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A Pentagonal Fuzzy Number in Solving Fuzzy Sequencing Problem

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Abstract: In this paper, we proposed a solution procedure to solve fuzzy job sequencing problem, where processing time being taken as pentagonal fuzzy numbers. The optimal solution of the job and idle time for each machine is obtained by solving corresponding crisp sequencing problems. The procedure adopted was the fuzzy sequencing problems that are defuzzified using Pascal's triangular graded mean for pentagonal fuzzy ranking index method. The fuzzy sequencing problem has been transformed in to crisp one, using linguistic variable and solved by Johnson's Algorithm. The linguistic variable helps to convert qualitative data in to quantitative data which will be effective in dealing with fuzzy sequencing problems at qualitative nature.

Keywords: Fuzzy sequencing problem, pentagonal fuzzy number, Pascal's Triangular graded mean technique.

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1. Introduction

In our real life decision making problems viz, job sequencing problem, game theory replacement of items of c. It is sometimes required to take decision where the values of parameter are ambiguous (i.e) parameters involved. A sequencing problem in the problem are imprecise is to determine the optimal sequence in which 'n' jobs to be performed by 'm' machine and various optimality criteria like minimum elapsed time, minimum idle time. Generally in job sequencing problems, the processing times are precise valued. But in reality, it is observed that the processing times during performance of the job are imprecise. To handle impreciseness fuzzy set theory plays an important role as fuzzy set is a best mathematical way for representing impreciseness (or) vagueness. In this paper the processing times are considered to be pentagonal fuzzy number, using Pascal's triangular graded mean method, the fuzzy sequencing problem, the processing time can be converted to a crisp valued, which can be solved using Johnson's algorithm. The optimal order and idle time for each machine is obtained by solving corresponding crisp sequencing problem.

2. Preliminaries

Definition 2.1 (Fuzzy Set [10]). *Let X be a set. A fuzzy set A on X is defined to be a function $A : X \to [0,1]$ or $\mu_A : X \to [0,1]$. Equivalently, a fuzzy set A is defined to be the class of objects having the following representation $A = \{(x, \mu_A x) : x \in X)\}$ where $\mu_A : X \to [0,1]$, is a function called the membership function of A.*

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Definition 2.2 (Fuzzy Number [4]). *The fuzzy number A is a fuzzy set whose membership function $\mu_A(x)$ satisfies the following conditions:*

(1). $\mu_A(x)$ is piecewise continuous;

(2). A fuzzy set A of the universe of discourse X is convex;

(3). A fuzzy set of the universe of discourse X is called a normal fuzzy set if $\exists\ x_i \in X,\ \mu_A(x_i) = 1$.

3. Pentagonal Fuzzy Number [6]

A Pentagonal Fuzzy Number of a fuzzy set A is defined as $\tilde{A}_p = \{a_1, a_2, a_3, a_4, a_5\}$, where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership function is given by

$$\mu_{\tilde{A}_p}(x) = \begin{cases} 0, & \text{for } x < a_1; \\ \frac{(x-a_1)}{(a_2-a_1)}, & \text{for } a_1 \le x \le a_2; \\ \frac{(x-a_2)}{(a_3-a_2)}, & \text{for } a_2 \le x \le a_3; \\ 1, & \text{for } x = a_3; \\ \frac{(a_4-x)}{(a_4-a_3)}, & \text{for } a_3 \le x \le a_4; \\ \frac{(a_5-x)}{(a_5-a_4)}, & \text{for } a_4 \le x \le a_5; \\ 0, & \text{for } x > a_5. \end{cases}$$

3.1. Condition on Pentagonal Fuzzy Number

A Pentagonal Fuzzy Number $\tilde{A}_p$ should satisfy the following conditions:

(1). $\mu_{\tilde{A}_p}(x)$ is a continuous function in the interval [0,1].

(2). $\mu_{\tilde{A}_p}(x)$ is strictly increasing and continuous function on $[a_1,\ a_2]$ and $[a_2,\ a_3]$.

(3). $\mu_{\tilde{A}_p}(x)$ is strictly decreasing and continuous function on $(a_3,\ a_4)$ and $(a_4,\ a_5)$.

3.2. Arithmetic Operations on Pentagonal Fuzzy Number

If $\tilde{A}_p = \{a_1, a_2, a_3, a_4, a_5\}$ and $\tilde{B}_p = \{b_1, b_2, b_3, b_4, b_5\}$ Then

Addition : $\tilde{A}_p + \tilde{B}_p = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3,\ a_4 + b_4,\ a_5 + b_5)$

Subtraction : $\tilde{A}_p - \tilde{B}_p = (a_1 - b_1,\ a_2 - b_2,\ a_3 - b_3,\ a_4 - b_4,\ a_5 - b_5)$

Multiplication : $\tilde{A}_p \times \tilde{B}_p = (a_1 \times b_1,\ a_2 \times b_2,\ a_3 \times b_3,\ a_4 \times b_4,\ a_5 \times b_5)$

4. Linguistic Variable [9]

A Linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variable is applied in dealing with situations which are too complex (or) too ill-defined to be reasonably described in conventional quantitative expressions. For example; Weight – is a linguistic variable, its values can be very high, high, medium, low, very low etc. These values can be represented by fuzzy numbers.

5. Processing of ‘n’ Jobs through 2 Machines [5]

Let there are ‘n’ jobs say $A_1, A_2, \ldots, A_n$ be processed through ‘2’ machines say M_1, M_2 in the Order M_1M_2. Let t_{ij} be the fuzzy processing time taken by i^{th} job to be completed by j^{th} Machine. The well-known Johnson method can be extended to this problem, then we find Optimal sequence, total elapsed time and Idle time on machines. Job machine fuzzy time for ‘n’ jobs and 2 machines are given below. Here fuzzy times are considered as pentagonal fuzzy number.

