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New Type of Generalized Closed Sets in Topological Spaces

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Abstract: In this paper two new classes of sets, namely $B\delta g$-closed sets and $B\delta g$-open sets are to be introduced and studied in topological spaces. We prove that this class of $B\delta g$-closed sets lies between the class of δ-closed sets and the class of δg-closed sets. Also we find some relations between $B\delta g$-closed sets and already existing closed sets. Further we discuss their characterisations and obtain their applications.

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1. Introduction

The concept of generalized closed sets plays a significant role in topology. In 1970, Levine [6] introduced the concept of generalized closed sets in topological spaces and a class of topological spaces called $T_{1/2}$ space. Extensive research on generalizing closedness was done in recent years by many Mathematicians. Arya and Nour [1], Maki [7], Dontchev and Ganster [3] Tong [11] and Veerakumar [12] introduced generalized semi-closed sets, α-generalized closed sets, δ-generalized closed sets and $\hat{g}$-closed sets in topological spaces. The purpose of this present paper is to define a new class of generalized closed sets called $B\delta g$-closed sets and also we obtain the basic properties of $B\delta g$-closed sets in topological spaces. Applying this set, we obtain a new type of spaces called ${}_BT_{\delta g}$-space.

2. Preliminaries

Throughout this paper (X, τ) (or simply X) represent topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, $cl(A)$, $int(A)$ and A^c denote the closure of A, the interior of A and the complement of A respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 2.1. *A subset A of a space (X, τ) is called*

(1). a semi-open set [5] if $A \subseteq cl(int(A))$.

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(2). a pre-open set [8] if $A \subseteq int(cl(A))$.

(3). an α*-open set [9] if* $A \subseteq int(cl(int(A)))$.

(4). a regular open set [10] if $A = int(cl(A))$.

The complement of a semi-open (respectively a pre-open, an α*-open, a regular) set is called semi-closed (respectively pre-closed,* α*-closed, regular closed). The intersection of all semi-closed (respectively* α*-closed) sets of X containing A is called the semi-closure [2] (respectively* α*-closure [9]) of A and it is denoted by* $scl(A)$ *(respectively* $\alpha cl(A)$*).*

Definition 2.2. *The* δ*-interior [13] of a subset A of X is the union of all regular open sets of X contained in A and it is denoted by* $Int_\delta(A)$*. A subset A is called* δ*-open [13] if* $A = Int_\delta(A)$*, i.e., a set is* δ*-open if it is the union of regular open sets. The complement of a* δ*-open set is called* δ*-closed. Alternatively, a set* $A \subseteq (X, \tau)$ *is called* δ*-closed [13] if* $A = cl_\delta(A)$*, where* $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.

Definition 2.3. *A subset A of* (X, τ) *is called*

(1). generalized closed (briefly g-closed) set [6] if $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open in* (X, τ).

(2). generalized semi-closed (briefly gs-closed) set [1] if $scl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open in* (X, τ).

(3). α*-generalized closed (briefly* αg*-closed) set [7] if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open in* (X, τ).

(4). δ*-generalized closed (briefly* δg*-closed) set [3] if* $cl_\delta(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is open in* (X, τ).

(5). $\hat{g}$*-closed set [12] if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is semi-open in* (X, τ).

(6). δ*-*$\hat{g}$*-closed (briefly* $\delta\hat{g}$*-closed) set [4] if* $cl_\delta(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is* $\hat{g}$*-open in* (X, τ).

The complement of a g-closed (respectively gs-closed, αg*-closed,* δg*-closed,* $\hat{g}$*-closed and* $\delta\hat{g}$*-closed) set is called g-open (respectively gs-open,* αg*-open,* δg*-open,* $\hat{g}$*-open and* $\delta\hat{g}$*-open).*

Definition 2.4 ([11])**.** *A subset A of a space* (X, τ) *is called*

(1). a t-set if $int(A) = int(cl(A))$.

