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Edge Difference Cordial Labeling of Graphs

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Abstract: We put up a dissimilar of difference cordial labeling namely as edge difference cordial labeling. As interchange the roles of vertices and edges in difference cordial labeling. Let G be a (p, q) graph. Let k be an integer with $1 \leq k \leq q$ and $f : E(G) \to \{1, 2, \ldots, k\}$ be a map. For each vertex v, assign the label $\min |(f(e_i) - f(e_j))|$. The function f is called an edge difference cordial labeling of G if f is one-to-one map and $|v_f(1) - v_f(0)| \leq 1$ where $v_f(x)$ denotes the number of vertices labeled with x ($x \in \{1, 2, \ldots, k\}$), $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labeled with 1 and not labeled with 1. A graph with an edge difference cordial labeling is called an edge difference cordial graph. In this paper we investigate some results on newly defined idea.

Keywords: Cordial labeling, edges in difference, cordial graph.

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1. Introduction

Getting motivated from the concept of difference cordial labeling we define here edge difference cordial labeling which is defined as follows: Let G be a (p, q) graph. Let k be an integer with $1 \leq k \leq q$ and $f : E(G) \to \{1, 2, \ldots, k\}$ be a map. For each vertex v, assign the label $min|f(e_i) - f(e_j)|$. The function f is called an edge difference cordial labeling of G if f is one-to-one map and $|v_f(1) - v_f(0)| \leq 1$ where $e_f(x)$ denotes the number of vertices labelled with x ($x \in \{1, 2, \ldots, k\}$), $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with an edge difference cordial labeling is called an edge difference cordial graph. We introduce in this paper edge difference cordial labeling of graphs. We consider simple, finite, undirected graph $G = (V, E)$. For the standard terminology and notations we follow Harary [1].

Definition 1.1. *Let G be a (p, q) graph. A bijective vertex labeling function $f : V(G) \to \{1, 2, \ldots, p\}$ is called a difference cordial labeling if for each edge uv, assign the label $|f(u) - f(v)|$ then $|e_f(0) - e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph.*

Definition 1.2. *The wheel $W_n (n \in \mathbb{N}, n \geq 3)$ is the join of the graphs C_n and K_1. i.e. $W_n = C_n + K_1$. Here vertices corresponding to C_n are called rim vertices and C_n is called rim of W_n. The vertex corresponding to K_1 is called apex vertex.*

Definition 1.3. *A helm $H_n (n \geq 3)$ is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the rim of W_n.*

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Definition 1.4. *A gear graph $G_n (n \geq 3)$ is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices on rim of W_n.*

Definition 1.5. *A one point union $C_n^{(k)}$ of k copies of cycles is the graph obtained by taking v as a common vertex such that any pair cycles are edge disjoint and do not have any vertex in common except v.*

Definition 1.6. *A closed helm CH_n is the graph obtained by taking a helm H_n and adding edges between the pendant vertices.*

Definition 1.7. *The flower fl_n is the graph obtained from a helm H_n by joining each pendent vertex to the apex of the H_n. It contains three types of vertices, an apex of degree $2n$, n vertices of degree 4 and n vertices of degree 2.*

2. Edge Difference Cordial Labeling of Graphs

Here we have prove that cycle, wheel for odd n, helm, gear, closed helm, $C_n^{(t)}$ and flower graph are edge difference cordial graphs.

Theorem 2.1. *C_n admits an edge difference cordial labeling.*

Proof. Let $E(C_n) = \{e_1, e_2, \ldots, e_n\}$ and $V(C_n) = \{v_1, v_2, \ldots, v_n\}$, where $e_i = v_i v_{i+1}, 1 \leq i \leq n-1$ and $e_n = v_n v_1$. We define labeling function $f : E(C_n) \to \{1, 2, \ldots, n\}$ as follows.

