



Computation of Leap Zagreb Indices of Some Windmill Graphs

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Abstract: Recently, A. M. Naji [13], introduced leap Zagreb indices of a graph based on the second degrees of vertices (number of their second neighbours). The first leap Zagreb index $LM_1(G)$ is equal to the sum of squares of the second degrees of the vertices, the second leap Zagreb index $LM_2(G)$ is equal to the sum of the products of the second degrees of pairs of adjacent vertices of G and the third leap Zagreb index $LM_3(G)$ is equal to the sum of the products of the first degrees with the second degrees of the vertices. In this paper, we computing Leap Zagreb indices of windmill graphs such as French windmill graph F_n^m , Dutch windmill graph D_n^m , Kulli cycle windmill graph C_{n+1}^m , and Kulli path windmill graph P_{n+1}^m .

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1. Introduction

In this paper, we are concerned only with simple graphs, i.e., finite graphs having no loops, multiple and directed edges. Let $G = (V, E)$ be such a graph with vertex set $V(G)$ and edges set $E(G)$. As usual, we denote by $n = |V|$ and $m = |E|$ to the number of vertices and edges in a graph G , respectively. The distance $d_G(u, v)$ between any two vertices u and v of a graph G is equal to the length of (number of edges in) a shortest path connecting them. For a vertex $v \in V(G)$ and a positive integer k , the open k -neighborhood of v in a graph G is denoted by $N_k(v)$ and is defined as $N_k(v) = \{u \in V(G) : d_G(u, v) = k\}$. The k -distance degree of a vertex v in G is denoted by $d_k(v)$ (or simply $d_k(v)$ if no misunderstanding) and is defined as the number of k -neighbours of the vertex v in G , i.e., $d_k(v) = |N_k(v)|$. It is clear that $d_1(v) = d(v)$ for every $v \in V(G)$. For a vertex v of G , the eccentricity $e(v) = \max\{d_G(v, u) : u \in V(G)\}$. The diameter of G is $diam(G) = \max\{e(v) : v \in V(G)\}$ and the $rad(G) = \min\{e(v) : v \in V(G)\}$. The join $G+H$ of two graphs G and H is a graph with vertex set $V(G+H) = V(G) \cup V(H)$ and edge set $E(G+H) = E(G) \cup E(H) \cup \{uv : u \in V(G), v \in V(H)\}$. For any terminology or notation not mention here, we refer to [10]. A topological index of a graph is a graph invariant number calculated from a graph representing a molecule and applicable in chemistry. The Zagreb indices have been introduced, more than forty year ago, by I. Gutman and Trinajestic [6], in 1972, and elaborated in [7]. They are defined as:

$$M_1(G) = \sum_{v \in V(G)} d_1^2(v) \text{ and } M_2 = \sum_{uv \in E} d_1(u)d_1(v)$$

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For properties of the two Zagreb indices see [2, 3, 7, 8, 15, 18] and the papers there in. In recent years, some novel variants of ordinary Zagreb introduced and studied, such as Zagreb coincides [1, 9], multiplicative Zagreb indices [6, 17, 18], multiplicative sum Zagreb index [4] and multiplicative Zagreb coincides [19] and ect. Recently, A. M. Naji [13], have been introduced a new distance-degree-based topological indices conceived depending on the second degrees of vertices (number of their second neighbours),and are so-called leap Zagreb indices of graph G and are defined as, respectively.

$$LM_1(G) = \sum_{v \in V} d_2^2(v)$$

$$LM_2(G) = \sum_{uv \in E(G)} d_2(u)d_2(v)$$

$$LM_3(G) = \sum_{v \in V} d(u)d_2(v).$$

For properties of the leap Zagreb indices see [13, 14, 16]. In this paper, a molecular graph is a graph in which, it corresponds the vertices and edges as atoms and bonds, respectively. In chemical sciences, chemical graph theory made very effective development, and also a topological index for a graph is used to determine some property of a graph of molecular by a singular number.

2. Main Result

In this section, we present the exact values of leap Zagreb indices of some windmill graphs.

Definition 2.1. *The French windmill graph F_n^m is the graph obtained by taking $m \geq 2$ copies of the complete graph K_n , $n \geq 2$ with a vertex in common. That is $F_n^m = K_1 + \cup_{j=1}^m K_{n-1}$.*

The French windmill graph F_2^m is called a star graph, the French windmill graph F_3^m is called a friendship graph and the French windmill graph F_3^2 is called a butterfly graph. Further, note that F_3^m is same as D_3^m . For more details on windmill graph, [5].

