Int. J. Math. And Appl., 6(2–A)(2018), 357–364
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Completely Generalized Semi-precontinuous Mapping in *IVF*-topological Spaces

G. Kumar[1] and R. Jeyabalan[2,*]

1 Department of Mathematics, Alagappa University Evening College, Ramnad, Tamilnadu, India.

2 Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu, India.

Abstract: In this paper we introduce ivf-completely generalized semi-pre continuous mappings. We investigate some of its properties. Also we provide some characterization of ivf-completely generalized semi-pre continuous mappings.

MSC: 54A40, 08A72.

Keywords: ivf-set, ivf-topological space, ivf-point, ivf-generalized semi-preclosed set, ivf-generalized semi-precontinuous mappings, ivf-semi-pre$T_{1/2}$ space.

Accepted on: 17.03.2018

1. Introduction

The concept of fuzzy subset was introduced and studied by L. A. Zadeh [14] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper: C. L. Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. Andrijevic [1] introduced semi-preclosed sets and Dontchev [4] introduced generalized semi-preclosed sets in general topology. After that the set was generalized to fuzzy topological spaces by Saraf and Khanna [13]. Tapas Kumar Mondal and S. K. Samantha [10] introduced the topology of interval valued fuzzy sets. Jeyabalan. R, Arjunan. K, [6] introduced ivf-generalaized semi-preclosed sets and Jeyabalan. R, Arjunan. K, [7] introduced ivf-generalaized semi-precontinuous mappings. In this paper, we introduce ivf-completely generalized semi-precontinuous mappings and some of its properties are investigated.

2. Preliminaries

Definition 2.1 ([10]). *Let X be a non empty set. A mapping $\bar{A} : X \to D[0,1]$ is called an interval valued fuzzy set (briefly $IVFS$) on X, where $D[0,1]$ denotes the family of all closed subintervals of [0,1] and $\bar{A}(x) = [A^-(x), A^+(x)]$, for all $x \in X$, where $A^-(x)$ and $A^+(x)$ are fuzzy sets of X such that $A^-(x) \le A^+(x)$, for all $x \in X$. Thus $\bar{A}(x)$ is an interval (a closed subset of [0,1]) and not a number fom the interval [0,1] as in the case of fuzzy set.*

* *E-mail: jeyram84@gmail.com*

Obviously any fuzzy set A on X is an $IVFS$.

Notation 2.2. *D^X denotes the set of all interval valued fuzzy subsets(ivf) of a non empty set X.*

Definition 2.3 ([10]). *Let X be a non empty set. Let $x_0 \in X$ and $\alpha \in D[0,1]$ be fixed such that $\alpha \neq [0,0]$. Then the interval valued fuzzy subset (ivf) $p_{x_0}^{\alpha}$ is called an interval valued fuzzy point(ivf) defined by,*

$$p_{x_0}^{\alpha} = \begin{cases} \alpha, & \text{if } x = x_0; \\ [0,0], & \text{if } x \neq x_0. \end{cases}$$

Definition 2.4 ([10]). *Let $\bar{A}$ and $\bar{B}$ be any two $IVFS$ of X, that is $\bar{A} = \{\langle x, [A^-(x), A^+(x)] \rangle : x \in X\}$, $\bar{B} = \{\langle x, [B^-(x), B^+(x)] \rangle : x \in X\}$. We define the following relations and operations:*

(1). $\bar{A} \subseteq \bar{B}$ if and only if $A^-(x) \leq B^-(x)$ and $A^+(x) \leq B^+(x)$, for all $x \in X$.

(2). $\bar{A} = \bar{B}$ if and only if $A^-(x) = B^-(x)$, and $A^+(x) = B^+(x)$, for all $x \in X$.

(3). $(\bar{A})^c = \bar{1} - \bar{A} = \{\langle x, [1 - A^+(x), 1 - A^-(x)] \rangle : x \in X\}$.

(4). $\bar{A} \cap \bar{B} = \{\langle x, [\min[A^-(x), B^-(x)], \min[A^+(x), B^+(x)]] \rangle : x \in X\}$.

(5). $\bar{A} \cup \bar{B} = \{\langle x, [\max[A^-(x), B^-(x)], \max[A^+(x), B^+(x)]] \rangle : x \in X\}$.

We denote by $\bar{0}_X$ and $\bar{1}_X$ for the interval valued fuzzy sets $\{\langle x, [0,0] \rangle$, for all $x \in X\}$ and $\{\langle x, [1,1] \rangle$, for all $x \in X\}$ respectively.

Definition 2.5 ([10]). *Let X be a set and $\Im$ be a family of interval valued fuzzy sets $(IVFSs)$ of X. The family $\Im$ is called an interval valued fuzzy topology $(IVFT)$ on X if and only if $\Im$ satisfies the following axioms:*

(1). $\bar{0}_X, \bar{1}_X \in \Im$,

(2). If $\{\bar{A}_i : i \in I\} \subseteq \Im$, then $\bigcup\limits_{i \in I} \bar{A}_i \in \Im$,

(3). If $\bar{A}_1, \bar{A}_2, \bar{A}_3, \ldots, \bar{A}_n \in \Im$, then $\bigcap\limits_{i=1}^{n} \bar{A}_i \in \Im$.

The pair $(X, \Im)$ is called an interval valued fuzzy topological space $(IVFTS)$. The members of $\Im$ are called interval valued fuzzy open sets $(IVFOS)$ in X.