Case 1: n is odd.

$$f(e_{2i-1}) = \begin{cases} 4i-3; \ 1 \leq i \leq \left\lceil \frac{n+1}{4} \right\rceil. \\ 2(n-2i)+5; \left\lceil \frac{n+1}{4} \right\rceil + 1 \leq i \leq \frac{n+1}{2}. \end{cases}$$

$$f(e_{2i}) = \begin{cases} 4i-2; \ 1 \leq i \leq \left\lfloor \frac{n+1}{4} \right\rfloor. \\ 2(n-2i)+2; \left\lfloor \frac{n+1}{4} \right\rfloor + 1 \leq i \leq \frac{n-1}{2}. \end{cases}$$

Case 2: n is even.

$$f(e_{2i-1}) = \begin{cases} 4i-3; \ 1 \leq i \leq \left\lceil \frac{n}{4} \right\rceil. \\ 2(n-2i)+4; \left\lceil \frac{n}{4} \right\rceil + 1 \leq i \leq \frac{n}{2}. \end{cases}$$

$$f(e_{2i}) = \begin{cases} 4i-2; \ 1 \leq i \leq \left\lceil \frac{n}{4} \right\rceil. \\ 2(n-2i)+3; \left\lceil \frac{n}{4} \right\rceil + 1 \leq i \leq \frac{n}{2}. \end{cases}$$

Then C_n satisfies the conditions for edge difference cordial labeling i.e. $|v_f(1) - v_f(0)| \leq 1$ and hence C_n is edge difference cordial graph. □

Example 2.2. *Edge difference cordial labeling of C_8 is shown in Figure 1.*

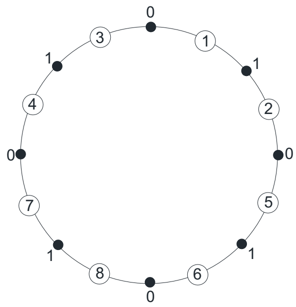

Figure 1.

Theorem 2.3. *W_n admits an edge difference cordial labeling when n is odd.*

Proof. Let $V_{W_n} = \{v_0, v_1, \ldots, v_n\}$ and $E(W_n) = E_1 \cup E_2$, where $E_1 = \{e_i \mid 1 \le i \le n\}$ is the set of rim edges and $E_2 = \{e'_i \mid 1 \le i \le n\}$ is the set of spoke edges. Here $e_i = \{v_i v_{i+1} \mid 1 \le i \le n-1\}$, $e_n = v_n v_1$ and $e'_i = \{v_0 v_i \mid 1 \le i \le n\}$, where v_0 is apex vertex. We define labeling function $f : E(W_n) \to \{1, 2, \ldots, 2n\}$ as follows.

$$f(e_{2i-1}) = \begin{cases} 4i-3; \ 1 \le i \le \lceil \frac{n}{4} \rceil. \\ 2(n-2i)+4; \lceil \frac{n}{4} \rceil + 1 \le i \le \frac{n}{2}. \end{cases}$$
$$f(e_{2i}) = \begin{cases} 4i-2; \ 1 \le i \le \lceil \frac{n}{4} \rceil. \\ 2(n-2i)+3; \quad \lceil \frac{n}{4} \rceil + 1 \le i \le \frac{n}{2}. \end{cases}$$
$$f(e'_i) = n+i; \ 1 \le i \le n.$$

Then W_n satisfies the conditions for edge difference cordial labeling i.e. $|v_f(1) - v_f(0)| \le 1$ and hence W_n is edge difference cordial graph. □

Example 2.4. *Edge difference cordial labeling of W_6 is shown in Figure 2.*

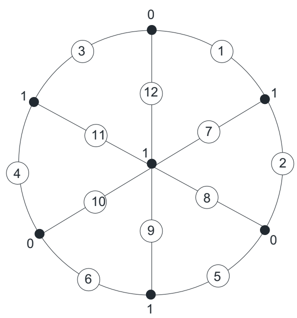

Figure 2.

