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# A Study on the Optimization of Transportation Problem 

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#### Abstract

Determining efficient solutions for large scale transportation problems is an important task in operational research. In this study, Least Cost Method (LCM) which is well known transportation methods in the literature was investigated to obtain more efficient initial solutions.


Keywords: Transportation problem, Least Cost Method, Odd cost, Even cost, Initial Basic feasible solution.
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## 1. Introduction

The transportation problem is a special kind of network optimization problems. It has the special data structure in solution characterized as a transportation graph. Transportation problem play an important role in logistics and supply chains. The problem basically deals with the determination of a cost plan for transporting a single commodity from a number of sources to a number of destinations. The purpose is to minimize the cost of shipping goods from one location to another so that the needs of each arrival area are met and every shipping location operates within its capacity. Therefore for solving transportation problem, finding minimal total cost, would be remarkable. Proposing optimal solution needs to start from a feasible solution as an initial basic feasible solution (IBFS). Therefore initial feasible solution is basic solution which affects to optimal solution for the problem. In other words, finding IBFS would be significant. There are few methods in references which can find IBFS for the problem. These methods are as follows: Northwest Corner Method (NCM), this method begins from the northwest cell in the transportation table. The algorithm allocates the possible maximum amount to the cell and regulates supply demand quantities. Then each supply or demand which attains zero, corresponds row or column will exit from the rest of calculations. This process will be continued until just a row or column is left from the table. During all these calculations the northwest cell of the table, without regarding removed rows and columns, will be selected each time. Least Cost Method (LCM), in this method, a cell which has lower cost is selected sooner than a cell with higher cost. In fact, the appropriation starts from the cell which includes the least cost of the table. The Minimum Cost Method finds IBFS better than NCM because the algorithm regards costs during the allocation unlike NCM. All process in NCM will be repeated here just with this difference that always the cell which includes minimum cost is selected instead the northwest cell. Minimum Row Method, Vogel's Approximation Method (VAM), this method has the best results according to literature. Summary of VAM steps are as follows: at the first, the difference between two least costs is calculated for each row and column as a penalty. Then a row or column which has the biggest penalty will be chosen. The possible maximum amount is allocated

[^0]to the cell with the minimum cost in the chosen row or column. Each row which has no supply or the column which the demand has been totally satisfied for that will be removed from the rest of calculations. The difference between two least costs will be calculated for the remaining rows and columns and the process is continued to find an initial basic feasible solution.

In this paper, a new method has been proposed in reference of odd $\&$ even cost for getting better IBFS. The proposed algorithm has been compared with Least Cost Method and the proposed algorithm gives better IBFS than Least Cost Method for different types of transportation problems.

## 2. Algorithm

(a). Balance the given transportation problem if it is not balanced.
(b). Check whether there is a zero cost cell in the given transportation cost matrix or not. If it is, then allocate the minimum of demand and supply to the cell. If not, then go to step (c).
(c). Identify the least even \& least odd cost from the transportation cost matrix.
(d). Subtract the least even cost from the even costs and least odd cost from the odd costs in the transportation cost matrix.
(e). There must be atleast one zero cost cell in the table. Allocate the cell with possible allocation of minimum of demand and supply.
(1). The tie in two or more zeros can be broken by selecting the cell with minimum possible allocation.
(2). The tie in the allocation can be broken by selecting the extreme left cell when the tie is in a row, and by selecting the uppermost cell when the tie is in a column.
(f). Eliminate the exhausted row or column and again go to step (c).
(g). Repeat steps (c)-(f) until all requirements have been met.
(h). Compute total transportation cost for the initial basic feasible allocation using the original balanced transportation cost matrix.

## Example 2.1.

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | SUPPLY |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $O_{1}$ | 19 | 30 | 50 | 10 | $\mathbf{7}$ |
| $O_{2}$ | 70 | 30 | 40 | 60 | $\mathbf{9}$ |
| $O_{3}$ | 40 | 8 | 70 | 20 | $\mathbf{1 8}$ |
| DEMAND | $\mathbf{5}$ | $\mathbf{8}$ | $\mathbf{7}$ | $\mathbf{1 4}$ |  |

Solution:
Step 1:

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | $D_{4}$ | SUPPLY |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $O_{1}$ | $0(5)$ | 22 | 42 | 2 | $\mathbf{7 / 2}$ |
| $O_{2}$ | 62 | 22 | 32 | 52 | $\mathbf{9}$ |
| $O_{3}$ | 32 | 0 | 62 | 12 | $\mathbf{1 8}$ |
| DEMAND | $\mathbf{5} / \mathbf{0}$ | $\mathbf{8}$ | $\mathbf{7}$ | $\mathbf{1 4}$ |  |