An interval valued fuzzy set $\bar{A}$ in X is said to be interval valued fuzzy closed set $(IVFCS)$ in X if and only if $(\bar{A})^c$ is an $IVFOS$ in X.

Definition 2.6 ([10]). *Let $(X, \Im)$ be an $IVFTS$ and $\bar{A} = \{\langle x, [A^-(x), A^+(x)] \rangle : x \in X\}$ be an $IVFS$ in X. Then the interval valued fuzzy interior and interval valued fuzzy closure of $\bar{A}$ denoted by $ivfint(\bar{A})$ and $ivfcl(\bar{A})$ are defined by*

$$ivfint(\bar{A}) = \bigcup \{\bar{G} : \bar{G} \text{ is an } IVFOS \text{ in } X \text{ and } \bar{G} \subseteq \bar{A}\},$$
$$ivfcl(\bar{A}) = \bigcap \{\bar{K} : \bar{K} \text{ is an } IVFCS \text{ in } X \text{ and } \bar{A} \subseteq \bar{K}\}.$$

For any $IVFS \bar{A}$ in $(X, \Im)$, we have $ivfcl(\bar{A}^c) = (ivfint\ (\bar{A}))^c$ and $ivfint(\bar{A}^c) = (ivfcl(\bar{A}))^c$.

Definition 2.7. *An $IVFS$ $\bar{A} = \{\langle x, [A^-(x), A^+(x)] \rangle : x \in X\}$ in an $IVFTS$ $(X, \Im)$ is said to be an*

(1). interval valued fuzzy regular closed set $(IVFRCS)$ if $\bar{A} = ivfcl(ivfint(\bar{A}))$;

(2). interval valued fuzzy semi-closed set (IVFSCS) if $ivfint\ (ivfcl(\bar{A})) \subseteq \bar{A}$;

(3). interval valued fuzzy preclosed set (IVFPCS) if $ivfcl(ivfint\ (\bar{A})) \subseteq \bar{A}$;

(4). interval valued fuzzy α *closed set (IVF*α*CS) if* $ivfcl(ivfint\ (ivfcl\ (\bar{A}))) \subseteq \bar{A}$;

(5). interval valued fuzzy β *closed set (IVF*β*CS) if* $ivfint(ivfcl\ (ivfint(\bar{A}))) \subseteq \bar{A}$.

Definition 2.8. *An IVFS* $\bar{A} = \{\langle x, [A^-(x), A^+(x)]\rangle : x \in X\}$ *in an IVFTS* $(X, \Im)$ *is said to be an*

(1). interval valued fuzzy generalized closed set (IVFGCS) if $ivfcl(\bar{A}) \subseteq \bar{U}$, *whenever* $\bar{A} \subseteq \bar{U}$ *and* $\bar{U}$ *in an IVFOS;*

(2). interval valued fuzzy generalized regular closed set (IVFGRCS) if $ivfcl(\bar{A}) \subseteq \bar{U}$, *whenever* $\bar{A} \subseteq \bar{U}$ *and* $\bar{U}$ *is an IVFROS.*

Definition 2.9. *An IVFS* $\bar{A} = \{\langle x, [A^-(x), A^+(x)]\rangle : x \in X\}$ *is an IVFTS*$(X, \Im)$ *is said to be an*

(1). interval valued fuzzy semi-preclosed set (IVFSPCS) if there exist on IVFPCS$\bar{B}$, *such that* $ivfint\bar{B} \subseteq \bar{A} \subseteq \bar{B}$;

(2). interval valued fuzzy semi-preopen set (IVFSPOS) if there exist on IVFPOS$\bar{B}$, *such that* $\bar{B} \subseteq \bar{A} \subseteq ivfcl(\bar{B})$.

Definition 2.10. *Let* $\bar{A}$ *be an IVFS is an IVFTS* $(X, \Im)$. *Then the interval valued fuzzy semi-preinterior of* $\bar{A}$ $(ivfspint(\bar{A}))$ *and the interval valued fuzzy semi-preclosure of* $\bar{A}(ivfspcl(\bar{A}))$ *are defined by*

$$ivfspint(\bar{A}) = \bigcup\{\bar{G} : \bar{G} \text{ is an } IVFSPOS \text{ in } X \text{ and } \bar{G} \subseteq \bar{A}\},$$
$$ivfspcl(\bar{A}) = \bigcap\{\bar{K} : \bar{K} \text{ is an } IVFSPCS \text{ in } X \text{ and } \bar{A} \subseteq \bar{K}\}.$$

For any $IVFS$ $\bar{A}$ in $(X, \Im)$, we have $ivfspcl(\bar{A}^c) = (ivfspint(\bar{A}))^c$ and $ivfspint\ (\bar{A}^c) = (ivfspcl(\bar{A}))^c$.

Definition 2.11 ([6])**.** *An IVFS* $\bar{A}$ *in IVFTS* $(X, \Im)$ *is said to be an interval valued fuzzy generalized semi-preclosed set (IVFGSPCS) if* $ivfspcl\ (\bar{A}) \subseteq \bar{U}$, *whenever* $\bar{A} \subseteq \bar{U}$ *and* $\bar{U} \in \Im$.