(2). a B-set if $A = G \cap F$ *where G is open and F is a t-set in X.*

Definition 2.5. *A space* (X, τ) *is called*

(1). $T_{1/2}$*-space [6] if every g-closed set in it is closed.*

(2). $T_{3/4}$*-space [3] if every* δg*-closed set in it is* δ*-closed.*

(3). $\hat{T}_{3/4}$*-space [4] if every* $\delta\hat{g}$*-closed set in it is* δ*-closed.*

3. $B\delta g$-closed Sets

In this section we introduce $B\delta g$-closed sets in topological spaces and study some relations between $B\delta g$-closed sets and other existing closed sets.

Definition 3.1. *A subset A of* (X, τ) *is called* $B\delta g$*-closed if* $cl_\delta(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is a B-set.*

The complement of a $B\delta g$-closed set is called $B\delta g$-open.

Theorem 3.2. *Every δ-closed set is $B\delta g$-closed.*

Proof. Let A be a δ-closed set in X. Let U be any B-set such that $A \subseteq U$. Since A is δ-closed, $cl_\delta(A) = A$ for every subset A of (X, τ). Therefore $cl_\delta(A) \subseteq U$ and hence A is $B\delta g$-closed. □

Remark 3.3. *The converse of Theorem 3.2 need not be true as shown by the following Example.*

Example 3.4. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then $\{a, c\}$ is $B\delta g$-closed set but not δ-closed.*

Theorem 3.5. *Every $B\delta g$-closed set is δg-closed.*

Proof. Let A be a $B\delta g$-closed set in X. Let U be any open set containing A in X. Since every open set is a B-set, U is a B-set of X. Since A is $B\delta g$-closed, $cl_\delta(A) \subseteq U$. Hence A is a δg-closed set of X. □

Remark 3.6. *The converse of Theorem 3.5 need not be true as shown by the following Example.*

Example 3.7. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{c\}, \{b, c\}, X\}$. Then $\{a, b\}$ is δg-closed set but not $B\delta g$-closed.*

Theorem 3.8. *Every $B\delta g$-closed set is g-closed.*

Proof. Let A be a $B\delta g$-closed set in X. Let U be an open set in X such that $A \subseteq U$. Since every open set is a B-set, U is a B-set of X. Since A is $B\delta g$-closed, $cl_\delta(A) \subseteq U$. Since $cl(A) \subseteq cl_\delta(A) \subseteq U$, we obtain that $cl(A) \subseteq U$ and hence A is g-closed. □

Remark 3.9. *The converse of Theorem 3.8 need not be true as shown by the following Example.*

Example 3.10. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, X\}$. Then $\{b\}$ is g-closed set but not $B\delta g$-closed.*

Theorem 3.11. *Every $B\delta g$-closed set is αg-closed.*

Proof. It is true that $\alpha cl(A) \subseteq cl_\delta(A)$ for every subset A of X. □

Remark 3.12. *The converse of Theorem 3.11 need not be true as shown by the following Example.*

Example 3.13. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{a, c\}, X\}$. Then $\{b, c\}$ is αg-closed set but not $B\delta g$-closed.*

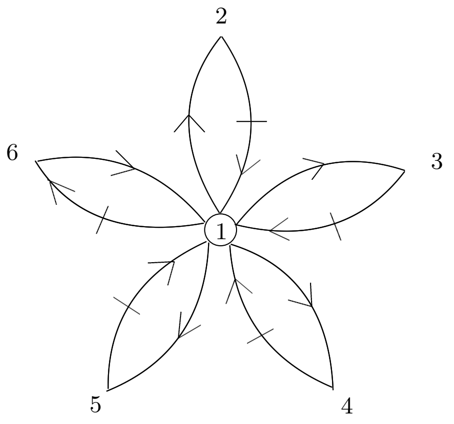
Theorem 3.14. *Every $B\delta g$-closed set is gs-closed.*

Proof. Let A be a $B\delta g$-closed set and U be any open set containing A in X. Since every open set is a B-set, $cl_\delta(A) \subseteq U$ for every subset A of X. Since $scl(A) \subseteq cl_\delta(A) \subseteq U$, $scl(A) \subseteq U$ and hence A is gs-closed. □

Remark 3.15. *A gs-closed set need not be $B\delta g$-closed as shown by the following Example.*

Example 3.16. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{c\}, X\}$. Then $\{b\}$ is gs-closed set but not $B\delta g$-closed.*

Remark 3.17. *From the above discussions we summarize the fundamental relationships between several types of generalized closed sets in the following diagram. None of the implications is reversible.*



(1). $B\delta g$-closed set (2). αg-closed set (3). δg-closed set (4). gs-closed set (5). g-closed set (6).δ-closed set.

