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Dominator Chromatic Number of Derived Graph of Some Graphs

R. Kalaivani[1,*] and D. Vijayalakshmi[1]

1 Department of Mathematics, Kongunadu Arts And Scienc College, Coimbatore, Tamilnadu, India.

Abstract: In a graph G, the distance $d(u, v)$ between a pair of vertices u and v is the length of a shortest path joining them. The Derived graph, $G^{\dagger}$ is the graph whose vertices are same as the vertices of G and two vertices in $G^{\dagger}$ are adjacent if and only if the distance between them in G is two. In this paper we obtain the exact value for χ_d for Derived Graph of Path, Cycle, Sunlet graph, Bistar graph and Triple Star graph respectively.

MSC: 05C15, 05C75.

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1. Introduction

The field of Graph theory plays an important role in various areas of pure and applied sciences. We begin with simple, connected, finite, undirected graph $G = (V(G), E(G))$ with $p = |V(G)|$ and $q = |E(G)|$. For all terminology and notations in graph theory especially defined in this paper, we refer the reader to the standard text books [8] respectively. The minimum and maximum degree of G are denoted by $\delta = \delta(G)$ and $\Delta = \Delta(G)$ respectively. The distance $d_G(v_i; v_j)$ between the vertices v_i and v_j is the length of a shortest path between them. If there is no path between v_i and v_j then we formally assume that $d_G(v_i; v_j) = \infty$.

A dominating set, D of a graph G is a subset of the vertices in G such that for each vertex v, $N_G[v] \cap D \neq \emptyset$ The domination number $\gamma(G)$ of G is the cardinality of a minimum dominating set. The concept of domination in graphs, with its many variations, has been well studied in graph theory [7]. A proper coloring of a graph G is a function from the set of vertices of a graph to a set of colors such that any two adjacent vertices have different colors. A subset of vertices colored with the same color is called a color class. The chromatic number is the minimum number of colors needed in a proper coloring of a graph and is denoted by χ (G). A dominator coloring is a coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other color class. The concept of a dominator coloring in a graph was introduced and studied by Gera [4] and studied further by Gera [3, 5] and Chellai and Maffray [1].

Let G be a simple graph. Its derived graph $G^{\dagger}$ is the graph whose vertices are same as the vertices of G and two vertices in $G^{\dagger}$ are adjacent if and only if the distance between them in G is two. Directly from this definition follows that $[G_1 \cup G_2]^{\dagger} = [G_1]\dagger \cup [G_2]\dagger$. Double graph [2] of a connected graph G is constructed by taking two copies of G say G' and G'', join each vertex u' in G' to the neighbour of the corresponding vertices u'' in G''. Double graph of G is denoted by $D(G)$.

* *E-mail: kalaivanirm@yahoo.com (Research Scholar)*

2. Dominator Chromatic Number of Derived Graphs

Theorem 2.1. *For $n \geq 3$, the dominator chromatic number of derived graph of Sunlet graph is.*

$$\chi_d(S_n)^\dagger = \begin{cases} \lfloor 2n/3 \rfloor + 1 & \textit{when } n \equiv 1 \mod 4 \\ n/2 + 3 & \textit{when } n \equiv 2 \mod 4 \\ \lceil n/2 \rceil + 2 & \textit{when } n \equiv 3 \mod 4 \\ n/2 + 2 & \textit{when } n \equiv 0 \mod 4 \end{cases}$$

Proof. Let us define the vertex set V of S_n as $V(S_n) = \{v_i : \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$ where v_i are the vertices of cycles taken in cyclic order and u_i are the pendent vertices. By the definition of derived graph, $V(S_n)^\dagger = \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$. A procedure to obtain dominator coloring of derived graph of sunlet graph as follows. Define a coloring function f on $V(S_n)^\dagger$ such that for all vertices of v_i and u_i

Case 1: When $n \not\equiv 1 \mod 4$

$$f(v_i) = \begin{cases} i \text{ for } v_{2i-2} \text{ when } i = 2k, 1 \leq k \leq \lceil n/4 \rceil \\ i \text{ for } v_{2i-1} \text{ when } i = 2k-1, 1 \leq k \leq \lceil n/4 \rceil \end{cases}$$

Remaining vertices are colored in the following sub cases.

