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A New Approach to Find an Optimal Solution of a Fuzzy Project Management Problem by Fuzzy Dynamic Programming

T. Nagalakshmi[1,2,*] and G. Uthra[3]

1 Department of Mathematics, S.A. Engineering College, Chennai, Tamilnadu, India.

2 Research Scholar, Bharathiar University, Coimbatore, Tamilnadu, India.

3 P.G. and Research Department of Mathematics, Pachaiyappa's College, Chennai, Tamilnadu, India.

Abstract: This paper mainly focuses on a new approach to find an optimal solution of a fuzzy Project Management problem with the help of Fuzzy Dynamic Programming. Project Management deals with the allocation of available foremen to the existing projects to complete the projects within the stipulated time. Moreover, it is known that fuzziness prevails in all fields. Hence, a general Project Management problem with fuzzy parameters is considered where the estimated time to complete each project is taken as Generalized Trapezoidal Fuzzy Numbers. The solution is obtained by the method of FDP by framing fuzzy forward and fuzzy backward recursive equations. It is observed that the solutions obtained by both the equations are the same. This approach is illustrated with a numerical example. This feature of the proposed approach eliminates the imprecision and fuzziness in Project Management models. The application of Fuzzy set theory in the field of dynamic Programming is called Fuzzy Dynamic Programming.

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1. Introduction

Dynamic Programming is a mathematical technique or tool which is used for making a sequence of interrelated decisions. It provides a systematic procedure to determine the combination of decisions which maximises the overall effectiveness. Dynamic Programming does not have a standard mathematical formulation like Linear Programming, but it is a general type of approach to solve a problem by framing equations to fit each individual situation. The solution of each stage is a decision and it is sequenced by all the stages as a decision policy. Each decision policy is combined together with some return in the form of costs or benefits. The objective in DP is to select a decision policy which will maximise the returns. In all our human activities, we come across multi stage decision making problems. FDP is an eminent tool to handle such type of uncertainty and fuzziness persisting in the analysis of MCDM problems. The application of fuzzy set theory in decision making process by Bellman [3] and Zadeh [10] attracted many researchers to contribute in this field. They started to apply fuzzy concepts in all the fields. Many researchers like Kacprzyk [6], Esogbue [5] contributed more to the literature. Kacprzyk [6] detailed the development in the theory and applications of FDP. Esogbue [5] provided a brief review of basic problem classes and developments of FDP. Schweickardt and Miranda [9] presented a FDP model in the evaluation of expansion

* *E-mail: nagalakshmi.1979@gmail.com*

distribution cost in fuzzy environments. Later on, Chung-Ching Su [4] developed a new approach using FDP for the unit commitment of a Power System. Abo-Sinna [1] discussed about his survey and some applications of multiple objective FDP problems. Other excellent applications of FDP also appear in the literature [7, 8].

The purpose of this paper is to find an optimal solution of a fuzzy project management problem by FDP. Here, a general Project Management problem with fuzzy parameters is considered where the estimated time to complete each project is taken as Generalized Trapezoidal Fuzzy Numbers. Fuzzy forward and fuzzy backward recursive equations are framed and fuzzy optimal solution is also obtained. It is observed that optimal solution obtained by both the equations are the same. The crisp value is also obtained by the Ranking method.

2. Definition and Preliminaries

Definition 2.1. *Let A be a classical set and $\mu_{\tilde{A}}(x)$ be a function from A to $[0,1]$. A fuzzy set $\tilde{A}$ with the membership function $\mu_{\tilde{A}}(x)$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A$ and $\mu_{\tilde{A}}(x) \in [0,1]\}$.*

Definition 2.2. *A Trapezoidal fuzzy number (TrFN) indicated by $\tilde{A}$ is defined as (a_1, a_2, a_3, a_4) where the membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a_1 \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & x \ge a_4 \end{cases} \tag{1}$$

Definition 2.3. *A Generalized Fuzzy Number $\tilde{A}$=$(a_1, a_2, a_3, a_4; w)$ is called a Generalized Trapezoidal Fuzzy Number if its membership function is given by*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a_1 \\ w\dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ w, & a_2 \le x \le a_3 \\ w\dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & x \ge a_4 \end{cases} \tag{2}$$

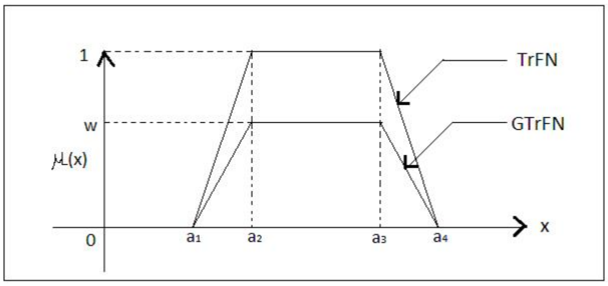

Figure 1. **Comparison between membership function of TrFN and GTrFN**

2.1. Properties of Generalized Trapezoidal Fuzzy numbers (GTrFN)

Let $\tilde{A} = (a, b, c, d; w_1)$ and $\tilde{B} = (p, q, r, s; w_2)$ be two GTrFNs, then

(1). **Equality of two GTrFNs :** $\tilde{A} = \tilde{B}$ if and only if $a = p, b = q, c = r, d = s$ and $w_1 = w_2$.