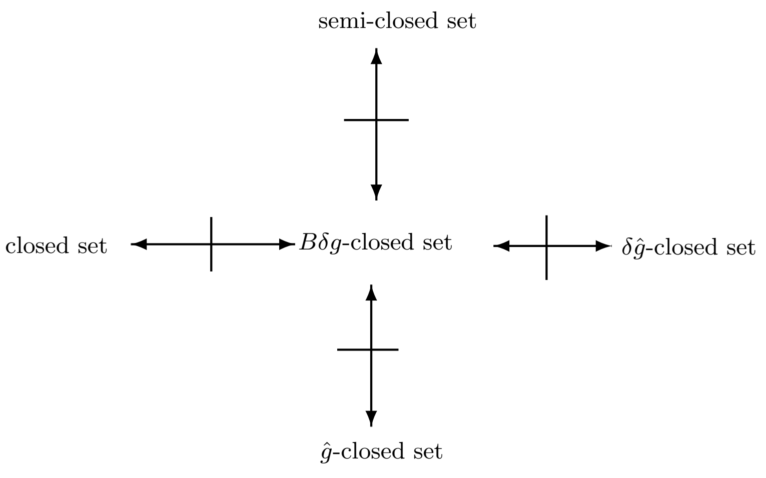
Remark 3.18. *The following Examples show that the concepts of $B\delta g$-closed set and closed set (respectively semi-closed set, $\hat{g}$-closed set and $\delta\hat{g}$-closed set) are independent.*

Example 3.19. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Then $\{a, b\}$ is $B\delta g$-closed set but it is neither closed nor semi-closed. Also $\{a, b\}$ is not $\delta\hat{g}$-closed.*

Example 3.20. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, X\}$. Then $\{b, c\}$ is closed, semi-closed and $\delta\hat{g}$-closed set. But it is not $B\delta g$-closed.*

Example 3.21. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{b\}, X\}$. Then $\{a, c\}$ is $\hat{g}$-closed set but not $B\delta g$-closed and $\{a, b\}$ is $B\delta g$-closed set but not $\hat{g}$-closed in.*

Remark 3.22. *From the above discussions we obtain the following diagram.*



4. Some Topological Properties

Theorem 4.1. *If A is both B-set and $B\delta g$-closed set of (X, τ), then A is δ-closed.*

Proof. Given A is both B-set and $B\delta g$-closed set of (X, τ). Then $cl_\delta(A) \subseteq A$ whenever A is a B-set and $A \subseteq A$. Therefore we obtain that $A = cl_\delta(A)$ and hence A is δ-closed. □

Proposition 4.2. *If A and B are $B\delta g$-closed sets, then $A \cup B$ is $B\delta g$-closed.*

Proof. Let $A \cup B \subseteq U$, where U is a B-set. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $B\delta g$-closed sets, $cl_\delta(A) \subseteq U$ and $cl_\delta(B) \subseteq U$, whenever $A \subseteq U$, $B \subseteq U$ and U is a B-set. Therefore $cl_\delta(A \cup B) = cl_\delta(A) \cup cl_\delta(B) \subseteq U$. So we obtain that $A \cup B$ is $B\delta g$-closed set of (X, τ). □

Remark 4.3. *The intersection of two $B\delta g$-closed sets need not be a $B\delta g$-closed set.*

Example 4.4. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, X\}$. Then $\{a, b\}$ and $\{a, c\}$ are $B\delta g$-closed sets. But $\{a, b\} \cap \{a, c\} = \{a\}$ is not $B\delta g$-closed.*

Proposition 4.5. *If A is a $B\delta g$-closed set of (X, τ) such that $A \subseteq B \subseteq cl_\delta(A)$, then B is also a $B\delta g$-closed set of (X, τ).*

