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Reliability Analysis of Time-Dependent System when the Number of Cycles Follow Geometric Distribution and Stress-Strength Follow Exponential Distribution

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Abstract: Failure of a system may occur due to certain type of stresses acting on them. If these stresses do not exceed a certain threshold value the system may work for a long period. On the other hand, if the stresses exceed the threshold they may fail within no time. There is uncertainty about stress and strength random variables at any instant of time and also about the behavior of the variables with respect to time and cycles. Time dependent stress- strength models are considered with repeated application of stress and also the change of the strength with time. Reliability of time dependent stress-strength system is carried out by considering each of stress and strength variables are random-fixed. In this paper to find the reliability, components are assumed to be identical and the number of cycles for any time period t is assumed to be random. Expression for system reliability have been attained when number of cycles can be follow Geometric distribution and stress and strength both follow exponential distribution & computations were also done.

Keywords: Stress-strength system, reliability, Geometric distribution, Exponential distribution, Time dependent.
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1. Introduction

Time dependent stress strength system is defined by [1]. A component fails if the stress on it is exceeding than its strength. In the present paper, the uncertainty about the stress and strength variables is classified into three categories:

(1). Deterministic: the variable assumes values that are exactly known a priory.

(2). Random fixed: the variable is random at any particular instant of time; the word fixed in this classification refers to the behaviour of the random variable with respect to time and/or cycles ; it means that the random variable changes or varies with time in a known manner.

(3). Random independent: the variable is not only random but unlike the random fixed case, the successive values assumed by the variables are statistically independent, in accordance with Kapur and Lamberson and Schatz, et al. Here, in this paper, the components are assumed to be identical and the number of cycles for any time period t is assumed to be random. Expression for system reliability have been attained when number of cycles can be follow geometric distribution and stress and strength both follow different distribution.

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1.1. Notations

R_n	: Reliability after n cycles
$R(t)$	: Reliability at time t
R	: Reliability, independent of the cycle number (fixed)
X	: Stress variable
Y	: Strength variable
x_0	: Deterministic stress
y_0	: Deterministic Strength
X_i	: Stress on the i^{th} cycle
Y_i	: Strength on the i^{th} cycle
$f(x)$	: Probability density function of a random variable X
$g(y)$	: Probability density function of a random variable Y
E_i	: Event no failure occurs on the i^{th} cycle
$f_0(x_0)$	: Probability density function of a random variable x_0
$g_0(y_0)$	: Probability density function of a random variable y_0
$p_i(t)$	: Probability of i cycles occurring in the time interval $[0, t]$
p	: Probability of success in any trial
$q = 1 - p$	: Probability of failure

2. Reliability Evaluation

If the cycles occur at random times, then

$$R(t) = \sum_{i=0}^{\infty} p_i(t) R_i$$

Where $p_i(t)$ is the probability of i cycles occurring in the time interval $[0, t]$ and R_i is, as before, the probability of all i success. Clearly the case of deterministic cycle times becomes a special case above equation. In some cases it is appropriate to assume that the number of cycles occurring in a given time interval are Geometric distributed. Hence

$$P(X = x) = q^x p, \quad x = 0, 1, 2, \ldots$$

Case 1: Deterministic Stress and random-fixed Strength

Let the stress be x_0, a constant, and the strength on the i^{th} cycle be Y_i given by

$$Y_i = Y_0 - a_i \ , \quad i = 1, 2, \ldots$$

Where $a_i \geq 0$ are known constants. Further, the a_i's are assumed nondecreasing in time. The probability density function of Y_0, $g_0(y_0)$ is assumed known. Then

$$\begin{aligned} P[E_n] &= P(x_n \leq Y_n) \\ &= P(x_0 \leq y_0 - a_n) \\ &= \int_{x_0 + a_n}^{\infty} g_0(y_0) dy_0 \end{aligned}$$