(2). **Addition of two GTrFNs :** $\tilde{A} + \tilde{B}$=$(a + p, b + q, c + r, d + s; w)$, where $w = min(w_1, w_2)$.

(3). **Subtraction of two GTrFNs:** $\tilde{A} - \tilde{B}$=$(a - s, b - r, c - q, d - p; w)$, where $w = min(w_1, w_2)$.

(4). **Multiplication of two GTrFNs:** $\tilde{A} * \tilde{B}$=$(ap, bq, cr, ds; w)$, where $w = min(w_1, w_2)$.

(5). **Division of two GTrFNs:** $\tilde{A} \div \tilde{B} = \left(\frac{a}{s}, \frac{b}{r}, \frac{c}{q}, \frac{d}{p}; w\right)$, where $w = min(w_1, w_2)$.

(6). **Scalar Multiplication of GTrFN :** $k\tilde{A} = (ka, kb, kc, kd; w)$, where $w = \min(w_1, w_2)$ if $k \geq 0$; $k\tilde{A} = (kd, kc, kb, ka; w)$, where $w = \min(w_1, w_2)$ if $k < 0$, k is a scalar.

2.2. Ranking Methodology of Generalized Trapezoidal Fuzzy number (GTrFN)

Various approaches for the ranking of fuzzy numbers have been proposed in the literature. The ranking Methodology of Trapezoidal Fuzzy Numbers is used to defuzzify the Trapezoidal Fuzzy Numbers to find the crisp solutions. The ranking method to defuzzify the Trapezoidal Fuzzy Numbers[2] is given below. That is, for every Generalized Trapezoidal Fuzzy number $\tilde{A} = (a, b, c, d; w)$ the ranking function is defined by the following function.

$$R(\tilde{A}) = w\left(\frac{a + b + c + d}{4}\right)$$

3. Fuzzy Dynamic Programming

3.1. Fuzzy Project Management Problem

With the development of global network economy and the increasing competition in the world market, some huge companies often meet a situation that multiple projects should be executed simultaneously. While project managers usually adopt a new approach of project management, that is multi-project management. So, the allocation of human resources to the project plays a vital role in the Project management. That is, it is the job of the project Manager to allocate the human resources to the projects by keeping in mind to minimise the time that is required to complete all the projects. Moreover, it is known that fuzziness prevails in all fields. Hence, a general Project Management problem where the estimated time to complete each project is taken as Generalized Trapezoidal Fuzzy Number. The solution is obtained by the method of FDP by framing fuzzy forward and fuzzy backward recursive equations.

3.2. Fuzzy Forward Recursive Equations

The Fuzzy Forward Recursive Equations are framed as follows:

$$\tilde{f}_1(s_1, x_1) = \min\{\tilde{t}(x_1)\}, \quad \text{where } x_1 \leq s_1.$$

$$\tilde{f}_i(s_i, x_i) = \min\{\tilde{t}(x_i) + \tilde{f}_{i-1}(s_i - x_i)\}, \quad \text{where } x_i \leq s_i \text{ for } i = 2, 3, 4, \cdots n.$$

3.3. Fuzzy Backward Recursive Equations

The Fuzzy Backward Recursive Equations are framed as follows:

$$\tilde{f}_n(s_n, x_n) = \min\{\tilde{t}(x_n)\}, \qquad \text{where } x_n \leq s_n.$$

$$\tilde{f}_i(s_i, x_i) = \min\{\tilde{t}(x_i) + \tilde{f}_{i+1}(s_i - x_i)\}, \text{ where } x_i \leq s_i \text{ for } i = 1, 2, 3, \cdots n-1.$$

4. Illustrative Example

The following project management problem is considered under a fuzzy environment. A construction contractor has four construction projects underway. He wants to minimise the time required to complete all the projects. Here, the estimated time to complete each project is considered as Generalized Trapezoidal Fuzzy Numbers. The following table gives the estimated time required to complete the project for a specified number of foremen assigned to the project.

Project	Number of foremen Assigned		
	1	2	3
A	(14,15,16,17;0.1)	(12,13,14,15;0.1)	(11,12,13,14;0.1)
B	(16,17,18,19;0.1)	(14,15,16,17;0.1)	(12,13,14,15;0.1)
C	(18,19,20,21;0.1)	(17,18,19,20;0.1)	(16,17,18,19;0.1)
D	(20,21,22,23;0.1)	(17,18,19,20;0.1)	(17,18,19,20;0.1)

Table 1. **Estimated Time to complete the Projects**

The contractor has only six foremen and each project must be assigned to at least one foreman. By considering each project as a stage, the problem has 4 stages.