Theorem 2.5. *H_n admits an edge difference cordial labeling.*

Proof. Let $E(H_n) = E_1 \cup E_2 \cup E_3$, where $E_1 = \{e_i \mid 1 \le i \le n\}$ is the set of spoke edges, $E_2 = \{e'_i \mid 1 \le i \le n\}$ is the set of rim edges(inner cycle) and $E_3 = \{e''_i \mid 1 \le i \le n\}$ is the set of pendant edges of W_n. Here $e_i = \{v_0 v_i \mid 1 \le i \le n\}$, $e'_{i+1} = \{v_i v_{i+1} \mid 1 \le i \le n\}$, $e'_1 = v_n v_1$ and $e''_i = \{v_i v'_i \mid 1 \le i \le n\}$, where v_0 is apex vertex, $v_1, v_2, \ldots, v_n$ are rim vertices corresponding to W_n and $v'_1, v'_2, \ldots, v'_n$ are pendant vertices. We define labeling function $f : E(H_n) \rightarrow \{1, 2, \ldots, 3n\}$ as follows.

$$f(e_i) = i; \qquad 1 \leq i \leq n.$$
$$f(e'_i) = n + i; \quad 1 \leq i \leq n.$$
$$f(e''_i) = 2n + i; \; 1 \leq i \leq n.$$

Then the helm graph H_n satisfies the conditions of an edge difference cordial labeling i.e. $|v_f(1) - v_f(0)| \leq 1$ and hence H_n is edge difference cordial graph. □

Example 2.6. *Edge difference cordial labeling of H_3 is shown in Figure 3.*

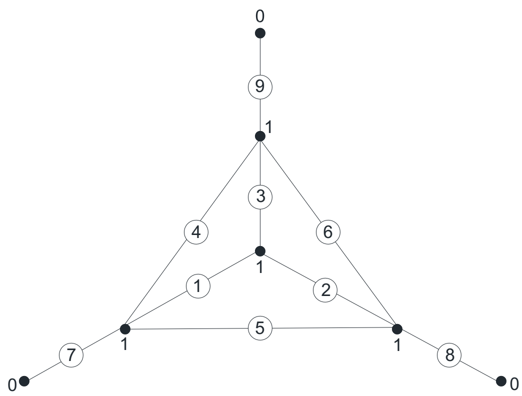

Figure 3.

Theorem 2.7. *Gear G_n admits an edge difference cordial labeling.*

Proof. Let $V(G_n) = \{v_0, v_1, \ldots, v_{2n}\}$, where v_0 is apex vertex, $\{v_1, v_3, \ldots, v_{2n-1}\}$ are the vertices of degree 3 and $\{v_2, v_4, \ldots, v_{2n}\}$ are the vertices of degree 2. Let $E(G_n) = E_1 \cup E_2$, where $E_1 = \{e_1, e_2, \ldots, e_{2n}\}$ and $E_2 = \{e'_1, e'_2, \ldots, e'_n\}$. Here $e_i = \{v_i v_{i+1} \mid 1 \leq i \leq (2n-1)\}$, $e_{2n} = v_{2n} v_1$ and $e'_i = \{v_0 v_{2i-1} \mid 1 \leq i \leq n\}$. We define labeling function $f : E(G_n) \rightarrow \{1, 2, \ldots, 3n\}$ as follows.

$$f(e_{2i+1}) = \begin{cases} 4i+1; \; 0 \leq i \leq \left\lfloor \frac{n-1}{2} \right\rfloor. \\ 4(n-i); \; \left\lfloor \frac{n-1}{2} \right\rfloor + 1 \leq i \leq (n-1). \end{cases}$$
$$f(e_{2i}) = \begin{cases} 4i-2; \; 1 \leq i \leq \frac{n+1}{2}. \\ 4(n-i)+3; \qquad \left\lfloor \frac{n+1}{2} \right\rfloor + 1 \leq i \leq n. \end{cases}$$
$$f(e'_i) = 2n + i; \; 1 \leq i \leq n.$$