Hence

$$R_n = P\left[E_n\right] = \int_{x_0+a_n}^{\infty} g_0(y_0)dy_0$$

Let $Y_i = Y_0$, $i = 1, 2, \ldots$ be the strength random variable with a known probability density function $f_0(y_0)$. Then

$$R_i = P\left[E_i\right] = \int_{x_0}^{\infty} f_0(y_0)dy_0 \triangleq R$$

The expression for R_i is independent of the cycle number i. Hence

$$\begin{aligned} R\left(t\right) &= \sum_{i=0}^{\infty} p_i\left(t\right)R_i \\ &= p_0\left(t\right)R_0 + \sum_{i=1}^{\infty} p_i\left(t\right)R_i \\ &= p + R(1-p_0) \\ R(t) &= p + R\left(1-p\right) = p + qR \end{aligned}$$

Case 2: Deterministic Stress and Random independent Strength

Let the stress be constant at x_0. Let $g_i(y)$ be the probability density function of the random variable strength Y_i during the cycle i. Since successive values of Y_i are independent, we get

$$R_n = P\left[E_1, E_2, \ldots . E_n\right] = P\left[E_1\right] * P\left[E_2\right] * \cdots * P\left[E_n\right]$$

Where

$$P\left[E_i\right] = P\left(x_0 \leq y_i\right) = \int_{x_0}^{\infty} g_i\left(y\right)dy$$

In particular, if the probability density function remains unchanged over time, that is, if

$$g_1\left(y\right) = g_2\left(y\right) = \cdots = g_n\left(y\right) = g(y)$$

Then

$$R_n = \left(P\left[E_i\right]\right)^n = \left\{\int_{x_0}^{\infty} g\left(y\right)dy\right\}^n$$

Let $R_i = R^i$, $i = 0, 1, 2, \ldots, n$, where $R = \int\limits_{x_0}^{\infty} g\left(y\right)dy$

$$\begin{aligned} R\left(t\right) &= \sum_{i=0}^{\infty} p_i\left(t\right)R_i \\ &= \sum_{i=0}^{\infty} q^i pR^i \\ &= p\sum_{i=0}^{\infty} \left(qR\right)^i \\ &= p\left[1 + qR + qR^2 + \ldots\right] \\ &= \frac{p}{(1-qR)} \end{aligned}$$

Case 3: Random-fixed Stress and deterministic Strength

Let the strength be y_0, a constant, and the stress on the i^{th} cycle be x_i given by

$$X_i = X_0 + b_i, \quad i = 1, 2, \ldots.$$

Where $b_i \geq 0$ are known constants. Further, the b_i's are assumed nondecreasing in time. The probability density function of X_0, $f_0(x_0)$ is assumed known. Then

$$\begin{aligned} R_n &= P\left[E_n\right] = P\left(X_n \leq y_n\right) \\ &= P\left(x_0 + b_n \leq y_0\right) \\ R_n &= \int_0^{y_0 - b_n} f_0(x_0) dx_0 \end{aligned}$$

Then

$$R_i = P\left[E_i\right] = \int_0^{y_0} f_0(x_0) dx_0 \triangleq R$$

The expression for R_i is independent of the cycle number i. Hence by reciprocity of Case 2, we get

$$R\left(t\right) = p + qR, \quad where \quad R = \int_0^{y_0} f_0\left(x_0\right) dx_0$$

Case 4: Random-fixed Stress and random-fixed Strength

Let the stress be given by

$$X_i = X_0 + b_i, \quad i = 1, 2, \ldots.$$

Where $b_i \geq 0$ are known constants. Further, the b_i's are assumed nondecreasing in time. Let the strength be given by

$$Y_i = Y_0 - a_i, \quad i = 1, 2, \ldots.$$

Where $a_i \geq 0$ are known constants. Further, the a_i's are assumed nondecreasing in time. The probability density functions $f_0\left(x_0\right)$ and $g_0(y_0)$ are assumed known. We have to required the stress to be nondecreasing and strength to be nonincreasing. Hence

$$\begin{aligned} R_n &= P\left[E_n\right] \\ &= P\left(X_n \leq Y_n\right) \\ &= P\left(x_0 + b_n \leq y_0 - a_n\right) \\ &= \int_0^{\infty} g_0(y_0) \left(\int_0^{y_0 - a_n - b_n} f_0\left(x_0\right) dx_0 \right) dy_0 \end{aligned}$$

