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The Approximate Solutions of Fredholm Integral Equations by Adomian Decomposition Method and its Modification

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Abstract: In this paper, a modification of Adomian decomposition method (MADM) for solving linear and nonlinear Fredholm integral equations of the second kind has been introduced. This new method is resulted from Adomian decomposition method (ADM) by a simple modification. This modification is based on the existence of Taylor expansion. To illustrate the modification on ADM, some examples are presented. Comparison of the result of applying the MADM and ADM revealing the new technique is very effective and convenient.

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1. Introduction

The Adomian decomposition method was first introduced and developed by Gorge Adomian and it has been proved to be reliable and efficient for a wide class of differential and integral equations of linear and nonlinear models [1]. The method provides the solution as an infinite series in which each term can be easily determined. The rapid convergence of the series obtained by this method is thoroughly discussed by Hamoud and Ghadle [12]. The concept of uniform convergence of the infinite series was addressed by Adomian [2, 3] and Rach [4] for linear problems and extended to nonlinear problems [6]. Modified Adomian decomposition method has been applied for a long time to solve Fredholm integral equation [16, 24]. Recently, another modification of decomposition method was proposed by Wazwaz and El-Sayed [10, 25]. In the new modification, the process of dividing into two components was replaced by a Taylor series of infinite components. The ideas have been shown to be computationally efficient in applying the proposed technique in several differential and integral equations. However, as will be seen from the examples below, the modified decomposition method will sometimes encounter obstacles to obtain the exact solution. After that, several authors further proposed expressing the function by the orthogonal polynomial [15, 17] and Jacobi and Gegenbauer series [8]. Some fundamental works on various aspects of modifications of the Adomian's decomposition method are given by Andrianov [7], Venkatarangan [18, 19] and Wazwaz [21]. Wazwaz [20] used the modified decomposition method and the traditional methods for solving nonlinear integral equations. A variety of powerful methods has been presented, such as the homotopy analysis method. Since Adomian firstly proposed the decomposition

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method [1] at the beginning of 1980s, the algorithm has been widely and effectively used for solving the analytic solutions of physically significant equations arranging from linear to nonlinear, from ordinary differential to partial differential, from integral to fractional integral equations. The authors have used several methods for the numerical or the analytical solution of linear and nonlinear Fredholm and Volterra integral and integro-differential equations of the second kind [14].

In this paper, we solve some of the linear and nonlinear Fredholm integral equations of second kind by ADM and MADM and then we obtained the exact solution. The article is organized as follows: In Section 2, we present some definitions of Fredholm integral equation. In Section 3, we describe the solution of linear and nonlinear Fredholm integral equations by using ADM. Section 4, describes the solution of linear and nonlinear Fredholm integral equations by using MADM. In Section 5, we solve some of the linear and nonlinear Fredholm integral equations of second kind by ADM and MADM and then we compare the approximation solution with the exact solution. Section 6, ends this paper with a brief conclusion.

2. Preliminaries

Definition 2.1. *The general form of linear Fredholm integral equation is defined as follows:*

$$f(x)y(x) = g(x) + \lambda \int_a^b k(x,t)y(t)dt, \tag{1}$$

where a and b are both constants. $f(x)$, $g(x)$, and $k(x,t)$ are known functions while $y(x)$ is the unknown function. A (nonzero parameter) is called eigenvalue of the integral equation. The function $k(x,t)$ is known as kernel of the integral equation.

Definition 2.2. *The linear integral equation is formed (by setting $f(x) = 1$ in Equation (1)*

$$y(x) = g(x) + \lambda \int_a^b k(x,t)y(t)dt, \tag{2}$$

Equation (2) is known as Fredholm integral equation of the second kind.

Definition 2.3. *The nonlinear Fredholm integral equation of second kind is defined as follows:*

$$y(x) = g(x) + \lambda \int_a^b k(x,t)F(y(t))dt, \tag{3}$$

where $k(x,t)$ is the kernel of the integral equation, $g(x)$ and $k(x,t)$ are known functions, and $y(x)$ is the unknown function that is to be determined.