Then gear graph G_n satisfies the conditions of edge difference cordial labeling i.e. $|v_f(1) - v_f(0)| \leq 1$ and hence G_n is edge difference cordial graph. □

Example 2.8. *Edge difference cordial labeling of G_4 is shown in Figure 4.*

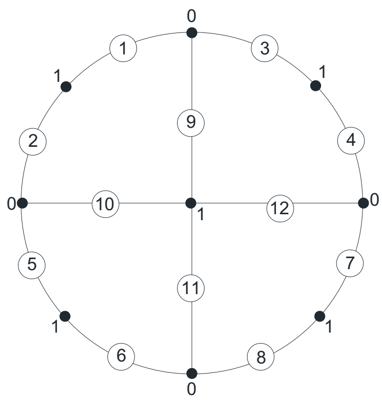

Figure 4.

Theorem 2.9. *$C_3^{(t)}$ admits an edge difference cordial labeling.*

Proof. Let $V(C_n^{(t)}) = \{v_0, v_1, \dots, v_{2n}\}$, where v_0 is apex vertex. Let $E(C_3^{(t)}) = \{e_1, e_2, \dots, e_{3n}\}$, where $e_{3i+1} = \{v_{2i+1}v_{2i+2} \mid 0 \leq i \leq (n-1)\}$, $e_{3i+2} = \{v_0 v_{2i+1} \mid 0 \leq i \leq (n-1)\}$, $e_{3i} = \{v_0 v_{2i} \mid 1 \leq i \leq n\}$. We define labeling function $f : E(C_3^{(t)}) \to \{1, 2, \dots, 3n\}$ as follows.

$$f(e_i) = i;\ 1 \leq i \leq 3n.$$

Then $C_3^{(t)}$ satisfies the conditions of edge difference cordial labeling i.e. $|v_f(1) - v_f(0)| \leq 1$ and hence $C_3^{(t)}$ is edge difference cordial graph. □

Example 2.10. *Edge difference cordial labeling of $C_3^{(5)}$ is shown in Figure 5.*

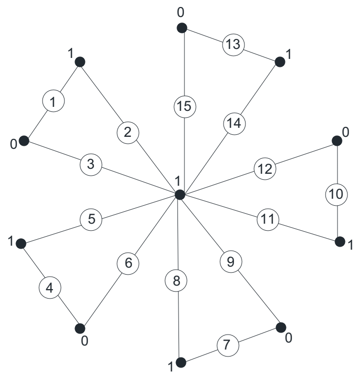

Figure 5.

Theorem 2.11. *CH_n admits an edge difference cordial labeling.*

Proof. Let $V(CH_n) = \{v_0\} \cup V_1 \cup V_2$, where v_0 is apex vertex, $V_1 = \{v_i \mid 1 \leq i \leq n\}$ is the set of vertices of inner cycle and $\{v_i' \mid 1 \leq i \leq n\}$ is the set of vertices of outer cycle. $E(CH_n) = E_1 \cup E_2 \cup E_3 \cup E_4$, where $E_1 = \{e_i = v_0 v_i \mid 1 \leq i \leq n\}$ is the set of spoke edges, $E_2 = \{e_i' = v_i v_{i+1} \mid 1 \leq i \leq (n-1)\} \cup \{e_n' = v_1 v_n\}$ is the set of edges of inner cycle, $E_3 = \{e_i'' = v_i v_i' \mid 1 \leq i \leq n\}$ is the set of edges which is join by corresponding inner cycle and outer cycle and $E_4 = \{e_i''' = v_i' v_{i+1}' \mid 1 \leq i \leq (n-1)\} \cup \{e_n''' = v_1' v_n'\}$ is the set of edges of outer cycle. We define labeling function $f : E(G_n) \to \{1, 2, \dots, 4n\}$ as follows.

