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Nano $\mathcal{O}_{16}$-closed Sets

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Abstract: In this paper, we investigated a new class of sets called nano $\mathcal{O}_{16}$-closed sets and nano $\mathcal{O}_{16}$-open sets in nano topological spaces and its properties are studied.

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1. Introduction

Lellis Thivagar [5] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space. Bhuvaneswari [4] introduced and investigated nano g-closed sets in nano topological spaces. Recently, Parvathy and Bhuvaneswari the notions of nano gpr-closed sets which are implied both that of nano rg-closed sets. In this paper, we defined nano $\mathcal{O}_{16}$-closed sets and obtained some of its basic properties as results.

2. Preliminaries

Throughout this paper $(U, \tau_R(X))$ (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \tau_R(X))$, $Ncl(H)$ and $Nint(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1 ([7]). *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

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(2). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 ([5]). *If (U, R) is an approximation space and $X, Y \subseteq U$; then*

(1). $L_R(X) \subseteq X \subseteq U_R(X)$;

(2). $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$;

(3). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(4). $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(5). $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

(6). $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(7). $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$;

(8). $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$;

(9). $U_RU_R(X) = L_RU_R(X) = U_R(X)$;

(10). $L_RL_R(X) = U_RL_R(X) = L_R(X)$.

Definition 2.3 ([5]). *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by the Property 2.2, R(X) satisfies the following axioms:*

(1). U and $\phi \in \tau_R(X)$,

(2). The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3). The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and $[\tau_R(X)]^c$ is called as the dual nano topology of $[\tau_R(X)]$.

Remark 2.4 ([5]). *If $[\tau_R(X)]$ is the nano topology on U with respect to X, then the set $B = \{U, \phi, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.5 ([5]). *If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by $Nint(H)$. That is, $Nint(H)$ is the largest nano open subset of H. The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by $Ncl(H)$. That is, $Ncl(H)$ is the smallest nano closed set containing H.*

Definition 2.6. *A subset H of a nano topological space $(U, \tau_R(X))$ is called*

(1). nano semi-open [5] if $H \subseteq Ncl(Nint(H))$.

(2). nano α-open [6] if $H \subseteq N(int(Ncl(Nint(H))))$.

(3). nano β-open [10] if $H \subseteq Ncl(Nint(Ncl(H)))$.

(4). nano regular-open [5] if $H = Nint(Ncl(H))$.

(5). nano pre-open [5] if $H \subseteq Nint(Ncl(H))$.

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.7. *A subset H of a nano topological space $(U, \tau_R(X))$ is called;*

(1). nano g-closed [3] if $Ncl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(2). nano gp-closed set [4] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(3). nano gpr-closed set [6] if $Npcl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano regular open.

(4). nano sg-closed set [2] if $Nscl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano semi-open.

(5). nano αg-closed [12] if $N\alpha cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano open.

(6). nano $g\alpha$-closed [12] if $N\alpha cl(H) \subseteq G$, whenever $H \subseteq G$ and G is nano α-open.

3. On Nano $\mathcal{O}_{16}$-closed Sets

Definition 3.1. *A subset H of space $(U, \tau_R(X))$ is called nano $\mathcal{O}_{16}$-closed if $Ncl(Nint(Ncl(H))) \subseteq G$, whenever $H \subseteq G$ and G is nano open. The complement of nano $\mathcal{O}_{16}$-open if $H^c = U - H$ is nano $\mathcal{O}_{16}$-closed.*

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$. Then the nano topology $\tau_R(X) = \{\phi, \{d\}, \{b, c\}, \{b, c, d\}, U\}$.*

(1). then $\{a, b\}$ is nano $\mathcal{O}_{16}$-closed.

(2). then $\{c\}$ is nano $\mathcal{O}_{16}$-open.

Theorem 3.3. *In a space $(U, \tau_R(X))$, every nano $g\alpha$-closed set is nano $\mathcal{O}_{16}$-closed.*

Proof. Let H be a nano $g\alpha$-closed set in U and $H \subseteq G$ where G is nano α-open. Now nano α-open implies that G is nano open. Also $Ncl(Nint(Ncl(H))) \subseteq Ncl(H) \subseteq N\alpha cl(H) \subseteq G$. Thus H is nano $\mathcal{O}_{16}$-closed set in U. □

Remark 3.4. *The converse of the theorem 3.3 need not be true as seen from the following example.*