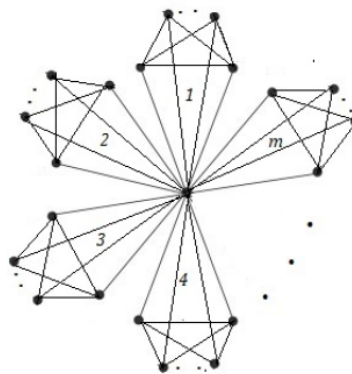


Figure 1. The French windmill graph F_n^m .

Lemma 2.2. *For the French windmill graph F_n^m , for $n \geq 2, m \geq 2$. Let v_0 be the central vertex (common) and K_{n-1}^j , $j = 1, 2, \dots, m$ be the j -th copy of K_{n-1} in F_n^m with vertex set $V_j = \{v_{1j}, v_{2j}, \dots, v_{(n-1)j}\}$. Then the following results are holding.*

- (1). $V(F_n^m) = \{v_0\} \cup (\cup_{i=1}^m V_j(K_{n-1}^j))$, where $V_j(K_{n-1}^j) = \{v_{ij} : 1 \leq i \leq n - 1\}$.

(2). $E(F_n^m) = E_0 \cup (\cup_{j=1}^m E_j)$, where $E_0 = \{v_0 v_{ij} : 1 \leq i \leq n-1, 1 \leq j \leq m\}$ and $E_j = E(K_{n-1}^j)$, for $1 \leq j \leq m$.

(3). $|V(F_n^m)| = 1 + m(n-1)$, $|E(F_n^m)| = \frac{mn(n-1)}{2}$, $|E_0| = m(n-1)$ and $|E_j| = \frac{(n-1)(n-2)}{2}$.

(4). $d(v_0) = m(m-1)$ and $d_2(v_0) = 0$
 $d(v_{ij}) = (n-1)$ and $d_2(v_{ij}) = (n-1)(m-1)$.

Theorem 2.3. The leap Zagreb indices of the French windmill graph $G = F_n^m$, for $n \geq 2, m \geq 2$ are

(1). $LM_1(F_n^m) = m(n-1)^3(m-1)^2$.

(2). $LM_2(F_n^m) = \frac{1}{2}m(m-1)^2(n-1)^3(n-2)$.

(3). $LM_3(F_n^m) = m(n-1)^3(m-1)$.

Proof. Let $G = F_n^m$, for $n \geq 2, m \geq 2$ be a French windmill graph as show in Figure 1. Then By Lemma 2.2, we obtain

$$\begin{aligned} (1). \quad LM_1(F_n^m) &= \sum_{v \in V} d_2^2(v) \\ &= d_2^2(v_0) + \sum_{j=1}^m \sum_{i=1}^{n-1} d_2^2(v_{ij}) \\ &= 0 + \sum_{j=1}^m \sum_{i=1}^{n-1} (n-1)^2(m-1)^2 \\ &= m(m-1)^2(n-2)^3. \end{aligned}$$

Therefore, $LM_1(F_n^m) = m(n-1)^3(m-1)^2$.

$$\begin{aligned} (2). \quad LM_2(F_n^m) &= \sum_{uv \in E(F_n^m)} d_2(u)d_2(v) \\ &= \sum_{v_{ij} \in E_0} d_2(v_0)d_2(v_{ij}) + \sum_{j=1}^m \sum_{uv \in E_j} d_2(u)d_2(v) \\ &= \sum_{j=1}^m \sum_{i=1}^{n-1} 0 \cdot d_2(v_{ij}) + \sum_{j=1}^m \sum_{i=1}^{n-1} (n-1)^2(m-1)^2 \\ &= 0 + \sum_{j=1}^m \frac{(n-1)(n-2)}{2} (n-1)^2(m-1)^2 \\ &= \frac{1}{2}m(m-1)^2(n-1)^3(n-2). \end{aligned}$$

Therefore, $LM_2(F_n^m) = \frac{1}{2}m(m-1)^2(n-1)^3(n-2)$.

$$\begin{aligned} (3). \quad LM_3(F_n^m) &= \sum_{v \in V(F_n^m)} d(v)d_2(v) \\ &= d(v_0)d_2(v_0) + \sum_{j=1}^m \sum_{i=1}^{n-1} d(v_{ij})d_2(v_{ij}) \\ &= 0 + \sum_{j=1}^m \sum_{i=1}^{n-1} (n-1)^2(m-1) \\ &= \sum_{j=1}^m (n-1)^3(m-1) \\ &= m(n-1)^3(m-1). \end{aligned}$$

Therefore, $LM_3(F_n^m) = m(n-1)^3(m-1)$. □

Definition 2.4. The Dutch windmill graph D_n^m , for $n \geq 5, m \geq 2$, is the graph obtained by taking m copies of the cycle C_n with a vertex in common.

