

Stochastic Non-Newtonian Lubrication in Roller Bearings Considering Viscosity Variation and Thermal Effects

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Abstract: In this paper the generalized Reynolds equation derived for power law fluid is applied to study the squeeze film lubrication of parallel plates considering surface roughness, viscosity variation and thermal effects. The consistency of the lubrication is assumed to vary across the film as two layer fluid and thermal effects by introducing a thermal parameter q in the consistency. Stochastic theory is applied to study roughness effect. Two types of roughness transversal roughness and longitudinal roughness are recognized and expected values of load capacity and time of squeezing are obtained. For all flow behavior index n the effects of high consistency near periphery on these parameters are also studied for both types of roughness.

Keywords: Power law lubrication, consistency, viscosity, thermal effects, load capacity, squeezing time, film thickness etc,

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1. Introduction

Lubrication is the Science of reducing frictional resistance by means of some kind of substance introduced between the two surfaces having relative motion. Such a substance which has some amount of viscosity is known as lubricant. A bearing is a system of machine elements whose function is to support an applied load by reducing friction between the relatively moving surfaces. It is used to avoid friction, which causes wear and tear of moving machine.

The load may be radial, axial or combination of these two. Bearing supports a radial load, it is called radial or journal bearing and supports a thrust or an axial load is called thrust bearing. Some bearings can support both radial and axial load and they are known as conical bearings. Many authors introduced the characteristics of power-law lubricants in various lubricated systems and it has been assumed that the bearing surfaces are smooth. However, in reality bearing surfaces would always have some roughness and it is very natural to study such effects on various bearing characteristics. Several attempts have been made to study such effects in bearing systems by both deterministic and stochastic approach [1–17].

In the deterministic approach the profile or roughness asperities are represented by a given shape function such as sine or cosine wave and thus modifying the film thickness in the usual study of the bearing characteristics. Using this approach, some studies have been conducted. It has been pointed out that the load capacity, frictional forces etc. are different from the corresponding case of smooth surfaces and they mainly depend upon the amplitude and the wave lengths of the representing the roughness surfaces. Recently a new deterministic theory has been proposed by Shukla [12] when the mean height of the asperities is of the same order as the minimum film thickness.

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In another method known as stochastic approach, the film thickness is assumed to be a stochastic or random function and hence the corresponding Reynolds equation becomes a stochastic differential equation. To study various characteristics, this equation is solved by taking the mean or the average of the stochastic variables involved. This concept has been used by Tzeng and Saibel [17] to study the effect of surface roughness in an infinite slider bearings and short journal bearings. Later Christensen and Tonder [10] derived generalized form of Reynolds equation for stochastic lubrication. Further refinement of this theory has been given by Christensen, Shukla and Kumar [12]. Since then several models including the effects of viscosity variation have been proposed and their applicability to various bearing systems have been investigated.

It may be noted that in the above mentioned studies, the lubricant is characterized by a Newtonian model. As pointed out earlier when additives of higher concentration are added to the base lubricant, it becomes Non-Newtonian and the simplest of these is the power-law model which describes the characteristics of several polymer solutions and silicone fluids. In the last two decades several attempts have been made to study the behavior of power-law lubricant in different bearing systems, but no attempt has been made to study the effects of surface roughness on bearing characteristics using such lubricants. In this chapter the generalized Reynolds equation derived for power law fluid is applied to study the squeeze film lubrication of parallel plates considering surface roughness, viscosity variation and thermal effects. The consistency of the lubrication is assumed to vary across the film as two layer fluid and thermal effects by introducing a thermal parameter q in the consistency. Stochastic theory is applied to study roughness effect. Two types of roughness transversal roughness and longitudinal roughness are recognized and expected values of load capacity and time of squeezing are obtained.

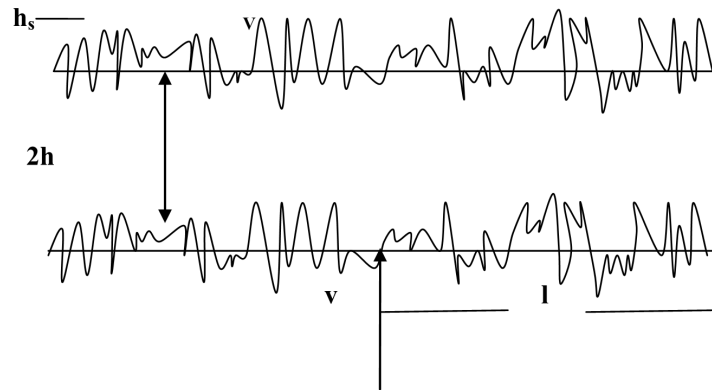


Figure 1. Squeezing between two rough parallel plates

2. Basic Equation

Consider the flow of a power-law lubricant with constant consistency between two symmetrically rolling and squeezing rough surfaces. The equation governing the pressure in the film is given from generalized Reynolds equations for power law lubricants as

$$\frac{d}{dx} \left[\frac{n}{n+1} H^{\frac{2n+1}{n}} f_0 \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \right] = -\frac{dH}{dt} - U \frac{dH}{dx} \quad (1)$$