Sub Case 1: $n \equiv 2 \mod 4$

$$f(v_i, u_i) = \begin{cases} n/2 + 2 \text{ for } u_i : 1 \leq i \leq n \\ n/2 + 3 \text{ otherwise} \end{cases}$$

Sub Case 2: $n \equiv 3 \mod 4$

$$f(v_i, u_i) = \begin{cases} \lceil n/2 \rceil + 1 \text{ for } v_{2i-1} \text{ when } i = 2k, 1 \leq k \leq \lceil n/4 \rceil \\ \lceil n/2 \rceil + 1 \text{ for } v_{2i} \text{ when } i = 2k, 1 \leq k \leq \lfloor n/4 \rfloor \\ \lceil n/2 \rceil + 2 \text{ for } u_i : 1 \leq i \leq n \end{cases}$$

Sub Case 3: $n \equiv 0 \mod 4$

$$f(v_i, u_i) = \begin{cases} n/2 + 2 \text{ for } v_{2i-1}, v_{2i} \text{ when } i = 2k, 1 \leq k \leq \lceil n/4 \rceil \\ n/2 + 1 \text{ for } u_i : 1 \leq i \leq n \end{cases}$$

Case 2: When $n \equiv 1 \mod 4$

$$f(v_i, u_i) = \begin{cases} i \text{ for } v_{2i-2} \text{ when } i = 2k, 1 \leq k \leq \lfloor n/4 \rfloor \\ i \text{ for } v_{2i-1} \text{ when } i = 2k-1, 1 \leq k \leq \lceil n/4 \rceil \\ \lfloor 2n/3 \rfloor \text{ for } u_i : 1 \leq i \leq n. \\ \lfloor 2n/3 \rfloor + 1 \text{ otherwise} \end{cases}$$

This Completes the proof of the theorem. □

Theorem 2.2. *For any n the dominator chromatic number of derived graph of double Star graph is,*

$$\chi_d(K_{1,n,n})^\dagger = n + 1$$

Proof. Let $V(K_{1,n,n})^{\dagger} = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$ be the vertices of derived graph of double star graph. The following procedure gives the dominator chromatic number of derived graph of double star graph. Consider the color class $C = \{c_1, c_2, c_3, \ldots, c_n, c_{n+1}\}$ Here $v_i : 1 \leq i \leq n$ forms a clique of order n so we have to assign c_i colors to $v_i : 1 \leq i \leq n$. Next assign the color c_{n+1} to v and c_{n-1} color to $u_i : 1 \leq i \leq n$. By the definition of dominator coloring every vertex of v_i dominates any one color class c_i and v dominates itself. Next the vertices $u_i : 1 \leq i \leq n$ dominates the color class c_{n+1}. Hence an easy observation shows that $\chi_d(K_{1,n,n})^{\dagger} = n+1$, where $c_i = \{v_i : 1 \leq n-1\}, c_n = \{v_n, u_i : 1 \leq i \leq n\}, c_{n+1} = v$. This completes the proof of theorem. □

Theorem 2.3. *For $n \geq 3$, the dominator chromatic number of derived graph of triple star graph is $n+2$ i.e.,*

$$\chi_d(K_{1,n,n,n})^{\dagger} = n+2$$

Proof. Let $V(K_{1,n,n,n})^{\dagger} = \{v\} \cup \{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq n\}$ be the vertices of derived graph of triple star graph. The following procedure gives the dominator chromatic number of derived graph of triple star graph. In $V(K_{1,n,n,n})^{\dagger}$ induced subgraph $v_i : 1 \leq i \leq n$ forms a clique of order n, so we have to assign color c_i to $v_i : 1 \leq i \leq n$. For $1 \leq i \leq n$, assign the color c_{n+1} to $w_i, u_i : 1 \leq i \leq n$ and assign the color c_{n+2} to v, By the definition of dominator coloring w_i dominates the color class $c_i : 1 \leq i \leq n$ and $u_i : 1 \leq i \leq n$ dominates the color class c_{n+2} and v dominates itself. Hence $\chi_d(K_{1,n,n,n})^{\dagger} = n+2$, where $c_i = \{v_i : 1 \leq i \leq n\}$, $c_{n+1} = \{u_i w_i : 1 \leq i \leq n\}$, $c_{n+2} = v$. □