Example 3.5. *In Example 3.2, then $\{b, c, d\}$ is nano $\mathcal{O}_{16}$-closed but not nano $g\alpha$-closed.*

Theorem 3.6. *In a spaces $(U, \tau_R(X))$, the union of two nano $\mathcal{O}_{16}$-closed is nano $\mathcal{O}_{16}$-closed.*

Proof. Assume that H and K are nano $\mathcal{O}_{16}$-closed set in U. Let G be a nano open in U such that $H \cup K \subseteq G$. Then $H \subseteq G$ and $K \subseteq G$. Since H and K are nano $\mathcal{O}_{16}$-closed, $Ncl(Nint(Ncl(H))) \subseteq G$ and $Ncl(Nint(Ncl(K))) \subseteq G$. Thus $Ncl(Nint(Ncl(H \cup K))) = Ncl(Nint(Ncl(H))) \cup Ncl(Nint(Ncl(K))) \subseteq G$. That is $Ncl(Nint(Ncl(H \cup K))) \subseteq G$. Hence $H \cup K$ is nano $\mathcal{O}_{16}$-closed set in U. □

Example 3.7. *In Example 3.2, then $P = \{a, c\}$ and $Q = \{a, d\}$ is nano $\mathcal{O}_{16}$-closed clearly $P \cup Q = \{a, c, d\}$ nano $\mathcal{O}_{16}$-closed.*

Remark 3.8. *In a space $(U, \tau_R(X))$, the intersection of two nano $\mathcal{O}_{16}$-closed is but not nano $\mathcal{O}_{16}$-closed.*

Example 3.9. *In Example 3.2, then* $P = \{a, b, d\}$ *and* $Q = \{b, c, d\}$ *is nano* $\mathcal{O}_{16}$*-closed clearly* $P \cap Q = \{b, d\}$ *is but not nano* $\mathcal{O}_{16}$*-closed.*

Theorem 3.10. *In a space* $(U, \tau_R(X))$*, if a subset* H *of* U *then* $Ncl(Nint(Ncl(H))) - H$ *does not contain any non empty nano open set.*

Proof. Suppose that H is nano $\mathcal{O}_{16}$-closed set in U. Let G be a nano open set such that $(Ncl(Nint(Ncl(H))) - H \subseteq G$ and $Ncl(Nint(Ncl(H))) \supseteq G$. Now $G \subseteq Ncl(Nint(Ncl(H))) - H$. Hence $G \subseteq U - G$. Since G is nano open set, $U - G$ is also nano open in U. Since H is nano $\mathcal{O}_{16}$-closed sets in U, by definition we have $Ncl(Nint(Ncl(H))) \subseteq U - G$. Therefore $G \subseteq U - Ncl(Nint(Ncl(H)))$. Also $G \subseteq Ncl(Nint(Ncl(H)))$. Hence $G \subseteq Ncl(Nint(Ncl(H))) \cap (U - Ncl(Nint(Ncl(H))) = \phi$. This shows that $G = \phi$ which is contradiction. Thus $Ncl(Nint(Ncl(H) - H$ does not contains any non empty nano open set in U. □

Remark 3.11. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.12. *In Example 3.2, let* $H = \{b, c, d\}$ *be a subset of nano* $\mathcal{O}_{16}$*-closed. Then* $Ncl(Nint(Ncl(H))) - H = U - \{b, c, d\} = \{a\}$ *does not contain any non-empty nano open set but* H *is not nano* $\mathcal{O}_{16}$*-closed .*

Theorem 3.13. *In a space* $(U, \tau_R(X))$*, if* H *is nano regular closed then nano* $\mathcal{O}_{16}$*-closed.*

Proof. Assume that $H \subseteq G$ and G is nano open in U. Now $G \subseteq U$ is nano open $\iff$ G is the union of a nano semi open set and nano pre open set. Let H be a nano regular closed subset of U. Hence $H = Ncl(Nint(Ncl(H)$. Every nano regular closed set is nano semi open set and every nano semi open set is nano open set. Hence $Ncl(Nint(Ncl(H) \subseteq G$ where G is nano open in U. Thus H is nano $\mathcal{O}_{16}$-closed in U. □

Remark 3.14. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.15. *Let* $U = \{a, b, c\}$ *with* $U/R = \{\{a, b\}, \{c\}\}$ *and* $X = \{b, c\}$*. Then the nano topology* $\tau_R(X) = \{\phi, \{a, b\}, U\}$*. We have* $H = \{a\}$*. Clearly* H *nano* $\mathcal{O}_{16}$*-closed set but not nano regular closed. Since* $H \neq Nrcl(H)$*. This implies that* H *is not nano regular closed.*