In the region $-X_1 \leq X \leq X_2$, $y > 0$, $\frac{dp}{dx} \leq 0$ and

$$\frac{d}{dx} \left[\frac{n}{n+1} H^{\frac{2n+1}{n}} f_0 \left(\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \right] = \frac{dH}{dt} + U \frac{dH}{dx} \quad (2)$$

In the region $-X_2 \leq X \leq X_1$, $y > 0$, $\frac{dp}{dx} \geq 0$, where

$$f_0 = 1 - \left(1 - k^{\frac{1}{n}} \right) \left\{ 1 - \left(1 - \frac{2a}{h} \right)^{\frac{2n+1}{n}} \right\} \quad (3)$$

and $2H$ is the total stochastic film thickness variables, U, V are the rolling and squeezing velocities respectively. Since equations (1) and (2) are stochastic differential equations for pressure. Taking the stochastic mean of equation (1) and (2) we get,

$$\frac{d}{dx} \left[\frac{n}{n+1} E \left\{ H^{\frac{2n+1}{n}} f_0 \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \right\} \right] = -E \frac{dH}{dt} - UE \frac{dH}{dx} \text{ for } \frac{dp}{dx} \leq 0 \text{ and} \tag{4}$$

$$\frac{d}{dx} \left[\frac{n}{n+1} E \left\{ H^{\frac{2n+1}{n}} f_0 \left(\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \right\} \right] = E \frac{dH}{dt} + UE \frac{dH}{dx} \text{ for } \frac{dp}{dx} \geq 0 \tag{5}$$

where

$$E(s) = \int_{-\infty}^{\infty} s f(s) ds \tag{6}$$

and $f(s)$ is the distribution of the stochastic variable s . The film thickness function H is given by

$$H = h(x, z) + h_s(x, z, \xi) \tag{7}$$

Where $2h$ is the nominal film thickness and $2h_s$ is the part of the film thickness due to surface roughness measured from the nominal level. It is assumed that h_s is a function of the random variable ξ , the mean value of which over the bearing surface is zero. To evaluate the terms on the left hand side of equations (4) and (5) we apply the some postulates as proposed by Christensen i.e.,

- (1). Let S_1 be the direction parallel to the roughness and S_2 the direction perpendicular to it. The pressure gradient in the roughness direction is assumed to be a stochastic variable with zero or negligible variance.
- (2). In the direction perpendicular to the direction of roughness the flux is assumed to have zero or negligible variance.

Using the above two postulates the Reynolds equations in the following cases are derived.

2.1. Longitudinal One-Dimensional Roughness

In this type of roughness the narrow ridges and valleys of the asperities run parallel to the direction of motion. The film thickness in this case is given by

$$H = h(x, z) + h_s(x, z, \xi) \tag{8}$$

Since by first postulate, $\frac{dp}{dx}$ has zero or negligible variance $\left(\frac{dp}{dx}\right)^{\frac{1}{n}}$ is also a stochastic variable having zero or negligible variance and

$$E \left[\left(\frac{dp}{dx} \right)^{\frac{1}{n}} \right] = \left[E \left(\frac{dp}{dx} \right) \right]^{\frac{1}{n}} \tag{9}$$

Since $\left(\frac{dp}{dx}\right)^{\frac{1}{n}}, H^{\frac{2n+1}{n}}$ are stochastically independent, we get from equation (4) and (9)

$$\frac{d}{dx} \left[\frac{n}{n+1} E \left\{ H^{\frac{2n+1}{n}} f_0 \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \right\} \right] = -\frac{dh}{dt} - U \frac{dh}{dx} \tag{10}$$

Where $\bar{p} = E(p), E(H) = h$. Similarly for the region $\frac{dp}{dx} > 0$ we get the corresponding Reynolds equation for the longitudinal roughness which can be written from equation (5) as follows

$$\frac{d}{dx} \left[\frac{n}{n+1} E \left\{ H^{\frac{2n+1}{n}} f_0 \left(\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \right\} \right] = \frac{dh}{dt} + U \frac{dh}{dx} \tag{11}$$

Equations (10) and (11) are generalized one dimensional form of Reynolds equation applicable for two symmetric rough rollers having longitudinal roughness using power-law lubricant.