This graph is shown in Figure 2. The Dutch windmill graph $D_3^m = F_3^m$ is called a friendship graph. For more details on windmill graph, [5].

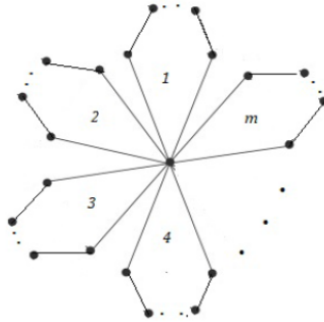


Figure 2. The Dutch windmill graph D_n^m

Lemma 2.5. For the Dutch windmill graph D_n^m , for $n \geq 5, m \geq 2$. Let v_0 be the central vertex (common) and C_n^j , $j = 1, 2, \dots, m$ be the j -th copy of C_n in F_n^m with vertex set $V_j = \{v_0, v_{1j}, v_{2j}, \dots, v_{(n-1)j}\}$. Then the following results are holding.

- (1). $V(D_n^m) = \{v_0\} \cup (\cup_{i=1}^m V_j(K_{n-1}))$, where $V_j = \{v_{ij} : 1 \leq i \leq n - 1\}$.
- (2). $E(D_n^m) = E_0 \cup (\cup_{j=1}^m E_j)$, where $E_0 = \{v_0 v_{1j}, v_0 v_{(n-1)j} : 1 \leq j \leq m\}$ and $E_j = \{v_{ij} v_{(i+1)j} : 1 \leq i \leq n - 1 \text{ and } 1 \leq j \leq m\}$.
- (3). $|V(D_n^m)| = 1 + m(n - 1)$, $|E(D_n^m)| = 2m + m(n - 1)$, $|E_0| = 2m$ and $|E_j| = m(n - 2)$.
- (4). $d(v_0) = d_2(v_0) = 2m$, $d(v_{ij}) = 2$ for $i = 1, 2, \dots, n - 1$ and $d_2(v_{ij}) = \begin{cases} 2m, & i = 1, n - 1; \\ 2, & i = 2, 3, \dots, n - 2. \end{cases}$

Theorem 2.6. The leap Zagreb indices of the Dutch windmill graph $G = D_n^m$, for $n \geq 5, m \geq 2$ are

- (1). $LM_1(D_n^m) = 4m[2m^2 + m + n - 2]$,
- (2). $LM_2(D_n^m) = 4m[2m^2 + 2m + n - 3]$,
- (3). $LM_3(D_n^m) = 4m[m + 3n - 3]$.

Proof. Let D_n^m be a Dutch windmill graph as shown in Figure 2. Then by Lemma 2.5, we obtain

$$\begin{aligned}
 (1). \quad LM_1(D_n^m) &= \sum_{v \in V(D_n^m)} d_2^2(v) \\
 &= d_2^2(v_0) + \sum_{j=1}^m \sum_{i=1}^{n-1} d_2^2(v_{ij}) \\
 &= d_2^2(v_0) + \sum_{j=1}^m d_2^2(v_{1j}) + \sum_{j=1}^m \sum_{i=2}^{n-2} d_2^2(v_{ij}) + \sum_{j=1}^m d_2^2(v_{(n-1)j}) \\
 &= (2m)^2 + \sum_{j=1}^m (2m)^2 + \sum_{j=1}^m \sum_{i=2}^{n-2} 2^2 + \sum_{j=1}^m (2m)^2 \\
 &= 4m^2 + 4m^3 + 4m(n - 3) + 4m^3 \\
 &= 8m^3 + 4m^2 + 4m(n - 3) \\
 &= 4m[2m^2 + m + n - 3].
 \end{aligned}$$

Therefore, $LM_1(D_n^m) = 4m[2m^2 + m + n - 2]$.