Proof. Let U be a B-set of (X, τ) such that $B \subseteq U$. Since $A \subseteq B$, $A \subseteq U$. Since A is $B\delta g$-closed, we have $cl_\delta(A) \subseteq U$. Now $cl_\delta(B) \subset cl_\delta(cl_\delta(A)) = cl_\delta(A) \subseteq U$. Therefore B is also a $B\delta g$-closed set of (X, τ). □

Proposition 4.6. *Let A be a $B\delta g$-closed set of (X, τ), then $cl_\delta(A) - A$ does not contain a non-empty complement of a B-set.*

Proof. Suppose that A is $B\delta g$-closed. Let F be the complement of a B-set and $F \subseteq cl_\delta(A) - A$. Since $F \subseteq cl_\delta(A) - A$, $F \subseteq X - A$, $A \subseteq X - F$ and $X - F$ is a B-set. Therefore $cl_\delta(A) \subseteq X - F$ and $F \subseteq X - cl_\delta(A)$. Also $F \subseteq cl_\delta(A)$. Therefore $F \subseteq (cl_\delta(A))^c \cap cl_\delta(A) = \phi$. Hence $F = \phi$. □

Theorem 4.7. *Let A be a $B\delta g$-closed set of X. Then A is δ-closed if and only if $cl_\delta(A) - A$ is the complement of a B-set.*

Proof. Necessity: Let A be a δ-closed subset of (X, τ). Then $cl_\delta(A) = A$ and so $cl_\delta(A) - A = \phi$ which is the complement of a B-set.

Sufficiency: Let $cl_\delta(A) - A$ be the complement of a B-set. Since A is $B\delta g$-closed, by Proposition 4.6, $cl_\delta(A) - A$ does not contain a non-empty complement of a B-set which implies $cl_\delta(A) - A = \phi$. Therefore $cl_\delta(A) = A$. Hence A is δ-closed. □

Proposition 4.8. *For each $x \in X$ either $\{x\}$ is the complement of a B-set or $\{x\}^c$ is $B\delta g$-closed in X.*

Proof. Suppose that $\{x\}$ is not the complement of a B-set in X, then $\{x\}^c$ is not a B-set and the only B-set containing $\{x\}^c$ is the space X itself. That is $\{x\}^c \subseteq X$. Therefore $cl_\delta(\{x\}^c) \subseteq X$ and so $\{x\}^c$ is $B\delta g$-closed. □

Definition 4.9. *The intersection of all B-sets of X containing A is called the B-kernel of A and is denoted by B-ker(A).*

Lemma 4.10. *A subset A of (X, τ) is $B\delta g$-closed iff $cl_\delta(A) \subseteq B$-ker(A).*

Proof. Assume that A is $B\delta g$-closed in X. Then $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is a B-set in X. Let $x \in cl_\delta(A)$. Suppose $x \notin B$-ker(A), then there is a B-set U such that $x \notin U$. Since U is a B-set containing A, $x \notin cl_\delta(A)$ which is a contradiction. Hence $x \in B$-ker(A). Conversely assume that $cl_\delta(A) \subseteq B$-ker(A). If U is any B-set containing A, then $cl_\delta(A) \subseteq B$-ker(A)$\subseteq U$. Therefore A is $B\delta g$-closed.

The intersection of all $B\delta g$-closed sets of X containing A is called the $B\delta g$-closure of A and it is denoted by $B\delta g$-cl(A). □

Lemma 4.11. *Let A and B be subsets of (X, τ). Then*

(1). $B\delta g$-cl$(\phi) = \phi$ and $B\delta g$-cl$(X) = X$.

(2). If $A \subset B$, then $B\delta g$-$cl(A) \subset B\delta g$-$cl(B)$.

(3). $B\delta g$-$cl(A) = B\delta g$-$cl(B\delta g$-$cl(A))$.

(4). $B\delta g$-$cl(A \cup B) = B\delta g$-$cl(A) \cup B\delta g$-$cl(B)$.