2.2. Transverse One-Dimensional Roughness

In this type of roughness, it is noted that the narrow ridges and valleys run perpendicular to the direction of motion. The film thickness function in this case is given by

$$H = h(x, z, t) + h_s(x, \xi) \quad (12)$$

To evaluate the term on the left hand side of equation (4) let us define the flux as follows

$$Q = UH + \frac{n}{2n+1} H^{\frac{2n+1}{n}} \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \quad (13)$$

This can also be written as

$$\frac{Q}{H^{\frac{2n+1}{n}}} = \frac{U}{H^{\frac{n+1}{n}}} + \frac{n}{2n+1} \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \quad (14)$$

Using postulate (2) and taking the expectation of equation (14) we get

$$E(Q)E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right) = UE\left(\frac{1}{H^{\frac{n+1}{n}}}\right) + \frac{n}{2n+1} E\left(-\frac{1}{m} \frac{dp}{dx}\right)^{\frac{1}{n}} \quad (15)$$

Taking again expectation of equation (14) and using equation (15) we get,

$$E\left[UH + \frac{n}{2n+1} H^{\frac{2n+1}{n}} \left(-\frac{1}{m} \frac{dp}{dx}\right)^{\frac{1}{n}}\right] = \frac{UE\left(\frac{1}{H^{\frac{n+1}{n}}}\right)}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} + \frac{n}{2n+1} \frac{E\left(-\frac{1}{m} \frac{dp}{dx}\right)^{\frac{1}{n}}}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} \quad (16)$$

Substituting this equation in (4) gives,

$$\frac{d}{dx} \left[\frac{n}{2n+1} f_0 \frac{E\left(-\frac{1}{m} \frac{dp}{dx}\right)^{\frac{1}{n}}}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} \right] = -\frac{dh}{dt} - U \frac{d}{dx} \left[\frac{E\left(\frac{1}{H^{\frac{n+1}{n}}}\right)}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} \right] \quad \text{for } \frac{dp}{dx} < 0 \quad (17)$$

Similarly for the region $\frac{dp}{dx} > 0$, we get

$$\frac{d}{dx} \left[\frac{n}{2n+1} f_0 \frac{E\left(\frac{1}{m} \frac{dp}{dx}\right)^{\frac{1}{n}}}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} \right] = \frac{dh}{dt} + U \frac{d}{dx} \left[\frac{E\left(\frac{1}{H^{\frac{n+1}{n}}}\right)}{E\left(\frac{1}{H^{\frac{2n+1}{n}}}\right)} \right] \quad (18)$$

Equations (17) and (18) are generalized one dimensional form of Reynolds equation applicable for symmetric rough surfaces in the case of transverse roughness. In equation (17) and (18) the term $E\left(-\frac{dp}{dx}\right)^{\frac{1}{n}}$ is yet to be expressed in terms of $\frac{d\bar{p}}{dx}$. This cannot be done exactly as $\frac{dp}{dx}$ is a stochastic function. To get an approximate relation we proceed as follows. By keeping $U = 0$ in equation (13) we get,

$$Q = \frac{n}{2n+1} H^{\frac{2n+1}{n}} \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \quad (19)$$

or

$$\frac{Q^n}{H^{\frac{2n+1}{n}}} = \left(\frac{n}{2n+1} \right)^n H^{2n+1} \left(-\frac{1}{m} \frac{dp}{dx} \right) \quad (20)$$

Taking expectation of equation (19) and (20), and using $E(Q^n) = [E(Q)]^n$, we get

$$E\left(\frac{dp}{dx}\right)^{\frac{1}{n}} = \frac{E\left[\frac{1}{H^{\frac{2n+1}{n}}}\right]}{E\left[\frac{1}{H^{2n+1}}\right]^{\frac{1}{n}}} \left(\frac{d\bar{p}}{dx}\right)^{\frac{1}{n}} \quad (21)$$

Using equation (21) in equations (17) and (18) we get the approximate Reynolds equation in the case of transverse roughness.

In the following we discuss the squeezing between two symmetric rough bearings.