3. Solution of Fredholm Integral Equation by Using ADM

(1). **Linear Equation:** The Adomian decomposition method consists of decomposing the unknown function $y(x)$ of any equation into a sum of an infinite number of components defined by the decomposition series

$$y(x) = \sum_{n=0}^{\infty} y_n(x), \tag{4}$$

where the components $y_n(x), n \geqslant 0$ of the unknown function $y(x)$ are completely determined in a recurrences manner. The decomposition method concerns itself with finding the components $y_0(x), y_1(x), y_2(x)$ individually. The determination of these components can be achieved in an easy way through a recurrence relation that usually involves simple integrals that can be easily evaluated. Substituting the decomposition Equation (4) into both sides of the Fredholm integral equation.

$$y(x) = g(x) + \lambda \int_a^b G(x,t)y(t)dt, \tag{5}$$

to obtain

$$\sum_{n=0}^{\infty} y_n(x) = g(x) + \lambda \int_a^b G(x,t) \sum_{n=0}^{\infty} y_n(t)dt. \tag{6}$$

The zeroth component $y_0(x)$ is identified by all terms that are not included under the integral sign. This means that the components $y_j(x), j \geqslant 0$ of the unknown function $y(x)$ are completely determined by setting the recurrence relation

$$\begin{aligned} y_0(x) &= g(x) \\ y_{n+1}(x) &= \lambda \int_a^b G(x,t)y_n(t)dt, \quad n \geqslant 0. \end{aligned} \tag{7}$$

As a result the components $y_0(x), y_1(x), y_2(x), \cdots$ are completely determined. Thus, the solution $y(x)$ of the Fredholm integral Equation (5) is readily obtained in a series form by using the series assumption in Equation (4). In view of Equation (7), the components $y_0(x), y_1(x), y_2(x), \cdots$ follow immediately. Once these components are determined, the solution $y(x)$ can be obtained using the series Equation (4). It may be noted that for some problems, the series give the closed-form solution. However, for other problems, we have to determine a few terms in the series such as $y(x) = \sum_{n=0}^{k} y_n(x)$ by truncating the series at certain term. Because of the uniformly convergence property of the infinite series a few terms will attain the maximum accuracy.

(2). **Nonlinear Equation:** In the decomposition method, we usually express the solution $y(x)$ of the integral equation

$$y(x) = b + \int_a^b f(x,y)dx, \tag{8}$$

in a series form defined by Equation (4). Substituting the decomposition Equation (4) into both sides of Equation (8) yields

$$\sum_{n=0}^{\infty} y_n(x) = b + \int_a^b f(x, \sum_{n=0}^{\infty} y_n(x))dx, \tag{9}$$

The components $y_0(x), y_1(x), y_2(x), \cdots$ of the unknown function $y(x)$ are completely determined in a recurrence manner if we set

$$\begin{aligned} y_0(x) &= b, \\ y_1(x) &= \int_a^b f(x, y_0)dx, \\ y_2(x) &= \int_a^b f(x, y_1)dx, \\ y_3(x) &= \int_a^b f(x, y_2)dx, \end{aligned}$$

and so on. The above decomposition scheme for determination of the components $y_0(x), y_1(x), y_2(x), \cdots$ of the solution $y(x)$ of Equation (8) can be written in a recurrence form by

$$y_0(x) = b; \qquad y_{n+1}(x) = \int_a^b f(x, y_n)dx. \tag{10}$$

In this new decomposition process, we expand the solution function in a Straightforward infinite series in Equation (4) assuming that the series converges to a finite limit as $n \to \infty$. Next we expand the function $f(x, y)$ which contains the solution function $y(x)$ by Taylor's expansion about $y_0(x)$ keeping x as it is such that

$$f(x, y) = f(x, y_0) + (y - y_0)f'(x, y_0) + \frac{(y - y_0)^2}{2!} f''(x, y_0) + \frac{(y - y_0)^3}{3!} f'''(x, y_0) + \cdots \tag{11}$$