Theorem 2.4. *Let $m, n \geq 3$, the dominator chromatic number of derived graph of Bi-star graph is,*

$$\chi_d(B_{m,n})^{\dagger} = \begin{cases} m+2 \text{ when } m > n \\ n+2 \text{ when } n \geq m \end{cases}$$

Proof. Consider the Bistar $B_{m,n}$ let $\{u_i : 1 \leq i \leq m\}$ be the m pendant edges attached to the vertex u and $\{v_i : 1 \leq i \leq n\}$ be the another n pendant edges attached to the vertex v. By the definition of derived,

$$V(B_{m,n})^{\dagger} = \{u, u_i : 1 \leq i \leq m\} \cup \{v, v_i : 1 \leq i \leq n\}.$$

In $(B_{m,n})^{\dagger}$ the vertices $u_i : 1 \leq i \leq m$ along with v forms a clique of order $n+1$. Also we see that the vertices $v_i : 1 \leq i \leq n$ together with u forms another clique of order $n+1$. Consider the following cases.

Case 1: When $m > n$. Consider the color class $C = \{c_1, c_2, c_3, \ldots, c_m, c_{m+1}, c_{m+2}\}$.

- Assign the color c_i to $u_i : 1 \leq i \leq m$ and assign color c_{m+1} for u.
- For $1 \leq i \leq n$ assign color c_i to v_i and assign color c_{m+2} to v.

By the definition of dominator coloring every vertex of u_i dominates the color class c_{m+2} and v_i dominates the color class c_{m+1}. Vertices u, v dominates itself. Hence $C = \{c_1, c_2, c_3, \ldots, c_m, c_{m+1}, c_{m+2}\}$ is a dominator coloring of $(B_{m,n})^{\dagger}$, where

$$c_i = \{v_i : 1 \leq i \leq n, u_i : 1 \leq i \leq m, \}$$
$$c_{m+1} = \{u\}, c_{m+2} = \{v\}.$$

Case 2: When $n \geq m$. Consider the color class $C = \{c_1, c_2, c_3, \ldots, c_n c_{n+1}, c_{n+2}\}$. Since $u_i : 1 \leq i \leq m$ along with v forms a clique of order $n+1$. Also we see that the vertices $v_i : 1 \leq i \leq n$ together with u forms another clique of order $n+1$ for the reason that assign color c_i to $u_i : 1 \leq i \leq m$ and $v_i : 1 \leq i \leq n$. Next assign the color c_{n+1} to the vertex v and c_{n+2} to u. Thus $\chi_d(B_{m,n})^{\dagger} \leq n+2$. On the other hand if we assign c_{n+1} colors to all the vertices in $(B_{m,n})^{\dagger}$ then it contradicts the definition of dominator coloring . Hence an easy observation shows that $\chi_d(B_{m,n})^{\dagger} = n+2$. □

The n-centipede C_n is a tree with $2n$ vertices and $2n-1$ edges obtained by joining the bottoms of n copies of the path graph P_2 laid in a row with edges.

Theorem 2.5. *For $n \geq 3$, the dominator chromatic number of derived graph of Centipede graph is.*

$$\chi_d(C_n)^\dagger = \begin{cases} \lceil n/2 \rceil + 2 & \textit{when } n \equiv 3,0 \mod 4 \\ n/2 + 3 & \textit{when } n \equiv 2 \mod 4 \\ \lfloor n/2 \rfloor + 3 & \textit{when } n \equiv 1 \mod 4. \end{cases}$$

Proof. Let us define the vertex set V of C_n as $(C_n)^\dagger = \{v_i :\leq i \leq n\} \cup \{u_i : 1 \leq i \leq n\}$. A procedure to obtain dominator coloring of derived graph of centipede graph as follows. Define a coloring function f on $V(C_n)^\dagger$ such that for any vertex v_i and u_i.