3. Parallel Plates (Squeeze Films)

Consider squeezing flow between symmetric parallel plates as shown in fig. Here we discuss the following cases:

3.1. Longitudinal One-Dimensional Roughness

In this type of roughness it may be noted again that the narrow ridges and valleys run in the direction of the flow of the lubricant. The equation governing the mean pressure can be written from equation (10) by putting $U = 0$ as follows:

$$\frac{d}{dx} \left[\frac{n}{2n+1} E \left\{ H^{\frac{2n+1}{n}} f_0 \left(-\frac{1}{m} \frac{dp}{dx} \right)^{\frac{1}{n}} \right\} \right] = V \quad (22)$$

Where $V = -\frac{dh}{dt}$, integrating equation (22) and using

$$\frac{d\bar{p}}{dx} = 0 \text{ at } x = 0, \quad \bar{p} = 0 \text{ at } x = l, \quad (23)$$

We get

$$\bar{p} = m \left(\frac{2n+1}{n} \right)^n \left[\frac{1}{f_0} \frac{v}{E \left(H^{\frac{2n+1}{n}} \right)} \right]^n \frac{l^{n+1} - x^{n+1}}{n+1} \quad (24)$$

Where $2l$ is the length of the bearing. The mean load capacity W_r is defined as

$$E(W) = W_r = 2b \int_0^l \bar{p} dx \quad (25)$$

Using equation (24) gives,

$$W_r = 2bm \left(\frac{2n+1}{n} \frac{v}{f_0} \right)^n \frac{l^{n+2}}{n+2} \left[\frac{1}{E \left(H^{\frac{2n+1}{n}} \right)} \right]^n \quad (26)$$

Where b is the width of the bearing the time of squeezing t_r for the surface to approach from an initial film thickness, $2h_i$ to a final film thickness $2h_f$ is given by

$$t_r = \left(\frac{2n+1}{n} \right) \left(\frac{2bm}{W_r} \frac{l^{n+2}}{n+2} \right)^{\frac{1}{n}} \int_{h_f}^{h_i} \frac{dh}{E \left(H^{\frac{2n+1}{n}} \right)} \quad (27)$$

Substituting the value of $E \left(H^{\frac{2n+1}{n}} \right)$ from equation (appendix) in equation (26) and (27) we can write W_r and t_r approximately as follows:

$$W_r = 2bm \left(\frac{2n+1}{n} \frac{v}{f_0} \right)^n \frac{l^{n+2}}{n+2} \frac{1}{\left[1 + \left(\frac{2n+1}{n} \right) \left(\frac{n+1}{n} \right) \frac{1}{2} \frac{\sigma^2}{h^2} \right]^n h^{2n+1}} \quad (28)$$

$$t_r = \left(\frac{2n+1}{n} \right) \left(\frac{2bm}{W_r} \frac{l^{n+2}}{n+2} \right)^{\frac{1}{n}} \int_{h_f}^{h_i} \frac{dh}{\left[1 + \left(\frac{2n+1}{n} \right) \left(\frac{n+1}{n} \right) \frac{1}{2} \frac{\sigma^2}{h^2} \right] h^{\frac{2n+1}{n}}} \quad (29)$$

By keeping $\sigma = 0$ in equation (28) and (29) we get the case of smooth surfaces. If W_s and t_s are the load capacity and the squeezing time respectively for this case we can write,

$$W_s = 2bm \left(\frac{2n+1}{n} \frac{v}{f_0} \right)^n \frac{l^{n+2}}{n+2} \frac{1}{h^{2n+1}} \quad (30)$$

$$t_s = \left(\frac{2n+1}{n}\right) \left(\frac{2bm}{W_s} \frac{l^{n+2}}{n+2}\right)^{\frac{1}{n}} \int_{h_f}^{h_i} \frac{dh}{h^{\frac{2n+1}{n}}} \quad (31)$$

Consider $m = m_0 \left(\frac{h}{h_0}\right)^q$, q is thermal factor

$$\frac{W_{r,q}}{W_{s,0}} = \bar{W} = \left(\frac{h}{h_0}\right)^q \frac{1}{\left[1 + \left(\frac{2n+1}{n}\right) \left(\frac{n+1}{n}\right) \frac{1}{2} \frac{\sigma^2}{h^2}\right]^n} \quad (32)$$

$$\frac{t_{r,q}}{t_{s,0}} = \bar{t} = \left[\frac{1}{1 - \bar{h}_f^{-\left(\frac{n+1}{n}\right)}} \right] \int_{h_f}^{h_i} \frac{\bar{d}h}{\left[1 + \left(\frac{2n+1}{n}\right) \left(\frac{n+1}{n}\right) \frac{1}{2} \frac{\sigma^2}{h^2}\right] h^{\frac{2n+1}{n}} - q} \quad (33)$$

$$\bar{h}_f = \frac{h_f}{h_i}, \quad \bar{h} = \frac{h}{h_0} \quad (34)$$

Equations (32) and (33) are evaluated numerically and graphs of $\frac{W_{r,q}}{W_{s,q}}$, $\frac{t_{r,q}}{t_{s,q}}$ are plotted with σ for various values in figures.