Definition 2.12 ([6])**.** *The complement* $\bar{A}^c$ *of an IVFGSPCS* $\bar{A}$ *in an IVFTS* $(X, \Im)$ *is called an interval valued fuzzy generalized semi-preopen set (IVFGSPOS) in X.*

Definition 2.13. *An IVFTS* $(X, \Im)$ *is called an interval valued fuzzy semi-pre* $T_{1/2}$ *space (IVFSPT*$_{1/2}$*) if every IVFGSPCS is an IVFSPCS in X.*

Definition 2.14 ([10])**.** *An IVFS* $\bar{A}$ *of a IVFTS of* $(X, \Im)$ *is said to be an interval valued fuzzy neighbourhood(IVFN) of an IVFP* $p^{\alpha}_{x_0}$ *if there exists an IVFOS* $\bar{B}$ *in X such that* $p^{\alpha}_{x_0} \in \bar{B} \subseteq \bar{A}$.

Definition 2.15. *Let* $(X, \Im)$ *and* (Y, σ) *be IVFTSs. Then a map* $g : X \to Y$ *is called an*

(1). interval valued fuzzy continuous (IVF continuous mapping) if $g^{-1}(\bar{B})$ *is IVFOS in X for all* $\bar{B}$ *in* σ.

(2). interval valued fuzzy semi-continuous mapping (IVFS-continuous mapping) if $g^{-1}(\bar{B})$ *is IVFSOS in X for all* $\bar{B}$ *in* σ.

(3). interval valued fuzzy α*-continuous mapping (IVF*α*-continuous mapping) if* $g^{-1}(\bar{B})$ *is IVF*α*OS in X for all* $\bar{B}$ *in* σ.

(4). interval valued fuzzy pre-continuous mapping (IVFP-continuous mapping) if $g^{-1}(\bar{B})$ *is IVFPOS in X for all* $\bar{B}$ *in* σ.

(5). interval valued fuzzy β*-continuous mapping (IVF*β*-continuous mapping) if* $g^{-1}(\bar{B})$ *is IVF*β*OS in X for all* $\bar{B}$ *in* σ.

(6). interval valued fuzzy semi-precontinuous mapping ($IVFSP$-continuous mapping) if $g^{-1}(\bar{B})$ is $IVFSPOS$ in X for all $\bar{B}$ in σ.

Definition 2.16. *Let $(X, \Im)$ and (Y, σ) be $IVFTSs$. Then a map $g : X \to Y$ is called interval valued fuzzy generalized continuous ($IVFG$ continuous) mapping if $g^{-1}(\bar{B})$ is $IVFGCS$ in X for all $\bar{B}$ in σ^c.*

Definition 2.17. *A mapping $g : (X, \Im) \to (Y, \sigma)$ is called an interval valued fuzzy completely generalized semi-precontinuous ($IVFcGSP$ continuous) mapping if $g^{-1}(\bar{V})$ is an $IVFRCS$ in X for every $IVFGSPCS$ $\bar{V}$ in Y.*

3. Main Results

Theorem 3.1. *Every $IVFcGSP$ continuous mapping is an $IVFGSP$ continuous mapping.*

Proof. Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFcGSP$ continuous mapping. Let $\bar{V}$ be an $IVFCS$ in Y. Hence $\bar{V}$ is an $IVFGSPCS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVFRCS$ in X. Since every $IVFRCS$ is an $IVFGSPCS$, $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFGSP$ continuous mapping. □

Remark 3.2. *The converse of the above Theorem 3.1 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $\bar{K_1} = \{\langle a, [0.1, 0.2]\rangle, \langle b, [0.3, 0.4]\rangle\}$, $\bar{L_1} = \{\langle u, [0.8, 0.9]\rangle, \langle v, [0.6, 0.7]\rangle\}$. Then $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ and $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ are $IVFT$ on X and Y respectively. Define a mapping $g : (X, \Im) \to (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFGSP$ continuous mapping but not an $IVFcGSP$ continuous mapping. Since $\bar{L}_1^c = \{\langle u, [0.1, 0.2]\rangle, \langle v, [0.3, 0.4]\rangle\}$ is an $IVFCS$ in Y, it is an $IVFGSPCS$ in Y but $g^{-1}(\bar{L}_1^c) = \{\langle a, [0.1, 0.2]\rangle, \langle b, [0.3, 0.4]\rangle\}$ is not an $IVFRCS$ in X, because $ivfcl(ivfint(g^{-1}(\bar{L_1}^c))) = ivfcl(\bar{0}_X) = \bar{0}_X \neq g^{-1}(\bar{L}_1^c)$.*

Theorem 3.3. *Every $IVFcGSP$ continuous mapping is an $IVFaGSP$ continuous mapping.*

Proof. Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFcGSP$ continuous mapping. Let $\bar{V}$ be an $IVFRCS$ in Y. Hence $\bar{V}$ is an $IVFGSPCS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVFRCS$ in X. Since every $IVFRCS$ is an $IVFGSPCS$, $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFaGSP$ continuous mapping. □

Remark 3.4. *The converse of the above Theorem 3.3 need not be true from the following example: Let $X = \{a, b\}$, $Y = \{u, v\}$ and*

$$\bar{K_1} = \{\langle a, [0.4, 0.5]\rangle, \langle b, [0.6, 0.7]\rangle\},$$
$$\bar{L_1} = \{\langle a, [0.6, 0.7]\rangle, \langle b, [0.7, 0.8]\rangle\},$$
$$\bar{M_1} = \{\langle u, [0.4, 0.8]\rangle, \langle v, [0.7, 0.9]\rangle\}.$$