Theorem 3.16. *In a space* $(U, \tau_R(X))$*, for an element* $x \in X$*, the set* $X - \{x\}$ *is nano* $\mathcal{O}_{16}$*-closed.*

Proof. Suppose $X - \{x\}$ is not nano open. Then X is the only nano open set containing $X - \{x\}$. This implies $Ncl(Nint(Ncl(X - \{x\}))) \subseteq X$. Hence $X - \{x\}$ is nano $\mathcal{O}_{16}$-closed. □

Theorem 3.17. *In a space* $(U, \tau_R(X))$*, if* H *is nano regular open and nano* $\mathcal{O}_{16}$*-closed then* H *is nano regular closed and nano clopen.*

Proof. Assume that H is nano regular open and nano $\mathcal{O}_{16}$-closed. As every nano regular open set is nano open and $H \subseteq H$, we have $Ncl(Nint(Ncl(H) \subseteq H$. Since $Ncl(H) \subseteq Ncl(Nint(Ncl(H)))$. We have $Ncl(H) \subseteq H$. So $H \subseteq cl(H)$. Therefore $cl(H) = H$ that means H is nano closed. Since H is nano regular open, H is nano open. Now $Ncl(Nint(H)) = Ncl(H) = H$. Hence H is nano regular closed and clopen. □

Theorem 3.18. *In a space* $(U, \tau_R(X))$*, if* H *is nano regular open and nano rg-closed then* H *is nano* $\mathcal{O}_{16}$*-closed.*

Proof. Let H be nano regular open and nano rg-closed in U. Let G be any nano open set in U such that $H \subseteq G$. Since H is nano regular open and nano rg-closed, we have $Ncl(H) \subseteq H$. Then $Ncl(H) \subseteq H \subseteq G$. Thus H is nano $\mathcal{O}_{16}$-closed. □

Theorem 3.19. *If* H *is nano* $\mathcal{O}_{16}$*-closed subset in* U *such that* $H \subseteq K \subseteq Ncl(H)$*, then* K *is nano* $\mathcal{O}_{16}$*-closed.*

Proof. Let H be nano $\mathcal{O}_{16}$-closed subset in U such that $H \subseteq K \subseteq Ncl(H)$. Let G be a nano open set of U such that $K \subseteq G$. Then $H \subseteq G$. Since H is nano $\mathcal{O}_{16}$-closed. We have $Ncl(H) \subseteq G$. Now $Ncl(K) \subseteq Ncl(Ncl(H)) = Ncl(H) \subseteq G$. Hence K is nano $\mathcal{O}_{16}$-closed. □

Remark 3.20. *The converse of the above theorem need not be true as seen from the following example.*

Example 3.21. *In Example 3.15, we have $S = \{a\}$ and $T = \{a, b\}$. Then S and T are nano $\mathcal{O}_{16}$-closed, but $S \subseteq T$ is not subset in $Ncl(S)$.*

Theorem 3.22. *In a space $(U, \tau_R(X))$, let H be nano $\mathcal{O}_{16}$-closed then H is closed $\iff$ $Ncl(H) - H$ is nano open.*

Proof. Assume that H is nano closed in U. Then $Ncl(H) = H$ and so $Ncl(H) - H = \phi$, which is nano open in U. Conversely, suppose $Ncl(H) - H$ is nano open in U. Since H is nano $\mathcal{O}_{16}$-closed, by Theorem 3.10, $Ncl(H) - H$ does not contain any non-empty nano open set in U. Then $Ncl(H) - H = \phi$, thus H is nano closed in U. □

Theorem 3.23. *If H is both nano open and nano g-closed in U then nano $\mathcal{O}_{16}$-closed*

Proof. Let H be nano open and nano g-closed in U. Let $H \subseteq G$ and let G be open in U. Now $H \subseteq H$, By hypothesis $Ncl(H) \subseteq H$. That is $Ncl(H) \subseteq G$. Hence H is nano $\mathcal{O}_{16}$-closed. □

Remark 3.24. *We obtain Definitions, Theorems, Remarks and Examples follows from the implications.*

$$\textbf{nano regular-closed} \rightarrow \textbf{nano } \mathcal{O}_{16}\textbf{-closed} \leftarrow \textbf{nano } g\alpha\textbf{-closed}$$

None of the above implications are reversible.

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