$$\begin{aligned}
 (2). \quad LM_2(D_n^m) &= \sum_{uv \in E_0} d_2(u)d_2(v) \\
 &= \sum_{vu \in E(D_n^m)} d_2(v)d_2(u) + \sum_{j=1}^m \sum_{uv \in E_j} d_2(u)d_2(v) \\
 &= \sum_{j=1}^m [d_2(v_0)d_2(v_{1j}) + d_2(v_0)d_2(v_{(n-1)j})] + \sum_{j=1}^m d_2(v_{1j})d_2(v_{2j}) \\
 &\quad + \sum_{j=1}^m d_2(v_{(n-1)j})d_2(v_{(n-2)j}) + \sum_{j=1}^m \sum_{i=2}^{n-2} d_2(v_{ij})d_2(v_{(i+1)(j+1)}) \\
 &= \sum_{j=1}^m (2m)(2m) + (2m)(2m) + \sum_{j=1}^m (2m)(2) + \sum_{j=1}^m (2m)(2) + \sum_{i=2}^{n-2} \sum_{j=1}^m (2)(2) \\
 &= 8m^3 + 4m^2 + 4m^2 + 4m(n-3) \\
 &= 4m[2m^2 + 2m + n - 3]
 \end{aligned}$$

Therefore, $LM_2(D_n^m) = 4m[2m^2 + 2m + n - 3]$.

$$\begin{aligned}
 (3). \quad LM_3(D_n^m) &= \sum_{v \in V} d(v)d_2(v) \\
 &= d(v_0)d_2(v_0) + \sum_{j=1}^m [d(v_{1j})d_2(v_{1j}) \\
 &\quad + d(v_{(n-1)j})d_2(v_{(n-1)j})] + \sum_{j=1}^m \sum_{i=2}^{n-2} d(v_{ij})d_2(v_{ij}) \\
 &= (2m)(2m) + \sum_{j=1}^m (2(2m) + 2(2m)) + \sum_{j=1}^m \sum_{i=2}^{n-2} (2)(2) \\
 &= 4m^2 + 8mn + 4m(n-3) \\
 &= 4m[m + 2n + n - 3] \\
 &= 4m[m + 3n - 3].
 \end{aligned}$$

Therefore, $LM_3(D_n^m) = 4m[m + 3n - 3]$. □

Definition 2.7. The Kulli cycle windmill graph C_{n+1}^m is the graph obtained by taking m copies of the graph $K_1 + C_n$ for $n \geq 3$ with a vertex K_1 in common.

This graph is shown in Figure 3. The Kulli cycle windmill graph C_4^m is a french windmill graph and it is denoted by F_3^m . This type of windmill graph is initiated by Kulli in [11].

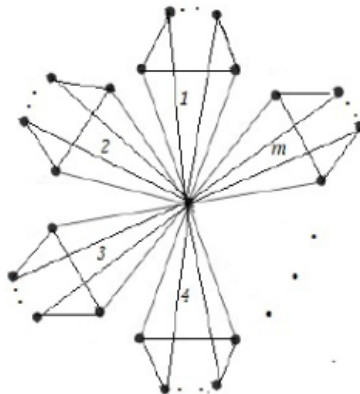


Figure 3. The Kulli cycle windmill graph C_{n+1}^m

Lemma 2.8. For the Kulli cycle graph C_{n+1}^m , for $n \geq 5, m \geq 2$. Let v_0 be the central vertex (common) and $C_n^j, j = 1, 2, \dots, m$ be the j -th copy of C_n in C_{n+1}^m with vertex set $V_j = \{V_{1j}, v_{2j}, \dots, v_{nj}\}$. Then the following results are holding.

- (1). $V(C_{n+1}^m) = \{v_0\} \cup (\cup_{i=1}^m V_j(C_n^i))$, where $V_j(C_n^j) = \{v_{ij} : 1 \leq i \leq n\}$.
- (2). $E(C_{n+1}^m) = E_o \cup (\cup_{j=1}^m E_j)$, where $E_o = \{v_0 v_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E_j = E(C_n^j)$, for $1 \leq j \leq m$.
- (3). $|V(C_{n+1}^m)| = 1 + mn, |E(C_{n+1}^m)| = mn^2, |E_o| = mn$ and $|E_j| = n$.
- (4). $d(v_0) = nm, d_2(v_0) = 0,$
 $d(v_{ij}) = 3, d_2(v_{ij}) = (nm - 2)$, where $1 \leq j \leq n$ and $1 \leq j \leq m$.