3.2. Transverse One-Dimensional Roughness

In this type of roughness it may be noted again that the narrow ridges and valleys run perpendicular to the direction of the flow of the lubricant. The one dimensional equation governing the pressure in this case can be written, from equations (17) and (21) as follows:

$$\frac{d}{dx} \left[\frac{n}{2n+1} \frac{1}{f_0} \frac{\left(-\frac{1}{m} \frac{dp}{dx}\right)^{\frac{1}{n}}}{\left[E \left(\frac{1}{H^{2n+1}}\right)\right]^{\frac{1}{n}}} \right] = V \quad (35)$$

Integrating the equation (35) and using boundary conditions (23) we get,

$$\bar{p} = m \left(\frac{2n+1}{n} \frac{v}{f_0}\right)^n E \left(\frac{1}{H^{2n+1}}\right) \frac{l^{n+1} - x^{n+1}}{n+1} \quad (36)$$

The mean load capacity W_r and squeezing time t_r can be written by using equations (25) and (35) as follows:

$$W_r = 2bm \left(\frac{2n+1}{n} \frac{v}{f_0}\right)^n E \left(\frac{1}{H^{2n+1}}\right) \frac{l^{n+2}}{n+2} \quad (37)$$

$$t_r = \left(\frac{2n+1}{n}\right) \left(\frac{2bm}{W_r} \frac{l^{n+2}}{n+2}\right)^{\frac{1}{n}} \int_{h_f}^{h_i} \left[E \left(\frac{1}{H^{2n+1}}\right)\right]^{\frac{1}{n}} dh \quad (38)$$

Substituting the values of $E \left(\frac{1}{H^{2n+1}}\right)$ from equations (appendix) the values of W_r and t_r can be written approximately as follows:

$$W_r = 2bm \left(\frac{2n+1}{n} \frac{v}{f_0}\right)^n \frac{1}{h^{2n+1}} \frac{l^{n+2}}{n+2} \left[1 + (n+1)(2n+1) \frac{\sigma^2}{h^2}\right] \quad (39)$$

$$t_r = \left(\frac{2n+1}{n}\right) \left(\frac{2bm}{W_r} \frac{l^{n+2}}{n+2}\right)^{\frac{1}{n}} \int_{h_f}^{h_i} \frac{\left[1 + (n+1)(2n+1) \frac{\sigma^2}{h^2}\right]^{\frac{1}{n}}}{h^{\frac{2n+1}{n}}} dh \quad (40)$$

By keeping $\sigma = 0$ in equation (39) and (40) we get the case of smooth surfaces. If W_s and t_s are the load capacity and the squeezing time respectively in this case of smooth surface, then we can write

$$W_s = 2bm \left(\frac{2n+1}{n} \frac{v}{f_0}\right)^n \frac{l^{n+2}}{n+2} \frac{1}{h^{2n+1}} \quad (41)$$

$$t_s = \left(\frac{2n+1}{n}\right) \left(\frac{2bm}{W_s} \frac{l^{n+2}}{n+2}\right)^{\frac{1}{n}} \int_{h_f}^{h_i} \frac{1}{h^{\frac{2n+1}{n}}} dh \quad (42)$$

Where $m = m_0 \left(\frac{h}{h_0} \right)^q$,

$$\frac{W_{r,q}}{W_{s,0}} = \bar{W} = \left(\frac{h}{h_0} \right)^q \left[1 + (n+1)(2n+1) \frac{\sigma^2}{h^2} \right] \tag{43}$$

$$\frac{t_{r,q}}{t_{s,0}} = \bar{t} = \left[\frac{1}{1 - \bar{h}_f - \left(\frac{n+1}{n} \right)} \right] \int_{h_f}^{h_i} \frac{\left[1 + (n+1)(2n+1) \frac{\sigma^2}{h^2} \right]^{\frac{1}{n}}}{h^{\frac{2n+1}{q} - q}} d\bar{h} \tag{44}$$

Equation (31), (32), (43) and (44) are evaluated numerically and graphs of are plotted for various parameters.

4. Results and Discussion

Here the results are discussed for the graphs from (2) to (11). Figure 2 is plotted for the load capacity with $\frac{\sigma}{h_i}$ for different values of q in the case of longitudinal roughness. From this we can say that the load capacity decreases with the increase of $\frac{\sigma}{h_i}$ and the load capacity decreases for increasing values of q. Figure 3 is plotted for the load capacity with $\frac{\sigma}{h_i}$ for different values of q in the case of transversal roughness. From this we can say that the load capacity increases with the increase of $\frac{\sigma}{h_i}$ and the load capacity decreases for increasing values of q.