Jobs	Machine M_1	Machine M_2
A_1	$t_{11} = (a_{11}, b_{11}, c_{11}, d_{11}, e_{11}, f_{11})$	$t_{12} = (a_{12}, b_{12}, c_{12}, d_{12}, e_{12}, f_{12})$
A_2	$t_{21} = (a_{21}, b_{21}, c_{21}, d_{21}, e_{21}, f_{21})$	$t_{22} = (a_{22}, b_{22}, c_{22}, d_{22}, e_{22}, f_{22})$
A_3	$t_{31} = (a_{31}, b_{31}, c_{31}, d_{31}, e_{31}, f_{31})$	$t_{32} = (a_{32}, b_{32}, c_{32}, d_{32}, e_{32}, f_{32})$
A_4	$t_{41} = (a_{41}, b_{41}, c_{41}, d_{41}, e_{41}, f_{41})$	$t_{42} = (a_{42}, b_{42}, c_{42}, d_{42}, e_{21}, f_{21})$
A_5	$t_{51} = (a_{51}, b_{51}, c_{51}, d_{51}, e_{51}, f_{51})$	$t_{21} = (a_{21}, b_{21}, c_{21}, d_{21}, e_{21}, f_{21})$

6. Pascal’s Triangular Graded Mean for Pentagonal Fuzzy Number [7]

Let $\tilde{A}_p = \{a_1\ , a_2\ ,\ a_3,\ a_4,\ a_5\}$ be a pentagonal fuzzy numbers, We can take the coefficient of fuzzy numbers from Pascal’s triangles and apply the simple probability approach we get the following formula

$$P(A) = \frac{a_1 + 4a_2 + 6\ a_3 + 4\ a_4 +\ a_5}{16}$$

The coefficients of $a_1\ , a_2\ ,\ a_3,\ a_4,\ a_5$ are 1, 4, 6, 4, 1. This Procedure is simply taken from the pascal’s triangles. These are useful to take the coefficients of fuzzy variables are Pascal triangular numbers and we just add and divided by the total of pascal numbers.

7. Procedure for Solving Fuzzy Sequencing Problems

Step 1: Using Pascal’s triangular graded mean approach, the fuzzy sequencing problem can be converted into crisp sequencing problem.

Step 2: The Optimal sequence for the crisp sequence problem is determined using crisp sequencing problem.

Step 3: After finding the optimal sequence, Determine the total elapsed fuzzy time and also the fuzzy idle time on machines.

8. Numerical Examples

Consider the fuzzy sequencing problem .Here the processing time of 5 jobs is given whose elements are fuzzy quantifiers which characterize the linguistic variables that are replaced by pentagonal fuzzy numbers. The problem is then solved by processing n jobs through two machines.

Jobs	**Machine M_1**	**Machine M_2**
J_1	Low	Medium
J_2	Medium	Very Low
J_3	Very Low	Good
J_4	Good	Very Good
J_5	Very Good	Low

Table 1. **Quantitative data**

The linguistic variables showing the qualitative data is converted into quantitative data using the full-table. As the processing time varies between 0 to 40 the minimum possible values is taken as 0 and the maximum possible value is taken as 40.

Very Low	(0,1,2,3,4)
Low	(6,8,10,12,14)
Medium	(15,16,17,18,19)
Good	(22,24,26,28,30)
Very Good	(31,33,35,37,39)

Table 2. **Problem Table**

Jobs	**Machine M_1**	**Machine M_2**
J_1	(6,8,10,12,14)	(15,16,17,18,19)
J_2	(15,16,17,18,19)	(0,1,2,3,4)
J_3	(0,1,2,3,4)	(22,24,26,28,30)
J_4	(22,24,26,28,30)	(31,33,35,37,39)
J_5	(31,33,35,37,39)	(6,8,10,12,14)

Table 3. **Quantitative Table**

Step 1: Apply Pascal's triangular graded mean for pentagonal fuzzy number, the fuzzy times can be converted in to crisp times

$$t_{11} = (6,8,10,12,14) = 10; \quad t_{12} = (15,16,17,18,19) = 17$$

$$t_{21} = (15,16,17,18,19) = 17; \quad t_{22} = (0,1,2,3,4) = 2$$

$$t_{31} = (0,1,2,3,4) = 2; \quad t_{32} = (22,24,26,28,30) = 26$$

$$t_{41} = (22,24,26,28,30) = 26; \quad t_{42} = (31,33,35,37,39) = 35$$

$$t_{51} = (31,33,35,37,39) = 35; \quad t_{52} = (6,8,10,12,14) = 10$$

Jobs	**Machine M_1**	**Machine M_2**
J_1	10	17
J_2	17	2
J_3	2	26
J_4	26	35
J_5	35	10

Optimum sequence

J_3	J_1	J_4	J_5	J_2

Step 2: Total elapsed time and Idle time

Jobs	Machine M_1		Machine M_2	
	Time in	**Time out**	**Time in**	**Time out**
J_3	0	2	2	28
J_1	2	12	28	45
J_4	12	38	45	80
J_5	38	73	80	90
J_2	73	**90**	90	**92**

$$\text{Total Elapsed time} = 92\ \ Hrs$$
$$\text{Idle time on Machine } M_1 = 2\ Hrs$$
$$\text{Idle time on Machine } M_2 = 2\ Hrs$$

9. Conclusion

In this paper, we have solved job sequencing problem with fuzzy processing times have considered pentagonal fuzzy numbers. A Numerical example has been considered and solved to illustrate the proposed method. Fuzzy sequencing problem is easy to understand and helps to formulate uncertainty decision makers in real life situation.

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