Let X_0 and Y_0 be the random fixed stress and strength with known probability density functions $f_0\left(x_0\right)$ and $g_0(y_0)$ respectively. X_0 and Y_0 will be assumed not vary with time that is $a_i = b_i = 0$, $i = 1, 2, \ldots.$ Hence

$$\begin{aligned} R_i &= \int_0^{\infty} g_0(y_0) \int_0^{y_0} f_0\left(x_0\right) dx_0 dy_0 = R, \quad i = 1, 2, \ldots. \\ R\left(t\right) &= \sum_{i=0}^{\infty} p_i\left(t\right) R_i \\ &= p_0\left(t\right) R_0 + \sum_{i=1}^{\infty} p_i\left(t\right) R_i \end{aligned}$$

$$= p(1) + R(1 - p_0)$$

$$= p + R\,(1 - p) = p + qR$$

Case 5: Random-independent Stress and deterministic Strength

Let the strength be constant at y_0. Let $f_i(x)$ be the probability density function of the random variable stress X_i during the cycle i. Since successive values of X_i are independent, we get

$$R_n = P\,[E_1, E_2, \ldots .E_n] = P\,[E_1] * P\,[E_2] * \ldots \cdots * P\,[E_n]$$

Where

$$P\,[E_i] = P\,(X_i \leq y_0) = \int_0^{y_0} f_i\,(x)\,dx$$

In particular, if the probability density function remains unchanged over time, that is, if

$$f_1\,(x) = f_2\,(x) = \cdots = f_n\,(x) = f(x)$$

Then

$$R_n = (P\,[E_i])^n = \left\{\int_0^{y_0} f\,(x)\,dx\right\}^n$$

Let $R_i = R^i$, we get

$$R\,(t) = \sum_{i=0}^{\infty} p_i\,(t)\,R_i$$

$$R\,(t) = \frac{p}{(1 - qR)} \quad where \quad R = \int_0^{y_0} f\,(x)\,dx$$

Case 6: Random-independent Stress and random-independent Strength

Let $f_i\,(x)$ and $g_i\,(y)$ be the probability density functions of stress X_i and strength Y_i respectively in cycle $i = 1, 2, \ldots$. Then, since X_i's and Y_i's are independent,

$$R_n = P\,[E_1, E_2, \ldots .E_n]$$

$$= P\,[E_1]\,.P\,[E_2]\,\ldots .P\,[E_n]$$

$$= \prod_{i=1}^{n} P(E_i)$$

Where

$$P = P(X_i < Y_i)$$

$$= \int_0^{\infty} f_i(x) \int_x^{\infty} g_i(y)dydx$$

Let $f\,(x)$ and $g(y)$ represent the probability density functions for stress X and strength Y respectively. Further the random variables be independent on each cycle. Then

$$R_i = R^i, \quad i = 0, 1, 2, \ldots, n$$

$$R\,(t) = \sum_{i=0}^{\infty} p_i\,(t)\,R_i$$

$$= \sum_{i=0}^{\infty} q^i p R^i$$
$$= p \sum_{i=0}^{\infty} (qR)^i$$
$$R(t) = p\left[1 + qR + qR^2 + \dots\right]$$
$$= \frac{p}{(1 - qR)}$$

where, $R = \int\limits_0^{\infty} f(x) \int_x^{\infty} g(y)\, dy dx$.