Then $\Im = \{\bar{0}_X, \bar{K_1}, \bar{L_1}, \bar{1}_X\}$ and $\sigma = \{\bar{0}_Y, \bar{M_1}, \bar{1}_Y\}$ are $IVFT$ on X and Y respectively. Define a mapping $g : (X, \Im) \to (Y, \sigma)$ by $g(a) = u$ and $g(b) = v$. Then g is an $IVFaGSP$ continuous mapping but not an $IVFcGSP$ continuous mapping, since $\bar{M}_1^c$ is $IVFGSPCS$ in Y but $g^{-1}(\bar{M}_1^c)$ is not an $IVFRCS$ in X, because $ivfcl(ivfint(g^{-1}(\bar{M}_1^c))) = \bar{0}_X \neq (g^{-1}(\bar{M}_1^c))$.

Theorem 3.5. *Every $IVFcGSP$ continuous mapping is an IVF continuous mapping.*

Proof. Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFcGSP$ continuous mapping. Let $\bar{V}$ be an $IVFCS$ in Y. Hence $\bar{V}$ is an $IVFGSPCS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVFRCS$ in X and hence an $IVFCS$ in X. Hence g is an IVF continuous mapping. □

Remark 3.6. *The converse of the above Theorem 3.5 need not be true from the following example: Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and* $\bar{K_1} = \{\langle a, [0.1, 0.2]\rangle, \langle b, [0.3, 0.4]\rangle\}$, $\bar{L_1} = \{\langle u, [0.3, 0.4]\rangle, \langle v, [0.4, 0.6]\rangle\}$. *Then* $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ *and* $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ *are* $IVFTs$ *on* X *and* Y *respectively. Define a mapping* $g : (X, \Im) \to (Y, \sigma)$ *by* $g(a) = u$ *and* $g(b) = v$. *Then* g *is an* IVF *continuous mapping but not an* $IVFcGSP$ *continuous mapping, since* $\bar{L}_1^c = \{\langle u, [0.6, 0.7]\rangle, \langle v, [0.4, 0.6]\rangle\}$ *is an* $IVFGSPCS$ *in* Y *but* $g^{-1}(\bar{L}_1^c) = \{\langle a, [0.6, 0.7]\rangle, \langle b, [0.4, 0.6]\rangle\}$ *is not an* $IVFRCS$ *in* X, *because* $ivfcl(ivfint(g^{-1}(\bar{L_1}^c))) = ivfcl(\bar{0}_X) = \bar{0}_X \neq g^{-1}(\bar{L}_1^c)$.

Theorem 3.7. *Every* $IVFcGSP$ *continuous mapping is an* $IVFP$ *continuous mapping.*

Proof. Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFcGSP$ continuous mapping. Let $\bar{V}$ be an $IVFCS$ in Y. Hence $\bar{V}$ is an $IVFGSPCS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVFRCS$ in X, by hypothesis. Since every $IVFRCS$ is an $IVFPCS$, $g^{-1}(\bar{V})$ is an $IVFPCS$ in X. Hence g is an $IVFP$ continuous mapping. □

Remark 3.8. *The converse of the above Theorem 3.7 need not be true from the following example: Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and* $\bar{K_1} = \{\langle a, [0.3, 0.4]\rangle, \langle b, [0.4, 0.7]\rangle\}$, $\bar{L_1} = \{\langle u, [0.1, 0.2]\rangle, \langle v, [0.3, 0.4]\rangle\}$. *Then* $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ *and* $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ *are* $IVFTs$ *on* X *and* Y *respectively. Define a mapping* $g : (X, \Im) \to (Y, \sigma)$ *by* $g(a) = u$ *and* $g(b) = v$. *Then* g *is an* $IVFaGSP$ *continuous mapping but not an* $IVFP$ *continuous mapping. Since* $\bar{L}_1^c = \{\langle u, [0.8, 0.9]\rangle, \langle v, [0.6, 0.7]\rangle\}$ *is an* $IVFCS$ *in* Y *and* $g^{-1}(\bar{L}_1^c) = \{\langle a, [0.8, 0.9]\rangle, \langle b, [0.6, 0.7]\rangle\}$ *is not an* $IVFPCS$ *in* X, *because* $ivfcl(ivfint(g^{-1}(\bar{L_1}^c))) = ivfcl(\bar{K_1}) = \bar{1}_X \not\subset g^{-1}(\bar{L}_1^c)$.

Theorem 3.9. *Every* $IVFSP$ *continuous mapping is an* $IVFaGSP$ *continuous mapping.*

Proof. Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFSP$ continuous mapping. Let $\bar{V}$ be an $IVFRCS$ in Y. Since every $IVFRCS$ is an $IVFSPCS$, $\bar{V}$ is an $IVFSPCS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVFSPCS$ in X. Since every $IVFSPCS$ is an $IVFGSPCS$, $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFaGSP$ continuous mapping. □