$$f(e_i) = 2i - 1; \qquad 1 \leq i \leq n.$$

$$f(e_i') = 2(n+i) - 1; \; 1 \leq i \leq n.$$

$$f(e_i'') = 2i; \qquad 1 \leq i \leq n.$$

$$f(e_i''') = 2(n+i); \qquad 1 \leq i \leq n.$$

Then CH_n satisfies the conditions of edge difference cordial labeling i.e. $|v_f(1) - v_f(0)| \leq 1$ and hence CH_n is edge difference cordial graph. □

Example 2.12. *Edge difference cordial labeling of CH_6 is shown in Figure 6.*

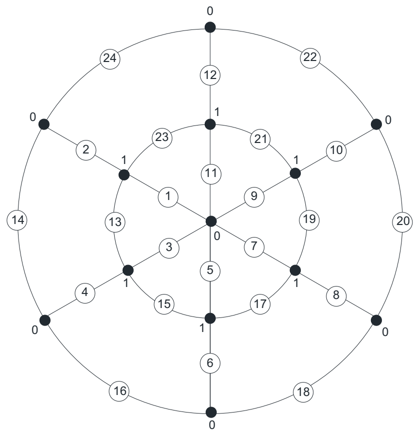

Figure 6.

Theorem 2.13. *fl_n admits an edge difference cordial labeling.*

Proof. Let $V(fl_n) = \{v_0\} \cup V_1 \cup V_2$, where v_0 is apex vertex, $V_1 = \{v_i \mid 1 \leq i \leq n\}$ is the set of vertices inner cycle and $\{v_i' \mid 1 \leq i \leq n\}$ is the set of pendant vertices of H_n, which are adjacent to apex vertex of fl_n. Let $E(fl_n) = E_1 \cup E_2 \cup E_3 \cup E_4$, where $E_1 = \{e_i = v_0 v_i / 1 \leq i \leq n\}$ is the set of spoke edges, $E_2 = \{e_i' = v_{i-1} v_i / 2 \leq i \leq n\} \cup \{e_1' = v_1 v_n\}$ is the set of inner cycle edges, $E_3 = \{e_i'' = v_i v_i' / 1 \leq i \leq n\}$ is the set of pendant edges of H_n and $E_4 = \{e_i''' = v_0 v_i' / 1 \leq i \leq n\}$ is the set of edges whose one end vertex is apex and other is a pendent vertex of H_n which is adjacent to apex in fl_n. We define labeling function $f : E(G_n) \to \{1, 2, \ldots, 4n\}$ as follows.

$$f(e_i) = i; \qquad 1 \leq i \leq n.$$

$$f(e_i') = n + i; \quad 1 \leq i \leq n.$$

$$f(e_i'') = 2n + i; \; 1 \leq i \leq n.$$

$$f(e_i''') = 3n + i; \; 1 \leq i \leq n.$$

Then fl_n satisfies the conditions of edge difference cordial labeling i.e. $|v_f(1) - v_f(0)| \leq 1$ and hence fl_n is an edge difference cordial graph. □

Example 2.14. *Edge difference cordial labeling of fl_4 is shown in Figure 7.*

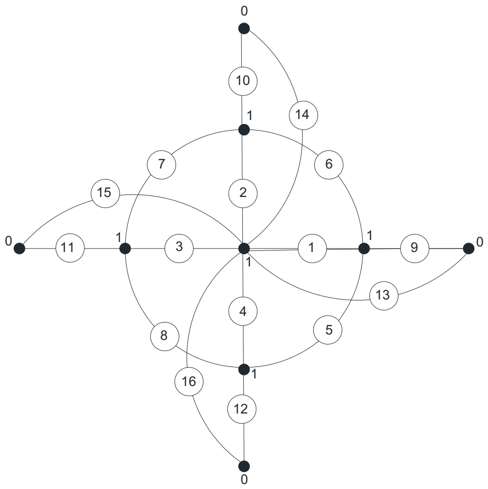

Figure 7.

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