We know that Taylor's expansion is absolutely and uniformly convergent in a given domain. Now using Equation (4) into Equation (11), yields

$$f(x, y) = f(x, y_0) + \sum_{n=0}^{\infty} y_n(x) f'(x, y_0) + \frac{1}{2!}(\sum_{n=0}^{\infty} y_n(x))^2 f''(x, y_0) + \frac{1}{3!}(\sum_{n=0}^{\infty} y_n(x))^3 f'''(x, y_0) + ... \tag{12}$$

which can subsequently be written as

$$f(x, y) = A_0(x) + A_1(x) + A_2(x) + A_3(x) + \cdots + A_n(x) = \sum_{n=0}^{\infty} A_n(x). \tag{13}$$

We define the different terms in $A_n(x, y)$ as follows:

$$\begin{aligned} &A_0 = f(x, y_0); \quad A_2 = y_2 f_y(x, y_0) + y_1^2 f_{yy}(x, y_0); \\ &A_1 = y_1 f(x, y_0); \; A_3 = y_3 f_y(x, y_0) + \frac{1}{2}(2y_1 y_2) f_{yy}(x, y_0) + \frac{1}{6} y_1^3 f_{yyy}(x, y_0). \end{aligned} \tag{14}$$

Substituting Equation (13) and Equation (4) into the integral Equation (8) we obtain

$$\sum_{n=0}^{\infty} y_n(x) = b + \int_a^b \sum_{n=0}^{\infty} A_n(x) dx.$$

The components $y_0, y_1, \cdots$ are completely determined by using the recurrence scheme

$$y_0(x) = b; \qquad y_{n+1}(x) = \int_a^b A_n(t) dt, \quad n \geqslant 0. \tag{15}$$

Consequently, the solution of Equation (14) in a series form is immediately determined by using Equation (4). As indicated earlier the series may yield the exact solution in a closed form, or a truncated series $\sum_{n=1}^{k} y_n(x)$ may be used if a numerical approximation is desired.

4. Solution of Fredholm Integral Equation by Using MADM

(1). **Linear Equation:** The Adomian decomposition method provides the solution in an infinite series of components. The components $y_j, j \geqslant 0$, are easily computed if the inhomogeneous term $g(x)$ in the Fredholm integral equation:

$$y(x) = g(x) + \lambda \int_a^b G(x, t) y(t) dt, \tag{16}$$

consists of a polynomial. However, if the function $g(x)$ consists of a combination of two or more of polynomials, trigonometric functions, hyperbolic functions, and others, the evaluation of the components $y_j, j \geqslant 0$, requires cumbersome work. A reliable modification of the Adomian decomposition method was developed by Wazwaz [22]. The modified decomposition method will facilitate the computational process and further accelerate the convergence of the series solution. The modified decomposition method will be applied, wherever it is appropriate, to all integral equations and differential equations of any order. It is interesting to note that the modified decomposition method depends mainly on splitting the function $g(x)$ into two parts; therefore it cannot be used if the function $g(x)$ consists of only one term [5, 9, 11, 13]. To give a clear description of the technique, we recall that the standard Adomian decomposition method admits the use of the recurrence relation:

$$y_0(x) = g(x); \quad y_{k+1}(x) = \lambda \int_a^b G(x,t) y_k(t) dt \quad k \geqslant 0, \tag{17}$$

where the solution $y(x)$ is expressed by an infinite sum of components defined by Equation (4), in view of Equation (17), the components $y_n(x), n \geqslant 0$, can be easily evaluated. The modified decomposition method introduces a slight variation to the recurrence relation (17) that will lead to the determination of the components of $y(x)$ in an easier and faster manner. For many cases, the function $g(x)$ can be set as the sum of two partial functions, namely $g_1(x), g_2(x)$. In other words, we can set

$$g(x) = g_1(x) + g_2(x),$$

In view of (16), we introduce a qualitative change in the formation of the recurrence relation (17). To minimize the size of calculations, we identify the zeroth component y_0 by one part of $g(x)$ namely $g_1(x)$ or $g_2(x)$. The other part of $g(x)$ can be added to the component $y_1(x)$ that exists in the standard recurrence relation (17). In other words, the modified decomposition method introduces the modified recurrence relation:

$$\begin{aligned} y_0(x) = g_1(x); \quad y_1(x) = g_2(x) + \lambda \int_a^b G(x,t) y_0(t) dt, \\ y_{k+1}(x) = \lambda \int_a^b G(x,t) y_k(t) dt, \quad k \geqslant 1, \end{aligned} \tag{18}$$