Remark 3.10. *The converse of the above Theorem 3.9 need not be true from the following example: Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and* $\bar{K_1} = \{\langle a, [0.3, 0.4]\rangle, \langle b, [0.4, 0.6]\rangle\}$, $\bar{L_1} = \{\langle u, [0.1, 0.2]\rangle, \langle v, [0.3, 0.4]\rangle\}$. *Then* $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ *and* $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ *are* $IVFTs$ *on* X *and* Y *respectively. Define a mapping* $g : (X, \Im) \to (Y, \sigma)$ *by* $g(a) = u$ *and* $g(b) = v$. *Then* g *is an* $IVFaGSP$ *continuous mapping but not an* $IVFSP$ *continuous mapping. Since* $\bar{L}_1^c = \{\langle u, [0.8, 0.9]\rangle, \langle v, [0.6, 0.7]\rangle\}$ *is an* $IVFCS$ *in* Y *but* $g^{-1}(\bar{L}_1^c) = \{\langle a, [0.8, 0.9]\rangle, \langle b, [0.6, 0.7]\rangle\}$ *is not an* $IVFSPCS$ *in* X, *because there exist no* $IVFPCS$ $\bar{B}$ *in* X *such that* $ivfint(g^{-1}(\bar{L}_1^c)) \subseteq \bar{B} \subseteq g^{-1}(\bar{L}_1^c)$.

Theorem 3.11. *Every* $IVF\beta$ *continuous mapping is an* $IVFaGSP$ *continuous mapping.*

Proof. Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFSP$ continuous mapping. Let $\bar{V}$ be an $IVFRCS$ in Y. Since every $IVFRCS$ is an $IVF\beta CS$, $\bar{V}$ is an $IVF\beta CS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVF\beta CS$ in X. Since every $IVF\beta CS$ is an $IVFGSPCS$, $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFaGSP$ continuous mapping. □

Remark 3.12. *The converse of the above Theorem 3.11 need not be true from the following example: Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and* $\bar{K_1} = \{\langle a, [0.5, 0.7]\rangle, \langle b, [0.3, 0.4]\rangle\}$, $\bar{L_1} = \{\langle u, [0.3, 0.4]\rangle, \langle v, [0.4, 0.6]\rangle\}$. *Then* $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ *and* $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ *are* $IVFTs$ *on* X *and* Y *respectively. Define a mapping* $g : (X, \Im) \to (Y, \sigma)$ *by* $g(a) = u$ *and* $g(b) = v$. *Then* g *is an* $IVFaGSP$ *continuous mapping but not an* $IVF\beta$ *continuous mapping. Since* $\bar{L}_1^c = \{\langle u, [0.6, 0.7]\rangle, \langle v, [0.4, 0.6]\rangle\}$ *is an* $IVFCS$ *in* Y *but* $g^{-1}(\bar{L}_1^c) = \{\langle a, [0.6, 0.7]\rangle, \langle b, [0.4, 0.6]\rangle\}$ *is not an* $IVF\beta CS$ *in* X, *because* $ivfint(ivfcl(ivfint(g^{-1}(\bar{L_1}^c)))) = ivfint(ivfcl(\bar{K_1})) = ivfint(\bar{1}_X) = \bar{1}_X \not\subset g^{-1}(\bar{L}_1^c)$.

Theorem 3.13. *Every* $IVF\alpha$ *continuous mapping is an* $IVFaGSP$ *continuous mapping.*

Proof. Let $g : (X, \Im) \to (Y, \sigma)$ be an $IVFSP$ continuous mapping. Let $\bar{V}$ be an $IVFRCS$ in Y. Since every $IVFRCS$ is an $IVF\alpha CS$, $\bar{V}$ is an $IVF\alpha CS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVF\alpha CS$ in X. Since every $IVF\alpha CS$ is an $IVFGSPCS$, $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFaGSP$ continuous mapping. □

Remark 3.14. *The converse of the above Theorem 3.13 need not be true from the following example: Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and* $\bar{K_1} = \{\langle a, [0.3, 0.4]\rangle, \langle b, [0.4, 0.6]\rangle\}$, $\bar{L_1} = \{\langle u, [0.1, 0.2]\rangle, \langle v, [0.3, 0.4]\rangle\}$. *Then* $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ *and* $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ *are* $IVFT$ *on* X *and* Y *respectively. Define a mapping* $g : (X, \Im) \to (Y, \sigma)$ *by* $g(a) = u$ *and* $g(b) = v$. *Then* g *is an* $IVFaGSP$ *continuous mapping but not an* $IVF\alpha$ *continuous mapping. Since* $\bar{L}_1^c = \{\langle u, [0.8, 0.9]\rangle, \langle v, [0.6, 0.7]\rangle\}$ *is an* $IVFCS$ *in* Y *but* $g^{-1}(\bar{L}_1^c) = \{\langle a, [0.8, 0.9]\rangle, \langle b, [0.6, 0.7]\rangle\}$ *is not an* $IVF\alpha CS$ *in* X, *because* $ivfcl(ivfint(ivfcl(g^{-1}(\bar{L_1}^c)))) = ivfcl(ivfint(\bar{1}_X)) = ivfcl(\bar{1}_X) = \bar{1}_X \not\subset g^{-1}(\bar{L}_1^c)$.