Theorem 2.9. The leap Zagreb indices of a Kulli cycle windmill graph is $C_{n+1}^m, n \geq 5, m \geq 2$, are

- (1). $LM_1(C_{n+1}^m) = nm(nm - 2)^2,$
- (2). $LM_2(C_{n+1}^m) = nm(nm - 2)^2,$
- (3). $LM_3(C_{n+1}^m) = 6m(nm - 2) + 3(n - 1)(nm - 3).$

Proof. Let $G = C_{n+1}^m$ be a Kulli cycle windmill graph as shown in Figure 3. Then by the Lemma 2.8, we obtain

$$\begin{aligned} (1). \quad LM_1(C_{n+1}^m) &= \sum_{v \in V(C_{n+1}^m)} d_2^2(v) \\ &= d_2^2(v_0) + \sum_{j=1}^m \sum_{i=1}^n d_2^2(v_{ij}) \\ &= 0 + \sum_{j=1}^m \sum_{i=1}^n (nm - 2)^2 \\ &= nm(nm - 2)^2. \end{aligned}$$

Therefore, $LM_1(C_{n+1}^m) = nm(nm - 2)^2$.

$$\begin{aligned} (2). \quad LM_2(C_{n+1}^m) &= \sum_{uv \in E(C_{n+1}^m)} d_2(u)d_2(v) \\ &= \sum_{v_0 v \in E_o} d_2(v_0)d_2(v) + \sum_{j=1}^m \sum_{uv \in E_j} d_2(u)d_2(v) \\ &= 0 + \sum_{j=1}^m \sum_{i=1}^n (nm - 2)(nm - 2) \\ &= nm(nm - 2)^2. \end{aligned}$$

Therefore, $LM_2(C_{n+1}^m) = nm(nm - 2)^2$.

$$\begin{aligned} (3). \quad LM_3(C_{n+1}^m) &= \sum_{v \in V(C_{n+1}^m)} d(v)d_2(v) \\ &= d(v_0)d_2(v_0) + \sum_{j=1}^m d(v_{ij})d_2(v_{ij}) + \sum_{i=2}^{n-1} d(v_{ij})d_2(v_{ij}) + d(v_{nj})d_2(v_{nj}) \\ &= 0 + \sum_{j=1}^m 3(nm - 2) + \sum_{i=2}^{n-1} 3(nm - 3) + 3(nm - 2) \\ &= 3m(nm - 2) + 3(n - 1)(nm - 3) + 3(nm - 2) \\ &= 6m(nm - 2) + 3(n - 1)(nm - 3). \end{aligned}$$

Therefore, $LM_3(C_{n+1}^m) = 6m(nm - 2) + 3(n - 1)(nm - 3)$. □

Definition 2.10. The Kulli path windmill graph P_{n+1}^m , for $n \geq 5, m \geq 2$ is the graph obtained by taking m copies of the graph $K_1 + P_n$ with a vertex v_0 in common.

This graph is shown in Figure 4. The Kulli path windmill graph P_3^m is a friendship graph and it is denoted by P_3^m . This type of windmill graph is initiated by Kulli in [12].

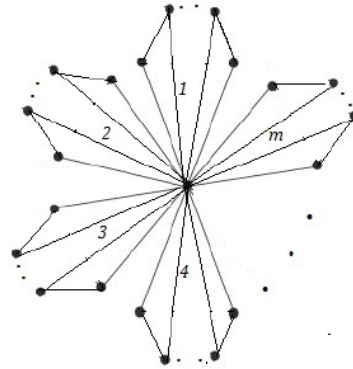


Figure 4. The Kulli path windmill graph P_{n+1}^m

Lemma 2.11. For the Kulli path graph P_{n+1}^m , for $n \geq 5, m \geq 2$. Let v_0 be the central vertex (common) and $P_n^j, j = 1, 2, \dots, m$ be the j -th copy of P_n in P_{n+1}^m with vertex set $V_j = \{v_{1j}, v_{2j}, \dots, v_{nj}\}$. Then the following results are holding.