This shows that the difference between the standard recurrence relation (17) and the modified recurrence relation (18) rests only in the formation of the first two components $y_0(x)$ and $y_2(x)$. The other Components $y_j, j \geqslant 0$ remain the same in the two recurrence relations. Although this variation in the formation of $y_0(x)$ and $y_1(x)$ is slight, it plays a major role in accelerating the convergence of the solution and in minimizing the size of computational work. Moreover, reducing the number of terms in $g_1(x)$ affects not only the component $y_1(x)$, but also the other components as well. This result was confirmed by several research works as in [23].

(2). **Nonlinear equation:** In this equation the modified decomposition method identifies the zero component $y_0(x)$ by one part of $g(x)$ and the other part of $g(x)$ can be added to the component $y_1(x)$. In this method the solution is expressed in a series which is given by (4) and after some iterations all the higher order derivatives are zero. By Taylor's expansion about $y = y_0, f(y)$ can be expressed as

$$f(y) = f(y_0) + (y_0 + y_1 + y_2 + \cdots) f_y(y_0) + \frac{1}{2!}(y_0 + y_1 + y_2 + \cdots)^2 f_{yy}(y_0) + \frac{1}{3!}(y_0 + y_1 + y_2 + \cdots)^3 f_{yyy}(y_0) + \cdots, \tag{19}$$

where we dene the decomposition coefficients as

$$A_0 = f(y_0); \quad A_1 = y_1 f_y(y_0); \quad A_2 = y_2 f_y(y_0) + \frac{1}{2} y_1^2 f_{yy}(y_0). \tag{20}$$

Then the subsequent terms of the series can be obtained as follows:

$$y_0 = g_1(x); \quad y_1 = g_2(x) + \int_a^b A_0(t) dt = 0; \quad y_2 = \int_a^b A_1(t) dt = 0; \quad y_n(x) = 0, \quad for \quad n \geqslant 1, \tag{21}$$

and hence the solution is $y(x) = g_1(x)$.

5. Application for Linear and Nonlinear Integral Equations

Example 5.1. *Consider the linear Fredholm integral equation*

$$y(x) = 1 + sec^2(x) - \int_0^{\frac{\pi}{4}} y(t)dt, \tag{22}$$

which has $y(x) = sec^2(x)$ as the exact solution.

(a). Applying the ADM we have

$$\begin{aligned} y_0 &= 1 + sec^2(x), \\ y_1(x) &= -\frac{\pi}{4} - 1, \\ y_2(x) &= \left(\frac{\pi}{4}\right)^2 + \frac{\pi}{4} \\ y_3 &= -\left(\frac{\pi}{4}\right)^3 - \left(\frac{\pi}{4}\right)^2, \\ &\vdots \end{aligned} \tag{23}$$

Hence the solution of eq(22) is given as

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \longrightarrow sec^2(x) \quad as \quad n \longrightarrow \infty. \tag{24}$$

(b). Applying the MADM we get

$$\begin{aligned} y_0 &= sec^2(x), \\ y_1(x) &= 1 - \int_0^{\frac{\pi}{4}} y_{0(t)dt} = 1 - \int_0^{\frac{\pi}{4}} sec^2(t)dt = 0 \\ y_2(x) &= -\int_0^{\frac{\pi}{4}} y_1(t)dt = 0 \\ y_n(x) &= 0 \end{aligned} \tag{25}$$

According to Equation (4) the solution is

$$y(x) = sec^2(x). \tag{26}$$

x	$y_{Exact}(x)$	$y_{Appr.}(x)$	$E_{50}(y)$
0.1	1.010067046	1.010061365	5.618×10^{-6}
0.2	1.041091358	1.041085677	5.681×10^{-6}
0.3	1.095688915	1.095683233	5.682×10^{-6}
0.4	1.178754106	1.178748424	5.682×10^{-6}
0.5	1.29844641	1.298440729	5.681×10^{-6}
0.6	1.468043173	1.468037491	5.682×10^{-6}
0.7	1.709449716	1.709444034	5.682×10^{-6}
0.8	2.060155558	2.060149876	5.682×10^{-6}
0.9	2.587998733	2.587993051	5.682×10^{-6}
1.0	3.425518821	3.425513139	5.682×10^{-6}