(5). $B\delta g$-$cl(A \cap B) \subset B\delta g$-$cl(A) \cap B\delta g$-$cl(B)$.

Remark 4.12. *If A is $B\delta g$-closed in (X, τ), then $B\delta g$-$cl(A) = A$ but the converse need not be true as shown by the following Example.*

Example 4.13. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{c\}, X\}$. Let $A = \{c\}$ then $B\delta g$-$cl(A) = \{c\}$. But $\{c\}$ is not a $B\delta g$-closed set.*

Remark 4.14. *In general, $B\delta g$-$cl(A) \cap B\delta g$-$cl(B) \nsubseteq B\delta g$-$cl(A \cap B)$. This can be shown from the following Example.*

Example 4.15. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$. Let $A = \{a, c\}$ and $B = \{b, c\}$, then $B\delta g$-$cl(A) \cap B\delta g$-$cl(B) = X \nsubseteq \{c\} = B\delta g$-$cl(A \cap B)$.*

5. $B\delta g$-open Sets

Definition 5.1. *A subset A of (X, τ) is called $B\delta g$-open if its complement A^c is $B\delta g$-closed in (X, τ).*

Theorem 5.2. *If a subset A of a topological space (X, τ) is δ-open then it is $B\delta g$-open in X.*

Proof. Let A be an δ-open set in X. Then A^c is δ-closed. By Theorem 3.2, A^c is $B\delta g$-closed in (X, τ). Hence A is $B\delta g$-open in X. □

Remark 5.3. *The converse of Theorem 5.2 need not be true as shown by the following Example.*

Example 5.4. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, X\}$. Then $\{b\}$ is $B\delta g$-open set but not δ-open in (X, τ).*

Proposition 5.5. *Every $B\delta g$-open set is δg-open (respectively g-open, αg-open, gs-open).*

Proof. Let A be an $B\delta g$-open set in X. Then A^c is $B\delta g$-closed. By Theorem 3.5, A^c is δg-closed. Hence A is δg-open in X. (respectively By Theorem 3.8, A^c is g-closed. Hence A is g-open in X, By Theorem 3.11, A^c is αg-closed. Hence A is αg-open in X, By Theorem 3.14, A^c is gs-closed. Hence A is gs-open in X).

Remark 5.6. *For a subset A of X, $cl_\delta(X - A) = X - int_\delta(A)$.*

Theorem 5.7. *A subset A of a topological space (X, τ) is $B\delta g$-open if and only if $G \subseteq int_\delta(A)$ whenever $X - G$ is a B-set and $G \subseteq A$*

Proof. Necessity: Let A be $B\delta g$-open. Let $X - G$ be a B-set and $G \subseteq A$. Then $X - A \subseteq X - G$. Since $X - A$ is $B\delta g$-closed, $cl_\delta(X - A) \subseteq X - G$. Hence $G \subseteq int_\delta(A)$.
Sufficiency: Suppose $X - G$ is a B-set and $G \subseteq A$ imply that $G \subseteq int_\delta(A)$. Let $X - A \subseteq U$ where U is a B-set. Then $X - U \subseteq A$ and $X - (X - U)$ is a B-set. By hypothesis $X - U \subseteq int_\delta(A)$. This implies $X - int_\delta(A) \subseteq U$ and $cl_\delta(X - A) \subseteq U$. So $X - A$ is $B\delta g$-closed. Hence A is $B\delta g$-open. □

Proposition 5.8. *If A is a $B\delta g$-open set in (X, τ) such that $int_\delta(A) \subseteq B \subseteq A$, then B is also a $B\delta g$-open set of (X, τ).*

Proof. $int_\delta(A) \subseteq B \subseteq A$ implies that $X - A \subseteq X - B \subseteq X - int_\delta(A)$. By Remark 5.6, $X - A \subseteq X - B \subseteq cl_\delta(X - A)$. Since $X - A$ is $B\delta g$-closed, by Proposition 4.5, $X - B$ is $B\delta g$-closed and hence B is $B\delta g$-open in (X, τ). □

Theorem 5.9. *If a set A is $B\delta g$-open in X then $G = X$ whenever G is a B-set and $int_\delta(A) \cup A^c \subseteq G$.*