4.1. Solution by Fuzzy Forward Recursive Equations

The DP model consists of the following steps.

- Let x_i be the number of foremen assigned to project i.
- Let s_i be the number of foremen still available for allocation.
- Let $\tilde{t}(x_i)$ be the completion times for project i when x_i foremen are assigned.
- The state variables $s_{i-1} = s_i - x_i$, from which the values of s_4, s_3, s_2, s_1 can be computed.
- The optimal fuzzy return function is given by $\tilde{f}_i(s_i, x_i) = \tilde{t}(x_i) + \tilde{f}_{i-1}(s_i - x_i)$ where, $\tilde{f}^*_{i-1}(s_i - x_i)$ is the minimum completion time for stage $(i = 2, 3, 4 \cdots n)$.

Stage 1: $\tilde{f}_1(s_1, x_1) = \min\{\tilde{t}(x_1)\}$, where $x_1 \leq s_1$.

s_1	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$\tilde{f}_1(s_1)$	x_1^*
1	(14,15,16,17;0.1)	-	-	(14,15,16,17;0.1)	1
2	-	(12,13,14,15;0.1)	-	(12,13,14,15;0.1)	2
3	-	-	(11,12,13,14;0.1)	(11,12,13,14;0.1)	3

Table 2. **Stage 1**

Stage 2: $\tilde{f}_2(s_2, x_2) = \min\{\tilde{t}(x_2) + \tilde{f}_1(s_2 - x_2)\}$, where $x_2 \leq s_2$.

s_2	$x_2=1$	$x_2=2$	$x_2=3$	$\tilde{f}_2(s_2)$	x_2^*
2	(30,32,34,36;0.1)	-	-	(30,32,34,36;0.1)	1
3	(28,30,32,34;0.1)	(28,30,32,34;0.1)	-	(28,30,32,34;0.1)	1 or 2
4	(27,29,31,33;0.1)	(26,28,30,32;0.1)	(26,28,30,32;0.1)	(26,28,30,32;0.1)	2 or 3

Table 3. Stage 2

Stage 3: $\tilde{f}_3(s_3,x_3)=\min\{\tilde{t}(x_3)+\tilde{f}_2(s_3-x_3)\}$, where $x_3\le s_3$.

s_3	$x_3=1$	$x_3=2$	$x_3=3$	$\tilde{f}_3(s_3)$	x_3^*
3	(48,51,54,57;0.1)	-	-	(48,51,54,57;0.1)	1
4	(46,49,52,55;0.1)	(47,50,53,56;0.1)	-	(46,49,52,55;0.1)	1
5	(44,47,50,53;0.1)	(45,48,51,54;0.1)	(46,49,52,55;0.1)	(44,47,50,53;0.1)	1

Table 4. Stage 3

Stage 4: $\tilde{f}_4(s_4,x_4)=\min\{\tilde{t}(x_4)+\tilde{f}_3(s_4-x_4)\}$, where $x_4\le s_4$.

s_4	$x_4=1$	$x_4=2$	$x_4=3$	$\tilde{f}_4(s_4)$	x_4^*
6	(64,68,72,76;0.1)	(63,67,71,75;0.1)	(65,69,73,77;0.1)	(63,67,71,75;0.1)	2

Table 5. Stage 4

4.2. Optimal Solution by Fuzzy Forward Recursive Equations

The fuzzy optimal solution is obtained by the following table.

$x_4{}^*$	$s_3=s_4-x_4{}^*$	$x_3{}^*$	$s_2=s_3-x_3{}^*$	$x_2{}^*$	$s_1=s_2-x_2{}^*$	$x_1{}^*$
2	$6-2=4$	1	$4-1=3$	1 or 2	$3-1=2$ or $3-2=1$	2 or 1

Table 6. Optimal Solution

From the above solution, it is observed that two foremen are assigned to Project D, one foreman is assigned to Project C. Two possibilities are observed here. That is, either two foremen are assigned to Project B and one foreman is assigned to Project A or one foreman is assigned to Project B and two foremen are assigned to Project A. The estimated fuzzy time is $(63,67,71,75;0.1)$. Its crisp value is given by 6.9. Now, let us solve the above fuzzy project management problem by fuzzy backward recursive equations method.

4.3. Solution by Fuzzy Backward Recursive Equations

The DP model consists of the following steps.