Figure 4 is plotted for the load capacity with $\frac{\sigma}{h_i}$ for different values of flow behavior index n in the case of longitudinal roughness. From this we can say that the load capacity decreases with the increase of $\frac{\sigma}{h_i}$ and the load capacity decreases for increasing values of n. Figure 5 is plotted for the load capacity with $\frac{\sigma}{h_i}$ for different values of n in the case of transversal roughness. From this we can say that the load capacity decreases with the increase of $\frac{\sigma}{h_i}$ and the load capacity decreases for increasing values of n.

Figure 6 is plotted for the squeezing time with $\frac{\sigma}{h_i}$ for different values of q in the case of longitudinal roughness. From this we can say that squeezing time decreases with the increase of $\frac{\sigma}{h_i}$ and the squeezing time decreases for increasing values of q. Figure 7 is plotted for the squeezing time with $\frac{\sigma}{h_i}$ for different values of q in the case of transversal roughness. From this we can say that the squeezing time increases with the increase of $\frac{\sigma}{h_i}$ and the squeezing time increases for increasing values of q.

Figure 8 is plotted for the squeezing time with $\frac{\sigma}{h_i}$ for different values of flow behavior index n in the case of longitudinal roughness. From this we can say that the squeezing time decreases with the increase of $\frac{\sigma}{h_i}$ and the squeezing time decreases for increasing of n. Figure 9 is plotted for the squeezing time with $\frac{\sigma}{h_i}$ for different values of flow behavior index n in the case of transversal roughness. From this we can say that the squeezing time increases with the increase of $\frac{\sigma}{h_i}$ and the squeezing time decreases for increasing of n.

Figure 10 is plotted for the squeezing time with $\frac{\sigma}{h_i}$ for different values of \bar{h}_f in the case of longitudinal roughness. From this we can say that the squeezing time decreases with the increase of $\frac{\sigma}{h_i}$ and the squeezing time decreases for increasing of \bar{h}_f . Figure 11 is plotted for the squeezing time with $\frac{\sigma}{h_i}$ for different values of \bar{h}_f in the case of transversal roughness. From this we can say that the squeezing time increases with the increase of $\frac{\sigma}{h_i}$ and the squeezing time increases for increasing of \bar{h}_f .

5. Graphs

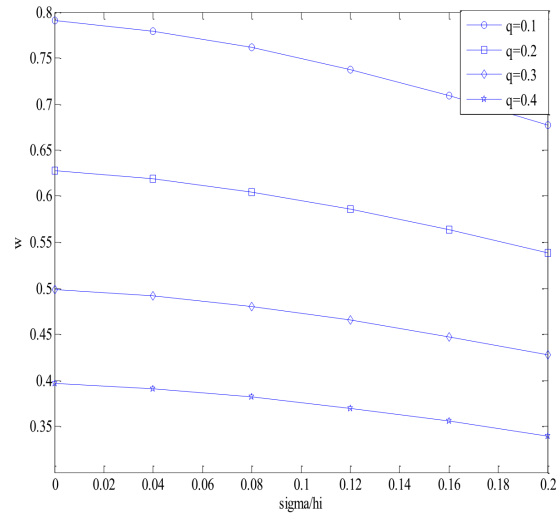


Figure 2. The load capacity Vs $\frac{\sigma}{h_i}$ for various q Case of transversal Roughness

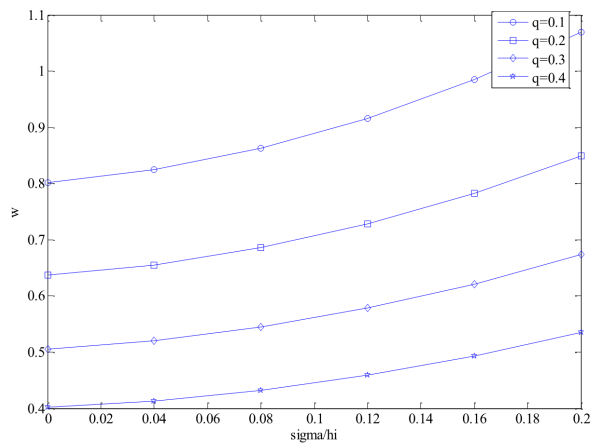


Figure 3. The load capacity Vs $\frac{\sigma}{h_i}$ for various q Case of longitudinal Roughness

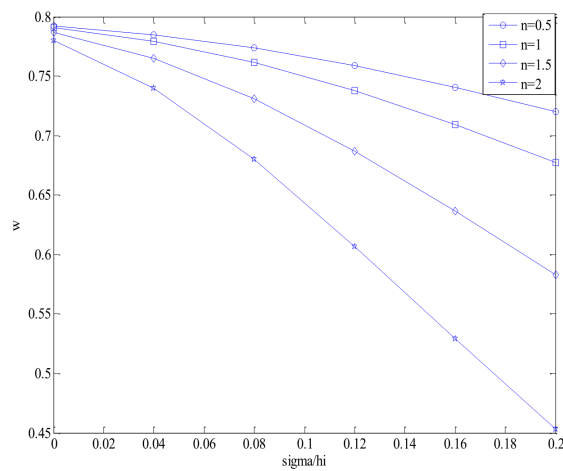


Figure 4. The load capacity Vs $\frac{\sigma}{h_i}$ for various n Case of transversal Roughness