Theorem 3.15. *Let* $g : (X, \Im) \to (Y, \sigma)$ *be a mapping where* $g^{-1}(\bar{V})$ *is an* $IVFRCS$ *in* X, *for every* $IVFCS$ $\bar{V}$ *in* Y. *Then* g *is an* $IVFaGSP$ *continuous mapping .*

Proof. Let $\bar{A}$ be an $IVFRCS$ in Y. Since every $IVFRCS$ is an $IVFCS$, $\bar{V}$ is an $IVFCS$ in Y. Then $g^{-1}(\bar{V})$ is an $IVFRCS$ in X. Since every $IVFRCS$ is an $IVFGSPCS$, $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFaGSP$ continuous mapping. Then $g^{-1}(\bar{V})$ is an $IVFRCS$ in X. Since every $IVFRCS$ is an $IVFGSPCS$, $g^{-1}(\bar{V})$ is an $IVFGSPCS$ in X. Hence g is an $IVFaGSP$ continuous mapping. □

Remark 3.16. *The converse of the above Theorem 3.15 need not be true from the following example: Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and* $\bar{K_1} = \{\langle a, [0.3, 0.4]\rangle, \langle b, [0.4, 0.7]\rangle\}$, $\bar{L_1} = \{\langle u, [0.1, 0.2]\rangle, \langle v, [0.3, 0.4]\rangle\}$. *Then* $\Im = \{\bar{0}_X, \bar{K_1}, \bar{1}_X\}$ *and* $\sigma = \{\bar{0}_Y, \bar{L_1}, \bar{1}_Y\}$ *are* $IVFT$ *on* X *and* Y *respectively. Define a mapping* $g : (X, \Im) \to (Y, \sigma)$ *by* $g(a) = u$ *and* $g(b) = v$. *Then* g *is an* $IVFGSP$ *continuous mapping but not a mapping as defined in Theorem 3.15, since* $\bar{L}_1^c = \{\langle u, [0.8, 0.9]\rangle, \langle v, [0.6, 0.7]\rangle\}$ *is an* $IVFCS$ *in* Y *and* $g^{-1}(\bar{L}_1^c) = \{\langle a, [0.8, 0.9]\rangle, \langle b, [0.6, 0.7]\rangle\}$ *is not an* $IVFRCS$ *in* X, *because* $ivfcl(ivfint(g^{-1}(\bar{L_1}^c))) = ivfcl(\bar{K}_1)) = \bar{1}_X \neq g^{-1}(\bar{L}_1^c)$.

Theorem 3.17. *Every* $IVFGSP$ *continuous mapping is an* $IVFaGSP$*- continuous mapping.*

Proof. Let $g : X \to Y$ be an $IVFGSP$-continuous mapping. Let $\bar{A}$ be an $IVFRCS$ in Y . Then $\bar{A}$ is an $IVFCS$ in Y. By hypothesis $g^{-1}(\bar{A})$ is an $IVFGSPS$ in X. Hence g is an $IVFaGSP$ continuous mapping. □

Remark 3.18. *The converse of the above Theorem 3.17 need not be true from the following example: Let* $X = \{a, b\}$, $Y = \{u, v\}$ *and*

$$\bar{K_1} = \{\langle a, [0.4, 0.5]\rangle, \langle b, [0.6, 0.7]\rangle\},$$
$$\bar{L_1} = \{\langle a, [0.6, 0.7]\rangle, \langle b, [0.7, 0.8]\rangle\},$$
$$\bar{M_1} = \{\langle u, [0.4, 0.8]\rangle, \langle v, [0.7, 0.9]\rangle\},$$
$$\bar{N_1} = \{\langle u, [0.3, 0.5]\rangle, \langle v, [0.5, 0.7]\rangle\},$$

Then $\Im = \{\bar{0}_X, \bar{K_1}, \bar{L_1}, \bar{1}_X\}$ *and* $\sigma = \{\bar{0}_Y, \bar{M_1}, \bar{N_1}, \bar{1}_Y\}$ *are* $IVFT$ *on* X *and* Y *respectively. Define a mapping* $g : (X, \Im) \to (Y, \sigma)$ *by* $g(a) = u$ *and* $g(b) = v$. *Then* g *is an* $IVFaGSP$ *continuous mapping but not an* $IVFGSP$ *continuous mapping, since* $\bar{M}_1^c = \{\langle u, [0.2, 0.6]\rangle, \langle v, [0.1, 0.3]\rangle\}$ *is* $IVFCS$ *in* Y *but* $g^{-1}(\bar{M}_1^c)$ *is not an* $IVFGSPCS$ *in* Y, *but* $g^{-1}(\bar{M}_1^c)$ *is not an* $IVFGSPCS$ *in* Y, $g^{-1}(\bar{M}_1^c) = \{\langle a, [0.2, 0.6]\rangle, \langle b, [0.1, 0.3]\rangle\} \subseteq \bar{K_1}$ *but* $ivfspcl(g^{-1}(\bar{M}_1^c)) = \bar{1}_X \not\subset \bar{K_1}$.

Theorem 3.19. *Let* $p_{x_0}^{\alpha}$ *be an* $IVFP$ *in* X. *A mapping* $g : X \to Y$ *is an* $IVFaGSP$ *continuous mapping, then for every* $IVFO$ $\bar{A}$ *in* Y *with* $g(p_{x_0}^{\alpha}) \in \bar{A}$, *there exists an* $IVFOS$ $\bar{B}$ *in* X *with* $p_{x_0}^{\alpha} \in \bar{B}$ *such that* $g^{-1}(\bar{A})$ *is* $IVFD$ *in* $\bar{B}$.