3. In Geometric Distribution, the Stress and Strength Follow Exponential Distribution

Case 1: Deterministic Stress and random-fixed Strength

$$R(t) = p + qR$$

where

$$R = \int_{x_0}^{\infty} f_0(y_0)\, dy_0$$
$$= \int_{x_0}^{\infty} \mu e^{-\mu y_0} dy_0 = e^{-\mu x_0}$$
$$R(t) = p + qe^{-\mu x_0}$$

Case 2: Deterministic Stress and random-independent Strength

$$R(t) = \frac{p}{(1 - qR)}$$

where

$$R = \int_{x_0}^{\infty} g(y)\, dy$$
$$= \int_{x_0}^{\infty} \mu e^{-\mu y} dy = e^{-\mu x_0}$$
$$R(t) = \frac{p}{(1 - q\ e^{-\mu x_0})}$$

Case 3: Random- fixed stress and deterministic Strength

$$R(t) = p + qR, \quad where \quad R = \int_0^{y_0} f_0(x_0)\, dx_0$$
$$R = \int_0^{y_0} \lambda e^{-\lambda x_0} dx_0 = \left(1 - e^{-\lambda y_0}\right)$$
$$R(t) = p + q\left(1 - e^{-\lambda y_0}\right) = 1 - qe^{-\lambda y_0}$$

Case 4: Random-fixed Stress and random- fixed Strength

$$R(t) = p + qR$$

where

$$
\begin{aligned}
R &= \int_0^{\infty} g_0(y_0) \int_0^{y_0} f_0\left(x_0\right) dx_0 dy_0 \\
&= \int_0^{\infty} \mu e^{-\mu y_0} \int_0^{y_0} \lambda e^{-\lambda x_0} dx_0 dy_0 \\
R &= \frac{\lambda}{(\lambda+\mu)} \\
R\left(t\right) &= p + \frac{q\lambda}{(\lambda+\mu)} \\
&= \frac{[\lambda + p\mu]}{(\lambda+\mu)}
\end{aligned}
$$

Case 5: Random-independent Stress and deterministic Strength

$$
\begin{aligned}
R\left(t\right) &= \frac{p}{(1-qR)} \quad where \;\; R = \int_0^{y_0} f\left(x\right) dx \\
R &= \int_0^{y_0} \lambda e^{-\lambda x} dx \\
&= \left(1 - e^{-\lambda y_0}\right) \\
R\left(t\right) &= \frac{p}{\left(1 - q\left(1 - e^{-\lambda y_0}\right)\right)} \\
&= \frac{p}{\left(p + qe^{-\lambda y_0}\right)}
\end{aligned}
$$

Case 6: Random-independent Stress and random-independent Strength

$$
R\left(t\right) = \frac{p}{(1-qR)}
$$

where

$$
\begin{aligned}
R &= \int_0^{\infty} f(x) \int_x^{\infty} g\left(y\right) dy dx \\
R &= \int_0^{\infty} \lambda e^{-\lambda x} \int_x^{\infty} \mu e^{-\mu y} dx dy \\
&= \frac{\lambda}{(\lambda+\mu)} \\
R\left(t\right) &= \frac{p}{\left(1 - q\frac{\lambda}{(\lambda+\mu)}\right)} \\
&= \frac{p\left(\lambda+\mu\right)}{\left(p\lambda+\mu\right)}
\end{aligned}
$$

4. Numerical Results

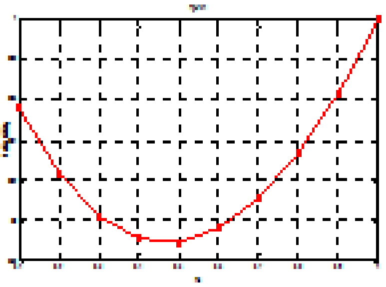
Case 1: Deterministic Stress and random- fixed Strength

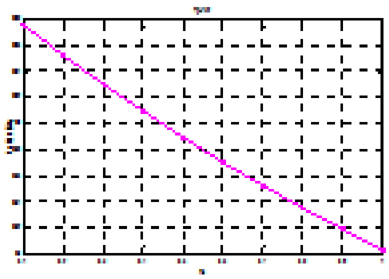
p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
R	0.9144	0.9239	0.9333	0.9429	0.9524	0.9619	0.9715	0.9809	0.9905	1

Table 1. ($\mu = 0.2$, $x_0 = 0.5$)

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.9561	0.9239	0.9025	0.8912	0.8894	0.8963	0.9114	0.9341	0.9638	1