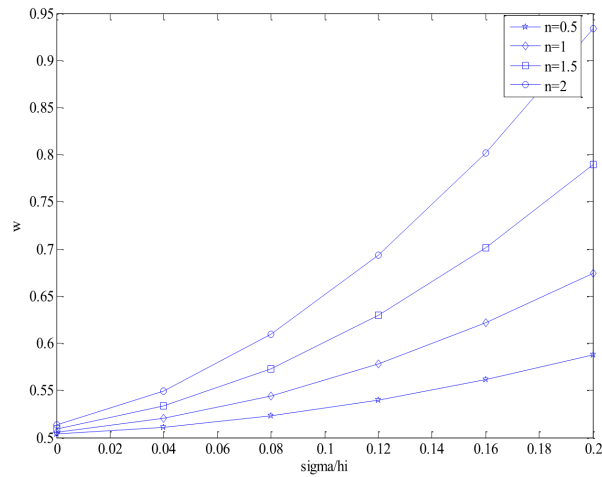


Figure 5. The load capacity W Vs $\frac{\sigma}{h_i}$ for various n Case of longitudinal Roughness

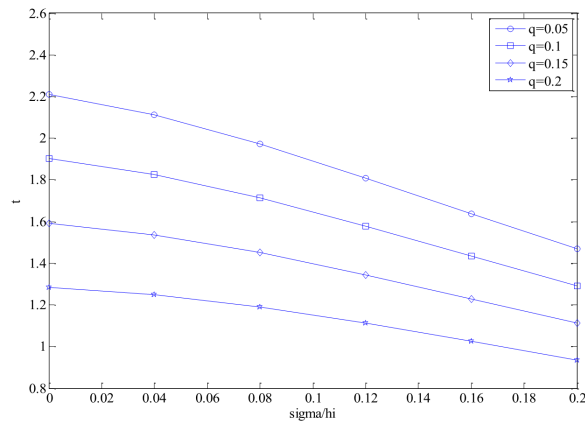


Figure 6. The squeezing time t Vs $\frac{\sigma}{h_i}$ for various q Case of transversal Roughness

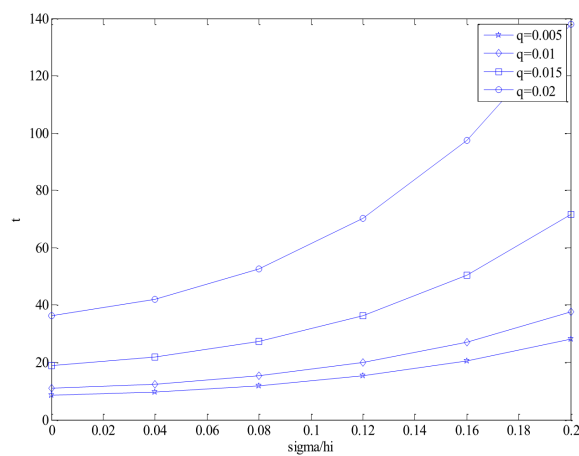


Figure 7. The squeezing time t Vs $\frac{\sigma}{h_i}$ for various q Case of longitudinal Roughness

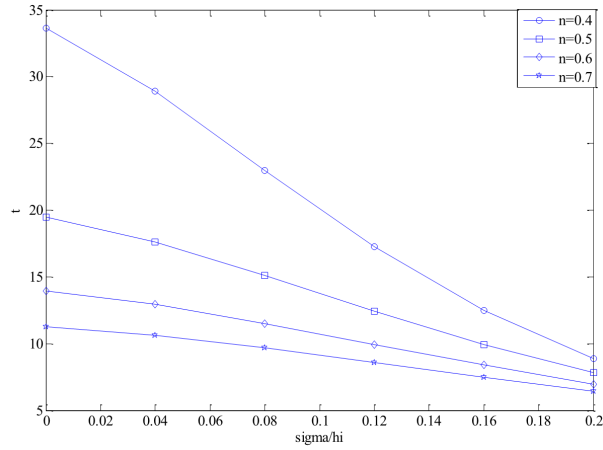


Figure 8. The squeezing time t Vs $\frac{\sigma}{h_i}$ for various n Case of transversal Roughness