Proof. Let $\bar{A}$ be an $IVFROS$ in Y. Then $\bar{A}$ is an $IVFOS$ in Y. Let $g(p^{\alpha}_{x_0}) \in \bar{A}$, then there exists an $IVFOS$ $\bar{B}$ in X such that $p^{\alpha}_{x_0} \in \bar{B}$ and $ivfcl(g^{-1}(\bar{A})) = \bar{B}$. Since $\bar{B}$ is an $IVFOS$, $ivfcl(g^{-1}(\bar{A}))$ is also an $IVFOS$ in X. Therefore $ivfint(ivfcl(g^{-1}(\bar{A}))) = ivfcl(g^{-1}(\bar{A}))$. Now $g^{-1}(\bar{A}) \subseteq ivfcl(g^{-1}(\bar{A})) = ivfint(ivfcl(g^{-1}(\bar{A}))) \subseteq ivfcl(ivfint(ivfcl(g^{-1}(\bar{A}))))$. This implies $g^{-1}(\bar{A})$ is an $IVF\beta OS$ in X and hence an $IVFGSPOS$ in X. Thus g is an $IVFaGSP$ continuous mappings. □

Theorem 3.20. *Let $f : X \to Y$ be a mapping where X is an $IVFSPT_{1/2}$ space. Then the following are equivalent:*

(1). g is an $IVFaGSP$ continuous mapping.

(2). $ivfspcl(g^{-1}(\bar{A})) \subseteq g^{-1}(ivfcl(\bar{A}))$ for every $IVFSPOS$ $\bar{A}$ in Y,

(3). $ivfspcl(g^{-1}(\bar{A})) \subseteq g^{-1}(ivfcl(\bar{A}))$ for every $IVFSOS$ $\bar{A}$ in Y,

(4). $g^{-1}(\bar{A}) \subseteq ivfspint(g^{-1}(ivfint(ivfcl(\bar{A}))))$ for every $IVFPOS$ $\bar{A}$ in Y

Proof. (1) $\Leftrightarrow$ (2) Let $\bar{A}$ be an $IVFSPOS$ in Y. Then by Definition 2.9, there exists an $IVFPOS$ $\bar{B}$ such that $\bar{B} \subseteq \bar{A} \subseteq ivfcl(\bar{B})$ and $\bar{B} \subseteq ivfint(ivfcl(\bar{B}))$. Now $ivfcl(ivfint(ivfcl(\bar{A}))) \supseteq ivfcl(ivfint(ivfcl(\bar{B}))) \supseteq ivfcl(\bar{B}) \supseteq \bar{A}$. Hence $\bar{A} \subseteq ivfcl(ivfint(ivfcl(\bar{A})))$. Therefore $ivfcl(\bar{A}) \subseteq ivfcl(ivfint(ivfcl(\bar{A})))$. But $ivfcl(ivfint(ivfcl(\bar{A}))) \subseteq ivfcl(\bar{A})$. Hence $ivfcl(ivfint(ivfcl(\bar{A}))) = ivfcl(\bar{A})$. This implies $ivfcl(\bar{A})$ is an $IVFRCS$ in $(X, \Im)$. By hypothesis $g^{-1}(ivfcl(\bar{A}))$ is an $IVFGSPCS$ in X and hence $g^{-1}(ivfcl(\bar{A}))$ is an $IVFSPCS$ in X, since X is an $IVFSPT_{1/2}$ space. This implies $ivfspcl(g^{-1}(ivfcl(\bar{A}))) = g^{-1}(ivfcl(\bar{A}))$. Now $ivfspcl(g^{-1}(\bar{A})) \subseteq ivfspcl(g^{-1}(ivfcl(\bar{A}))) = g^{-1}(ivfcl(\bar{A}))$. Thus $ivfspcl(g^{-1}(\bar{A})) \subseteq g^{-1}(ivfcl(\bar{A}))$.

(2) $\Leftrightarrow$ (3) Since every $IVFSOS$ is an $IVFSPOS$, proof is similar in $(i) \Rightarrow (ii)$.

(3) $\Leftrightarrow$ (1) Let $\bar{A}$ be an $IVFRCS$ in Y. Then $\bar{A} = ivfcl(ivfint(\bar{A}))$. Therefore $\bar{A}$ is an $IVFSOS$ in Y. By hypothesis, $ivfspcl(g^{-1}(\bar{A})) \subseteq g^{-1}(ivfcl(\bar{(}A))) = g^{-1}(\bar{A}) \subseteq ivfspcl(g^{-1}(\bar{A}))$. Hence $g^{-1}(\bar{A})$ is an $IVFSPCS$ and hence is an $IVFGSPCS$ in X. Thus g is an $IVFaGSP$ continuous mapping.

(1) $\Leftrightarrow$ (4) Let $\bar{A}$ be an $IVFPOS$ in Y. Then $\bar{A} \subseteq ivfint(ivfcl(\bar{A}))$. Since $ivfint(ivfcl(\bar{A}))$ is an $IVFROS$ in Y, by hypothesis, $g^{-1}(ivfint(ivfcl(\bar{A})))$ is an $IVFGSPOS$ in X. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(ivfint(ivfcl(\bar{A})))$ is an $IVFSPOS$ in X. Therefore $g^{-1}(\bar{A}) \subseteq g^{-1}(ivfint(ivfcl(\bar{A}))) = ivfspint(g^{-1}(ivfint(ivfcl(\bar{A}))))$.