Table 1. Comparison between exact and approximate solution for example 1

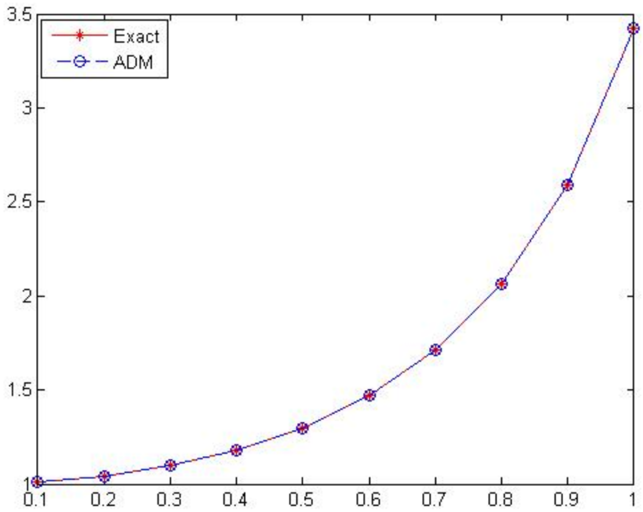

Figure 1. **Comparison between exact and approximate solution for example 1**

Figure 1 shows the comparison between the exact solution and the approximate solution obtained by the proposed methods.It is seen from Figure 1 that the solution obtained by the proposed methods nearly identical to the exact solution. The accuracy of the result can be improved by introducing more terms of the approximate solutions. ADM solution is compared with the exact solution of the Fredholm integral equation at the different value of x in Table 1 for Example 5.1.

Example 5.2. *Consider the linear Fredholm integral equation*

$$y(x) = x + e^x - \frac{4}{3} + \int_0^1 ty(t)dt. \tag{27}$$

Which has $y(x) = x + e^x$ as exact solution.

(a). Applying the ADM we have

$$y_0 = x + e^x - \frac{4}{3};\;\; y_1(x) = \int_0^1 ty_0(t)dt = \frac{2}{3};\;\; y_2(x) = \frac{1}{3};\;\; y_3 = -\frac{1}{6};\;\; y_{50} = \frac{1}{844424930131968}. \tag{28}$$

Hence the solution of Equation (27) is given as

$$y(x) = x + e^x - \frac{4}{3} + \frac{2}{3}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{562949953421312}\right) \tag{29}$$

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \longrightarrow x + e^x \quad as \quad n \longrightarrow \infty. \tag{30}$$

(b). Applying the MADM we get

$$y_0 = x + e^x;\;\; y_1(x) = -\frac{4}{3} + \int_0^1 tA_0(t)dt = -\frac{4}{3} + \int_0^1 t(t + e^t dt) = 0;\;\; y_2(x) = -\int_0^1 tA_1(t)dt = 0;\;\; y_n(x) = 0 \tag{31}$$

According to Equation (4) the solution is

$$y(x) = x + e^x. \tag{32}$$

x	$y_{Exact}(x)$	$y_{Appr.}(x)$	$E_{50}(y)$
0.1	1.20517091807564	1.20517091808679	1.114723 $\times 10^{-11}$
0.2	1.42140275816016	1.42140275817131	1.114723 $\times 10^{-11}$
0.3	1.64985880757600	1.64985880758715	1.114723 $\times 10^{-11}$
0.4	1.89182469764127	1.89182469765241	1.114723 $\times 10^{-11}$
0.5	2.14872127077128	2.14872127077112	1.114723 $\times 10^{-11}$
0.6	2.42211880039050	2.42211880040165	1.114723 $\times 10^{-11}$
0.7	2.71375270747047	2.71375270748162	1.114723 $\times 10^{-11}$
0.8	3.02554092849246	3.02554092850361	1.114723 $\times 10^{-11}$
0.9	3.35960311115694	3.35960311116809	1.114723 $\times 10^{-11}$
1.0	3.71828182845904	3.71828182847019	1.114723 $\times 10^{-11}$