Table 2. ($x_0 = 0.5$)





Case 2: Deterministic Stress and random-independent Strength

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
R	0.5387	0.7243	0.8183	0.8751	0.9131	0.9403	0.9608	0.9768	0.9895	1

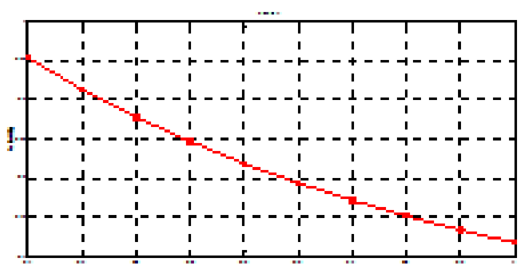
Table 3. ($\mu = \mathbf{0.2}$, $x_0 = 0.5$)

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.6949	0.7243	0.7547	0.7862	0.8189	0.8527	0.8876	0.9238	0.9613	1

Table 4. ($x_0 = 0.5$)

μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.9534	0.9131	0.8777	0.8465	0.8188	0.7942	0.7720	0.7520	0.7340	0.7176

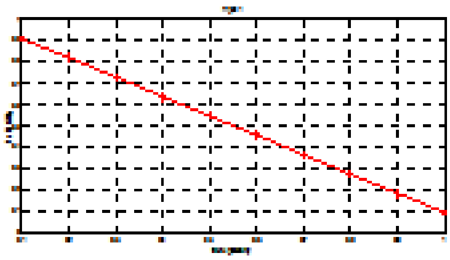
Table 5. ($p = 0.5$, $q = 0.5$, $x_0 = 0.5$)



Case 3: Random-fixed stress and deterministic Strength

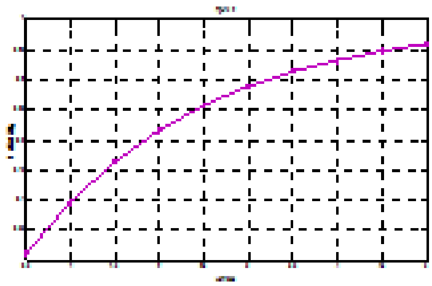
q	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.9095	0.8190	0.7285	0.6380	0.5475	0.4571	0.3666	0.2761	0.1856	0.0951

Table 6. ($\lambda = 0.2$, $y_0 = 0.5$)



λ	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
R	0.6106	0.6967	0.7638	0.8161	0.8567	0.8884	0.9131	0.9323	0.9473	0.9589

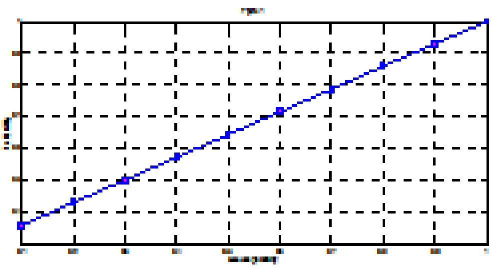
Table 7. $(q = 0.5,\ y_0 = 0.5)$



Case 4: Random-fixed Stress and random-fixed Strength

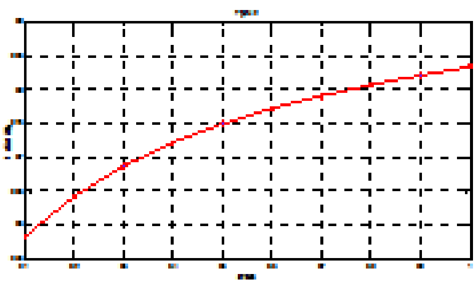
p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.3571	0.4286	0.5	0.5714	0.6428	0.7143	0.7857	0.8571	0.9286	1