- (1). $V(P_{n+1}^m) = \{v_0\} \cup (\cup_{i=1}^m V_j(P_n^j))$, where $V_j(P_n^j) = \{v_{ij} : 1 \leq i \leq n\}$.
- (2). $E(P_{n+1}^m) = E_0 \cup (\cup_{j=1}^m E_j)$, where $E_0 = \{v_0 v_{ij} : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E_j = E(P_n^j)$, for $1 \leq j \leq m$.
- (3). $|V(P_{n+1}^m)| = 1 + mn, |E(P_{n+1}^m)| = mn^2 - 1, |E_0| = mn$ and $|E_j| = n - 1$.
- (4). $d(v_0) = nm, d_2(v_0) = 0, d(v_{i,j}) = d(v_{n,j}) = 2,$
 $d(v_{i,j}) = 3$, where, $2 \leq i \leq n - 1$, and $1 \leq j \leq m,$
 $d_2(v_{i,j}) = d_2(v_{n,j}) = nm - 2$, where, $1 \leq j \leq m,$
 $d_2(v_{i,j}) = nm - 3$, where, $2 \leq i \leq n - 1$, and $1 \leq j \leq m.$

Theorem 2.12. The leap Zagreb indices of a Kulli path windmill graph P_{n+1}^m , for $n \geq 5, m \geq 2$ are

- (1). $LM_1(P_{n+1}^m) = 2m(nm - 2)^2 + m(nm - 3)^2(n - 2),$
- (2). $LM_2(P_{n+1}^m) = m(nm - 3)[2(nm - 2) + (n - 2)(nm - 3)],$
- (3). $LM_3(P_{n+1}^m) = 4m(nm - 2) + 3m(nm - 3)(n - 2).$

Proof. Let P_{n+1}^m be a Kulli path windmill graph as shown in Figure 4. Then by Lemma 2.11, we obtain

$$\begin{aligned}
 (1). \quad LM_1(P_{n+1}^m) &= \sum_{v \in V(P_{n+1}^m)} d_2^2(v) \\
 &= d_2^2(v_0) + \sum_{j=1}^m [d_2^2(v_{1j})] + \sum_{i=2}^{n-1} [d_2^2(v_{ij}) + d_2^2(v_{nj})] \\
 &= 0 + \sum_{j=1}^m (nm - 2)^2 + \sum_{i=2}^{n-1} [(nm - 3)^2 + (nm - 2)^2] \\
 &= m(nm - 2)^2 + m(nm - 3)^2(n - 2) + m(nm - 2)^2 \\
 &= 2m(nm - 2)^2 + m(nm - 3)^2(n - 2).
 \end{aligned}$$

Therefore, $LM_1(P_{n+1}^m) = 2m(nm - 2)^2 + m(nm - 3)^2(n - 2).$

$$\begin{aligned}
(2). \quad LM_2(P_{n+1}^m) &= \sum_{uv \in E(P_{n+1}^m)} d_2(u)d_2(v) \\
&= \sum_{v_0 v_{1j}} d_2(v_0)d_2(v_{1j}) + \sum_{v_{ij} v_{(i+1)j}} d_2(v_{ij})d_2(v_{i+1j}) \\
&= 0 + \sum_{j=1}^m [d_2(v_{1j})d_2(v_{2j})] + \sum_{i=2}^{n-1} [d_2(v_{ij})d_2(v_{i+1j}) + d_2(v_{(n-1)j})d_2(v_{nj})] \\
&= \sum_{j=1}^m (nm-2)(nm-3) + \sum_{i=2}^{n-1} [(nm-3)^2 + (nm-2)(nm-3)] \\
&= \sum_{j=1}^m [2(nm-2)(nm-3) + (n-2)(nm-3)^2] \\
&= 2m(nm-2)(nm-3) + m(n-2)(nm-3)^2 \\
&= m(nm-3)[2(nm-2) + (n-2)(nm-3)].
\end{aligned}$$

Therefore, $LM_2(P_{n+1}^m) = m(nm-3)[2(nm-2) + (n-2)(nm-3)]$.

$$\begin{aligned}
(3). \quad LM_3(P_{n+1}^m) &= \sum_{v \in V(P_{n+1}^m)} d(v)d_2(v) \\
&= d(v_0)d_2(v_0) + \sum_{j=1}^m [d(v_{1j})d_2(v_{1j})] + \sum_{j=2}^{n-1} [d(v_{ij})d_2(v_{ij}) + d(v_{nj})d_2(v_{nj})] \\
&= 0 + \sum_{j=1}^m [2(nm-2) + \sum_{j=2}^{n-1} 3(nm-3) + 2(nm-2)] \\
&= 4m(nm-2) + 3m(nm-3)(n-2).
\end{aligned}$$

Therefore, $LM_3(P_{n+1}^m) = 4m(nm-2) + 3m(nm-3)(n-2)$. □

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