- Let x_i be the number of foremen assigned to project i.
- Let s_i be the number of foremen still available for allocation.
- Let $\tilde{t}(x_i)$ be the completion times for project i when x_i foremen are assigned.
- The state variables $s_{i-1}=s_i-x_i$, from which the values of s_4,s_3,s_2,s_1 can be computed.
- The optimal fuzzy return function is given by $\tilde{f}_i(s_i,x_i)=\tilde{t}(x_i)+\tilde{f}_{i+1}(s_i-x_i)$ where, $\tilde{f}^*_{i+1}(s_i-x_i)$ is the minimum completion time for stage $(i=1,2,3\cdots n-1)$.

Stage 4: $\tilde{f}_4(s_4, x_4) = \min\{\tilde{t}(x_4)\}$, where $x_4 \leq s_4$.

s_4	$x_4 = 1$	$x_4 = 2$	$x_4 = 3$	$\tilde{f}_4(s_4)$	x_4^*
1	(20,21,22,23;0.1)	-	-	(20,21,22,23;0.1)	1
2	-	(17,18,19,20;0.1)	-	(17,18,19,20;0.1)	2
3	-	-	(17,18,19,20;0.1)	(17,18,19,20;0.1)	3

Table 7. **Stage 4**

Stage 3: $\tilde{f}_3(s_3, x_3) = Min\{\tilde{t}(x_3) + \tilde{f}_4(s_3 - x_3)\}$, where $x_3 \leq s_3$.

s_3	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$	$\tilde{f}_3(s_3)$	x_3^*
2	(38,40,42,44;0.1)	-	-	(38,40,42,44;0.1)	1
3	(35,37,39,41;0.1)	(37,39,41,43;0.1)	-	(35,37,39,41;0.1)	1
4	(35,37,39,41;0.1)	(34,36,38,40;0.1)	(36,38,40,42;0.1)	(34,36,38,40;0.1)	2

Table 8. **Stage 3**

Stage 2: $\tilde{f}_2(s_2, x_2) = Min\{\tilde{t}(x_2) + \tilde{f}_3(s_2 - x_2)\}$, where $x_2 \leq s_2$.

s_2	$x_2 = 1$	$x_2 = 2$	$x_2 = 3$	$\tilde{f}_2(s_2)$	x_2^*
3	(54,57,60,63;0.1)	-	-	(54,57,60,63;0.1)	1
4	(51,54,57,60;0.1)	(52,55,58,61;0.1)	-	(51,54,57,60;0.1)	1
5	(50,53,56,59;0.1)	(49,52,55,58;0.1)	(50,53,56,59;0.1)	(49,52,55,58;0.1)	2

Table 9. **Stage 2**

Stage 1: $\tilde{f}_1(s_1, x_1) = Min\{\tilde{t}(x_1) + \tilde{f}_2(s_1 - x_1)\}$, where $x_1 \leq s_1$.

s_1	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$	$\tilde{f}_1(s_1)$	x_1^*
6	(63,67,71,75;0.1)	(63,67,71,75;0.1)	(65,69,73,77;0.1)	(63,67,71,75;0.1)	1 or 2

Table 10. **Stage 1**

4.4. Optimal Solution by Fuzzy Backward Recursive Equations

The fuzzy optimal solution is obtained by the following table.

$x_1{}^*$	$s_2 = s_1 - x_1{}^*$	$x_2{}^*$	$s_3 = s_2 - x_2{}^*$	$x_3{}^*$	$s_4 = s_3 - x_3{}^*$	$x_4{}^*$
1 or 2	$6 - 1 = 5$ or $6 - 2 = 4$	2 or 1	$5 - 2 = 3$ or $4 - 1 = 3$	1	$3 - 1 = 2$	2

Table 11. **Optimal Solution**

Two possibilities are observed here from the above solution. That is, either one foreman is assigned to Project A and two foremen are assigned to Project B or two foremen are assigned to Project A and one foreman is assigned to Project B. It is observed that one foreman is assigned to Project C and two foremen are assigned to Project D. The estimated fuzzy time is $(63, 67, 71, 75; 0.1)$. Its crisp value is given by 6.9. It is observed from the above optimal solutions that both the fuzzy as well as crisp solutions obtained by fuzzy forward and backward recursive equation are the same.

5. Conclusion

In this paper, mathematical formulation of fuzzy Project Management problem and the procedure for finding optimal solution by fuzzy recursive equations are discussed with the numerical example. In particular, the estimated times to complete the project when the foremen are assigned to every project are expressed as generalized trapezoidal fuzzy numbers. Fuzzy Dynamic Programming (FDP) is an eminent tool for dealing with a large volume of multi-stage decision-making problems in a fuzzy environment. In the real world where the allocation of human resources is needed to complete a task with time as fuzzy constraints, this proposed approach will be used. Also, the proposed approach will help the researchers to solve the Project Management Problems involving multi-stage decision process in a fuzzy environment. The approach would also bring about many new interesting applications.

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