Table 8. $(\lambda = 0.2,\ \mu = 0.5)$



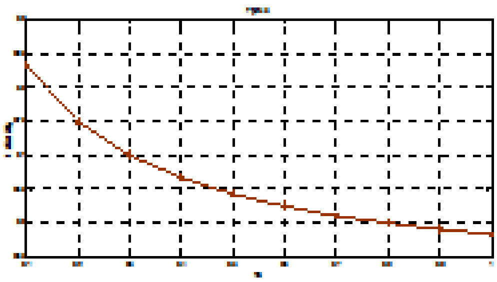
λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.5833	0.6429	0.6875	0.7222	0.75	0.7727	0.7917	0.8077	0.8214	0.8333

Table 9. $(p = 0.5,\ \mu = 0.5)$



μ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.8333	0.75	0.7	0.6667	0.6428	0.625	0.6111	0.6	0.5909	0.5833

Table 10. $(p = 0.5,\ \lambda = 0.2)$



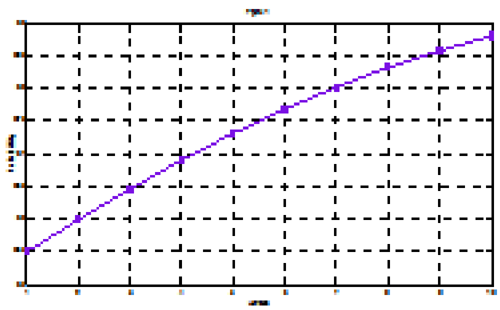
Case 5: Random-independent Stress and deterministic Strength

p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
R	0.1094	0.2165	0.3214	0.4242	0.5249	0.6237	0.7206	0.8155	0.9086	1

Table 11. $(\lambda = 0.5,\ y_0 = 0.2)$

λ	1	2	3	4	5	6	7	8	9	10
R	0.5498	0.5986	0.6456	0.6899	0.7310	0.7685	0.8021	0.8320	0.8581	0.8807

Table 12. $(p = q = 0.5,\ y_0 = 0.2)$



y_0	1	2	3	4	5	6	7	8	9	10
R	0.5498	0.5986	0.6456	0.6899	0.7310	0.7685	0.8021	0.8320	0.8581	0.8807

Table 13. $(p = q = 0.5,\ \lambda = 0.2)$

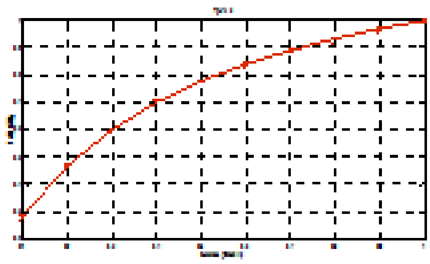
p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
q	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
λ	1	2	3	4	5	6	7	8	9	10
R	0.1194	0.2716	0.4384	0.5973	0.7310	0.8327	0.9044	0.9519	0.9819	1

Table 14. $(y_0 = 0.2)$

Case 6: Random-independent Stress and random-independent Strength

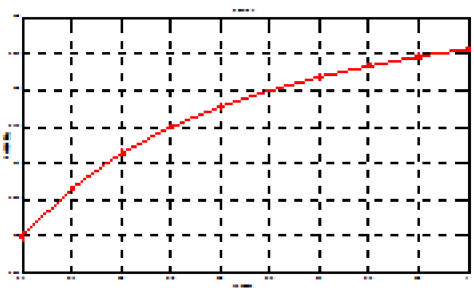
p	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.28	0.4667	0.6	0.7	0.7778	0.84	0.8909	0.9333	0.9692	1

Table 15. ($\lambda = 0.5$, $\mu = 0.2$)



λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
R	0.6	0.6667	0.7142	0.75	0.7778	0.8	0.8182	0.8333	0.8461	0.8571

Table 16. ($p = 0.5$, $\mu = 0.2$)



5. Conclusion

In this, Reliability of stress-strength system has been done when the number of cycles follows Geometric distribution. Numerical calculations for reliability have been carried for six models, where stress and strength follow Exponential distribution. Reliability computations have been done for dependent and independent of time. It is observed by the computations, the reliability increases when stress parameter (λ), strength parameter (y_0) increase and reliability decreases when mean no. of cycles (a), strength parameter (μ) and stress parameter (x_0) increase.

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