Proof. Let A be a $B\delta g$-open set, G be a B-set and $int_\delta(A) \cup A^c \subseteq G$. This implies $G^c \subseteq (int_\delta(A) \cup A^c)^c = (int_\delta(A))^c \cap A = (int_\delta(A))^c - A^c = cl_\delta(A^c) - A^c$. Since A^c is $B\delta g$-closed and G^c is the complement of a B-set, it follows from Proposition 4.6 that $G^c = \phi$. Hence $G = X$. □

Lemma 5.10. *Let A be a subset of (X, τ) and $x \in X$. Then $x \in B\delta g\text{-}cl(A)$ if and only if $V \cap A \neq \phi$ for every $B\delta g$-open set V containing x.*

Proof. Suppose that there exists a $B\delta g$-open set V containing x such that $V \cap A = \phi$. Since $A \subset X - V$, $B\delta g\text{-}cl(A) \subset X - V$ and then $x \notin B\delta g\text{-}cl(A)$. Conversely, assume that $x \notin B\delta g\text{-}cl(A)$. Then there exists a $B\delta g$-closed set F containing A such that $x \notin F$. Since $x \in X - F$ and $X - F$ is $B\delta g$-open, $(X - F) \cap A = \phi$. □

6. Applications

Definition 6.1. *A space X is called a ${}_BT_{\delta g}$-space if every $B\delta g$-closed set in it is δ-closed.*

Theorem 6.2. *Every $T_{3/4}$-space is ${}_BT_{\delta g}$-space.*

Proof. Let A be a $B\delta g$-closed set in X. Since every $B\delta g$-set is δg-closed by Theorem 3.5, A is δg-closed. Since X is $T_{3/4}$-space, A is δ-closed. Hence X is ${}_BT_{\delta g}$-space. □

Remark 6.3. *The converse of Theorem 6.2 need not be true as shown by the following Example.*

Example 6.4. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Then (X, τ) is ${}_BT_{\delta g}$-space but not $T_{3/4}$-space.*

Remark 6.5. *The concepts of ${}_BT_{\delta g}$-space and $\hat{T}_{3/4}$-space are independent of each another as shown by the following Examples.*

Example 6.6. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{a\}, \{b, c\}, X\}$. Then (X, τ) is $\hat{T}_{3/4}$-space but not ${}_BT_{\delta g}$-space.*

Example 6.7. *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$. Then (X, τ) is ${}_BT_{\delta g}$-space but not $\hat{T}_{3/4}$-space.*

Theorem 6.8. *For a topological space (X,τ), the following conditions are equivalent.*

(1). (X, τ) is a ${}_BT_{\delta g}$-space.

(2). Every singleton of X is either δ-open or $X - \{x\}$ is a B-set.

Proof. (1) ⇒ (2) Let $x \in X$. Suppose that $X - \{x\}$ is not a B-set of (X, τ). Then $X - \{x\}$ is a $B\delta g$-closed set of (X, τ). Since (X, τ) is ${}_BT_{\delta g}$-space, $X - \{x\}$ is an δ-closed set of (X, τ), i.e., $\{x\}$ is an δ-open set of (X,τ).

(2) ⇒ (1) Let A be an $B\delta g$-closed set of (X,τ). Let $x \in cl_\delta(A)$. By (ii), $\{x\}$ is either δ-open or $X - \{x\}$ is a B-set.

Case(a) : Let $\{x\}$ be δ-open. Since $x \in cl_\delta(A)$, then $\{x\} \cap A \neq \phi$. This shows that $x \in A$.

Case(b) : Suppose that $X - \{x\}$ is a B-set. If we assume that $x \notin A$, then we would have $x \in cl_\delta(A) - A$, which cannot be happen according to Proposition 4.6. Hence $x \in A$. So in both cases we have $cl_\delta(A) \subseteq A$. Trivially $A \subseteq cl_\delta(A)$. Therefore $A = cl_\delta(A)$ or equivalently A is δ-closed. Hence (X,τ) is a ${}_BT_{\delta g}$-space. □

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