Table 2. **Comparison between exact and approximate solution for example 2**

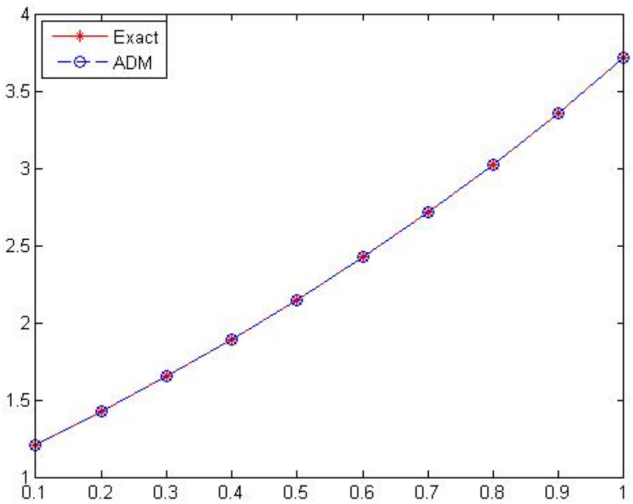

Figure 2. **Comparison between exact and approximate solution for example 2**

Figure 2 shows the comperison between the exact solution and the approximate solution obtained by the ADM. It is seen from Figure 2 the solution obtained by the proposed method nearly identical to the exact solution. In this example, the simplicity and accuracy of the proposed method is illustrated by computing the absolute error $E_{50}(X) = y_{exact}(x) - y_{appr.}(x)$ for the Example 5.2. The accuracy of the result can be improved by introducing more terms of the approximate solutions. In Table 2, ADM solution are compared with the exact solution of the Fredholm integral Equation (27). There is good agreement between exact and approximate solution obtained by proposed method. The table also shows the absolute error between the exact and approximate solution.

Example 5.3. *Consider the nonlinear Fredholm integral equation*

$$y(x) = x + \frac{1}{4} - \int_0^1 ty^2(t)dt. \tag{33}$$

Which has $y(x) = x$ *as exact solution.*

(a). Applying the ADM we have

$$y_0 = x + \frac{1}{4},$$
$$y_1(x) = -\int_0^1 tA_0(t)dt = -[\frac{1}{4} + \frac{1}{6} + \frac{1}{32}]$$

$$y_2(x) = -\int_0^1 tA_1(t)dt = -\int_0^1 2t(t+\frac{1}{4})(-[\frac{1}{4}+\frac{1}{6}+\frac{1}{32}])dt, \tag{34}$$

by canceling the noise terms from $y_{0(x)}$ and $y_1(x)$ verifying that the remaining non-canceled term in $y_0(x)$ identically satises the original Equation (33), the exact solution is therefore given as

$$y(x) = x. \tag{35}$$

(b). Applying the MADM we get

$$\begin{aligned} y_0 &= x, \\ y_1(x) &= \frac{1}{4} - \int_0^1 tA_0(t)dt = \frac{1}{4} - \int_0^1 t^3 dt = 0 \\ y_2(x) &= -\int_0^1 tA_1(t)dt = 0, \\ y_n(x) &= 0. \end{aligned} \tag{36}$$

According to Equation (4) the solution is

$$y(x) = x. \tag{37}$$

6. conclusion

In this paper, we carefully applied a reliable modification of Adomian decomposition method for solving linear and nonlinear Fredholm equations. The main advantage of this method is the fact that it gives the analytical solution. The method overcomes the difficulties arising in calculating the ADM. The efficiency of the MADM was tested on some numerical examples, and the results show that the method is easier than AMD and many other numerical techniques. The MADM requires much less computational work compared with ADM. From the above examples, it is obvious that the approximate solution of integral equations using MADM is simple and numerical results show that the methods are working well and the accuracy is comparable with exact solutions.

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