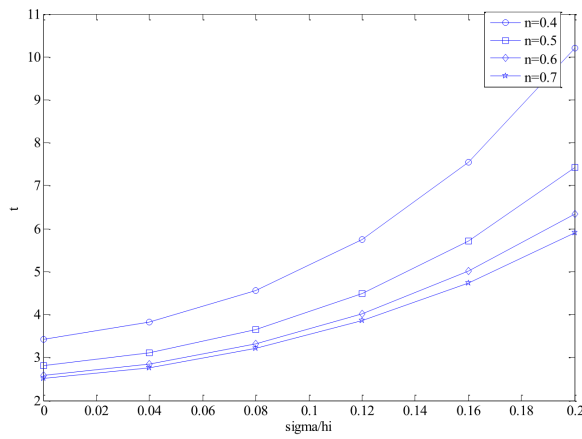


Figure 9. The squeezing time t Vs $\frac{\sigma}{h_i}$ for various n Case of longitudinal Roughness

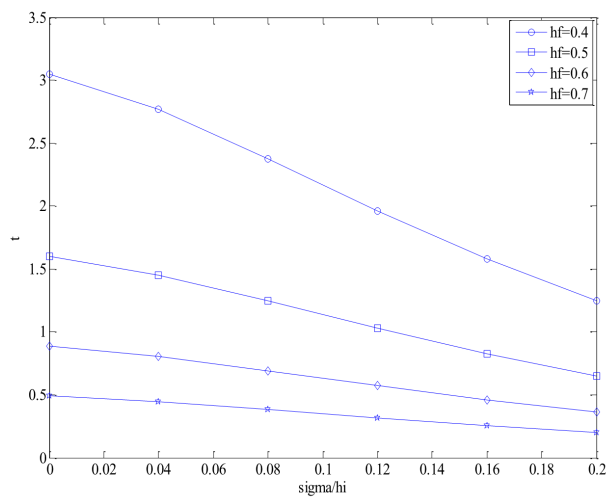


Figure 10. The squeezing time t Vs $\frac{\sigma}{h_i}$ for various \bar{h}_f Case of transversal Roughness

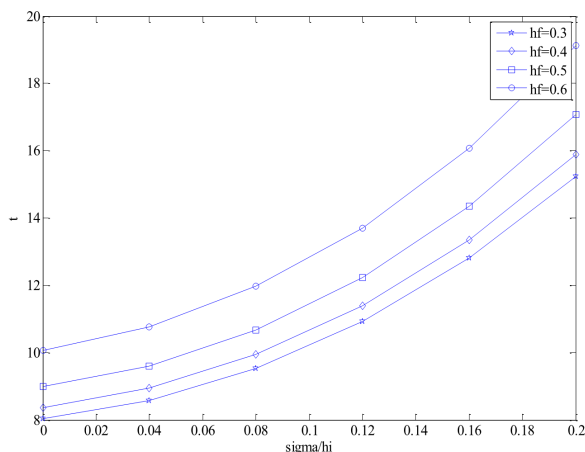


Figure 11. The squeezing time t Vs $\frac{\sigma}{\bar{h}_i}$ for various \bar{h}_f Case of longitudinal Roughness

6. Summary

- In this chapter the generalized Reynolds equation derived for power law fluid is applied to study the squeeze film lubrication of parallel plates considering surface roughness, viscosity variation and thermal effects.
- The consistency of the lubrication is assumed to vary across the film as two layer fluid and thermal effects by introducing a thermal parameter ‘q’ in the consistency.
- As thermal factor increases the load capacity decreases, but the time of squeezing decreases in the case of longitudinal and increases in the case of transversal
- Stochastic theory is applied to study roughness effect. Two types of roughness transversal roughness and longitudinal roughness are recognized and expected values of load capacity and time of squeezing are obtained.
- The load capacity and squeezing time increases as the mean film thickness increases in case of transversal roughness and decreases in case of longitudinal roughness.
- It is shown that for all flow behavior index ‘n’ the load capacity and squeezing time increases as surface roughness parameter increases in the case of transversal roughness and decreases in the case of longitudinal roughness.

Nomenclature

b	: Width of the bearing
c	: Half range random film thickness variable
E	: Expectation operator
H	: Half film thickness random variable
$2h$: Deterministic part of film thickness (nominal film thickness)
$2h_i$: Initial film thickness
$2h_s$: Random part of the film thickness
L	: Length of the bearing
M	: Consistency of the fluid

n	: Flow behavior index
p	: Hydrodynamic pressure, a random variable
\bar{p}	: Mean Hydrodynamic pressure
Q	: Flow flux
q	: Thermal factor
x, y, z	: Coordinate system

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