(4) $\Leftrightarrow$ (1) Let $\bar{A}$ be an $IVFROS$ in Y. Then $\bar{A}$ is an $IVFPOS$ in X. By hypothesis, $g^{-1}(\bar{A}) \subseteq ivfspint(g^{-1}(ivfint(ivfcl(\bar{A})))) = ivfspint(g^{-1}(\bar{A})) \subseteq g^{-1}(\bar{A})$. This implies $g^{-1}(\bar{A})$ is an $IVFSPOS$ in X and hence is an $IVFGSPOS$ in X. Therefore g is an $IVFaGSP$ continuous mapping. □

Theorem 3.21. *Let $g : X \to Y$ be a mapping. If g is an $IVFGSP$ continuous mapping, then $ivfgspcl(g^{-1}(\bar{A}) \subseteq g^{-1}(ivfcl(barA))$ for every $IVFSPOS$ $\bar{A}$ in Y.*

Proof. let $\bar{A}$ be an $IVFSPOS$ in Y. Then $ivfcl(\bar{A})$ is an $IVFRCS$ in Y. By hypothesis $g^{-1}(ivfcl(\bar{A}))$ is an $IVFGSPCS$ in X. Then $ivfgspcl(g^{-1}(ivfcl(\bar{A}))) = g^{-1}(ivfcl(\bar{A}))$. Now $ivfspcl(g^{-1}(\bar{A})) \subseteq ivfgspcl(g^{-1}(ivfcl(\bar{A}))) = g^{-1}(ivfcl(\bar{A}))$. That is $ivfgspcl(g^{-1}(\bar{A})) \subseteq g^{-1}(ivfcl(\bar{A}))$. □

Theorem 3.22. *Let $g : X \to Y$ be a mapping where X is an $IVFSPT_{1/2}$ space. If g is an $IVFaGSP$ continuous mapping, then $ivfint(ivfcl(ivfint(g^{-1}(\bar{B})))) \subseteq g^{-1}(ivfspcl(\bar{B}))$ for every $\bar{B} \in IVFRC(Y)$.*

Proof. Let $\bar{B} \subseteq Y$ be an $IVFRCS$. By hypothesis, $g^{-1}(\bar{B})$ is an $IVFGSPCS$ in X. Since X is an $IVFSPT_{1/2}$ space, $g^{-1}(\bar{B})$ is an $IVFSPCS$ in X. Therefore $ivfspcl(g^{-1}(\bar{B})) = g^{-1}(\bar{B})$. Now $ivfint(ivfcl(ivfint(g^{-1}(\bar{B})))) \subseteq g^{-1}(\bar{B}) \bigcup$

$ivfint(ivfcl(ivfint(g^{-1}(\bar{B})))) \subseteq ivfspcl(g^{-1}(\bar{B})) = g^{-1}(\bar{B}) = g^{-1}(ivfspcl(\bar{B}))$. Hence $ivfint(ivfcl(ivfint(g^{-1}(\bar{B})))) \subseteq g^{-1}(ivfspcl(\bar{B}))$. □

Theorem 3.23. *Let $g : X \to Y$ be a mapping where X is an $IVFSPT_{1/2}$ space. If g is an $IVFaGSP$ continuous mapping, then $g^{-1}(ivfspint(\bar{B})) \subseteq ivfcl(ivfint(ivfcl(g^{-1}(\bar{B}))))$ for every $\bar{B} \in IVFRO(Y)$.*

Proof. This theorem can be easily proved by taking complement in Theorem 3.22. □

References

[1] D. Andrijevic, *Semipreopen Sets*, Mat. Vesnik, 38(1986), 24-32.

[2] A. S. Bin Shahna, *Mappings in Fuzzy Topological spaces*, Fuzzy sets and Systems, 61(1994), 209-213.

[3] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl., 24(1968), 182-190.

[4] J. Dontchev, *On Generalizing Semipreopen sets*, Mem. Fac. sci. Kochi. Univ. Ser. A, Math., 16(1995), 35-48.

[5] S. Ganguly and S. Saha, *A Note on fuzzy Semipreopen Sets in Fuzzy Topological Spaces*, Fuzzy Sets and System, 18(1986), 83-96.

[6] R. Jeyabalan and K. Arjunan, *Notes on interval valued fuzzy generalaized semipreclosed sets*, International Journal of Fuzzy Mathematics and Systems, 3(3)(2013), 215-224.

[7] R. Jeyabalan and K. Arjunan, *Interval valued fuzzy generalaized semi-precontinuous mappings*, Global Journal of Pure and Applied Mathematics.

[8] N. Levine, *Generalized Closed Sets in Topology*, Rend. Circ. Math. Palermo, 19(1970), 89-96.

[9] N. Levine, *Strong Continuity in Topological Spaces*, Amer. Math. Monthly, 67(1960), 269.

[10] T. K. Mondal, *Topology of Interval Valued Fuzzy Sets*, Indian J. Pure Appl. Math., 30(1)(1999), 23-38.

[11] N. Palaniyappan and K. C. Rao, *Regular Generalized Closed Sets*, Kyunpook Math. Jour., 33(1993), 211-219.

[12] Pu Pao-Ming, J.H. Park and B. Y. Lee, *Fuzzy Semipreopensets and Fuzzy semiprecontinuos Mapping*, Fuzzy sets and system, 67(1994), 359-364.

[13] R.K. Saraf and K. Khanna, *Fuzzy Generalized semipreclosed sets*, Jour.Tripura. Math.Soc., 3(2001), 59-68.

[14] L. A. Zadeh, *Fuzzy sets*, Information and control, 8(1965), 338-353.