Step 2:

|  | $D_{2}$ | $D_{3}$ | $D_{4}$ | SUPPLY |
| :---: | :---: | :---: | :---: | :---: |
| $O_{1}$ | 20 | 40 | $0(2)$ | $\mathbf{2 / 0}$ |
| $O_{2}$ | 20 | 30 | 50 | $\mathbf{9}$ |
| $O_{3}$ | 0 | 60 | 10 | $\mathbf{1 8}$ |
| DEMAND | $\mathbf{8}$ | $\mathbf{7}$ | $\mathbf{1 4 / 1 2}$ |  |

Step 3:

|  | $D_{2}$ | $D_{3}$ | $D_{4}$ | SUPPLY |
| :---: | :---: | :---: | :---: | :---: |
| $O_{2}$ | 10 | 20 | 40 | $\mathbf{9}$ |
| $O_{3}$ | $0(8)$ | 50 | 0 | $\mathbf{1 8 / 1 0}$ |
| DEMAND | $\mathbf{8 / 0}$ | $\mathbf{7}$ | $\mathbf{1 2}$ |  |

Step 4:

|  | $D_{3}$ | $D_{4}$ | SUPPLY |
| :---: | :---: | :---: | :---: |
| $O_{2}$ | $0(7)$ | $20(2)$ | $\mathbf{9 / 2} / \mathbf{0}$ |
| $O_{3}$ | 30 | $0(10)$ | $\mathbf{1 0} / \mathbf{0}$ |
| DEMAND | $\mathbf{7 / 0}$ | $\mathbf{1 2 / 1 0} / \mathbf{0}$ |  |

Initial Basic Feasible Solutions- $x_{11}=5, x_{14}=2, x_{32}=8, x_{23}=7, x_{24}=2, x_{34}=10$.

$$
\begin{aligned}
\text { Total transportation cost } & =(5 \times 19)+(10 \times 2)+(8 \times 8)+(40 \times 7)+(60 \times 2)+(20 \times 10) \\
& =779
\end{aligned}
$$

## Example 2.2.

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | SUPPLY |
| :---: | :---: | :---: | :---: | :---: |
| $O_{1}$ | 6 | 8 | 4 | $\mathbf{1 4}$ |
| $O_{2}$ | 4 | 9 | 3 | $\mathbf{1 2}$ |
| $O_{3}$ | 1 | 2 | 6 | $\mathbf{5}$ |
| DEMAND | $\mathbf{6}$ | $\mathbf{1 0}$ | $\mathbf{1 5}$ |  |

## Solution:

## Step 1:

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | SUPPLY |
| :---: | :---: | :---: | :---: | :---: |
| $O_{1}$ | 4 | 6 | 2 | $\mathbf{1 4}$ |
| $O_{2}$ | 2 | 8 | 2 | $\mathbf{1 2}$ |
| $O_{3}$ | $0(5)$ | 0 | 4 | $\mathbf{5 / 0}$ |
| DEMAND | $\mathbf{6}$ | $\mathbf{1 0}$ | $\mathbf{1 5}$ |  |

Step 2:

|  | $D_{1}$ | $D_{2}$ | $D_{3}$ | SUPPLY |
| :---: | :---: | :---: | :---: | :---: |
| $O_{1}$ | 2 | 4 | 0 | $\mathbf{1 4}$ |
| $O_{2}$ | $0(1)$ | 6 | 0 | $\mathbf{1 2} / \mathbf{1 1}$ |
| DEMAND | $\mathbf{1} / \mathbf{0}$ | $\mathbf{1 0}$ | $\mathbf{1 5}$ |  |

## Step 3:

|  | $D_{2}$ | $D_{3}$ | SUPPLY |
| :---: | :---: | :---: | :---: |
| $O_{1}$ | $0(10)$ | $0(4)$ | $\mathbf{1 4 / 4 / 0}$ |
| $O_{2}$ | 2 | $0(11)$ | $\mathbf{1 1 / 0}$ |
| DEMAND | $\mathbf{1 0} / \mathbf{0}$ | $\mathbf{1 5} / \mathbf{0}$ |  |

Initial Basic Feasible Solutions- $x_{31}=5, x_{21}=1, x_{12}=10, x_{13}=4, x_{23}=11$.

$$
\begin{aligned}
\text { Total transportation cost } & =(1 \times 5)+(4 \times 1)+(8 \times 10)+(4 \times 4)+(11 \times 3) \\
& =138
\end{aligned}
$$

## 3. Result Analysis

| Transportation Problem | Problem Size | LCM | Proposed Method |
| :---: | :---: | :---: | :---: |
| Example 2.1 | $4 \times 3$ | 814 | 779 |
| Example 2.2 | $3 \times 3$ | 139 | 138 |

## 4. Conclusion

In this paper, we observe from the above examples of transportation problem that the algorithm defined in (2) gives better optimal solution than LCM.

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