Case 1: When $n \equiv 3,0 \mod 4$

$$f(v_i, u_i) = \begin{cases} i \text{ for } v_{2i} \text{ when } i = 2k-1, 1 \leq k \leq \lceil n/4 \rceil \\ i \text{ for } v_{2i-1} \text{ when } i = 2k, 1 \leq k \leq \lceil n/4 \rceil \\ \lceil n/2 \rceil + 1 \text{ for } u_i : 1 \leq i \leq n. \\ \lceil n/2 \rceil + 2 \text{ otherwise} \end{cases}$$

Case 2: When $n \equiv 2 \mod 4$

$$f(v_i, u_i) = \begin{cases} i \text{ for } v_{2i} \text{ when } i = 2k-1, 1 \leq k \leq \lfloor n/4 \rfloor \\ i \text{ for } v_{2i-1} \text{ when } i = 2k, 1 \leq k \leq \lfloor n/4 \rfloor \\ \lceil n/2 \rceil \text{ for } u_n \\ \lceil n/2 \rceil + 1 \text{ for } v_n \\ \lceil n/2 \rceil + 2 \text{ for } u_i : 1 \leq i \leq n \\ \lceil n/2 \rceil + 3 \text{ otherwise} \end{cases}$$

Case 3: When $n \equiv 1 \mod 4$

$$f(v_i, u_i) = \begin{cases} i \text{ for } v_{2i} \text{ when } i = 2k-1, 1 \leq k \leq \lfloor n/4 \rfloor \\ i \text{ for } v_{2i-1} \text{ when } i = 2k, 1 \leq k \leq \lfloor n/4 \rfloor \\ \lfloor n/2 \rfloor + 1 \text{ for } u_n : \\ \lfloor n/2 \rfloor + 2 \text{ for } u_i : 1 \leq i \leq n. \\ \lfloor n/2 \rfloor + 3 \text{ otherwise} \end{cases}$$

Hence this completes the proof of the theorem. □

Theorem 2.6. *For $n \geq 9$, the dominator chromatic number of derived graph of cycle is*

$$\chi_d(C_n^\dagger) = \begin{cases} \lceil n/3 \rceil + 2 & \textit{when } n \equiv 0,1 \mod 3 \\ 2\lceil n/6 \rceil + 2 & \textit{when } n \equiv 2 \mod 3 \end{cases}$$

Proof. By the definition of derived graph ,$\{v_1, v_2, v_3, \ldots, v_n\}$ be the vertices of derived graph of cycle graph. The following procedure gives the dominator chromatic number of derived graph of cycle graph. Consider the color classes $C = \{c_1, c_2, c_3, \ldots, \lceil n/3 \rceil + 2\}$.

Case 1: When $n \equiv (0, 1) \mod 3$

For v_i where $i = 3k - 2, 1 \leq k \leq \lceil n/3 \rceil$ are colored by color $c_{\lceil i/3 \rceil}$ and assign the color $c_{\lceil n/3 \rceil + 2}, c_{\lceil n/3 \rceil + 1}$ to the remaining vertices of v_i. By the definition of dominator coloring v_i where $i = 3k - 2, 1 \leq k \leq \lceil n/3 \rceil$ Dominate itself and the remaining vertices dominates at least any one color classes $c_i, i = 3k - 2, 1 \leq k \leq \lceil n/3 \rceil$.

Case 2: $n \equiv 2 \mod 3$

First Assign the color $c_{\lceil i/3 \rceil}$ to $v_i, i = 6k - 5$ where $1 \leq k \leq \lfloor n/3 \rfloor - 2$ and assign color$c_{\lceil i/3 \rceil} + 1$ to $v_i, i = 6k - 4, 1 \leq k \leq \lfloor n/3 \rfloor - 2$. Next the color $c_{\lceil n/3 \rceil + 2}$ and $c_{\lceil n/3 \rceil + 1}$ are assigned to the remaining vertices of v_i. Hence,

$$\chi_d(G^{\dagger}(C_n)) = \begin{cases} \lceil n/3 \rceil + 2 & \text{when } n \equiv (0, 1) \mod 3 \\ \lceil n/3 \rceil + 2 & \text{when } n \equiv 2 \mod 3 \end{cases}$$

□

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