposition 4.4. *For a function $f : (X, m, I) \to (Y, \tau)$, the following statements are equivalent:*

(a). f is a m-δ-I-continuous function.

(b). For each $x \in X$ and each open set V in Y with $f(x) \in V$, there exists $U \in m\delta IO(X, m)$ with $x \in U$ such that $f(U) \subset V$.

(c). The inverse image of each closed set in Y is m-δ-I-closed in X.

Proposition 4.5. *For a function $f : (X, m, I) \to (Y, \tau)$, the following statements are equivalent:*

(a). f is a m-δ-I-continuous function.

(b). $f^{-1}(V) = mInt(f^{-1}(V))$ for every open set V of Y.

(c). $f^{-1}(F) = mCl(f^{-1}(F))$ for every closed set F of Y.

(d). $mCl(f^{-1}(B)) \subset f^{-1}(mCl(B))$ for every subset B of Y.

(e). $f(mCl(A)) \subset mCl(f(A))$ for every subset A of X.

(f). $f^{-1}(mInt(B)) \subset mInt(f^{-1}(B))$ for every subset B of Y.

Proof. Proof follows from definitions. □

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