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Prime Labelings of Some Wheel And Crown Related Graphs

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Abstract: A Graph G with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper we investigate the existence of prime labeling of some graphs related to wheel W_n and crown C_n^*. We discuss prime labeling in the context of the graph operation namely corona product.

Keywords: Graph Labeling, Prime Labeling, corona product, Prime Graph.

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1. Introduction

In this paper, we consider only finite simple undirected graph. The graph G is a set of vertices $V(G)$, together with a set of edges $E(G)$ and incidence relation. If $u, v \in V(G)$ are connected by an edge, we say u and v are adjacent and the corresponding edge is denoted by uv or vu. The degree of a vertex u is the number of edges adjacent with u. A graph is connected if it does not consist of two or more disjoint "pieces". The path P_n is the connected graph consisting of two vertices of degree 1 and $n-2$ vertices of degree 2. An $n-$cycle C_n, is the connected graph consisting of n vertices each of degree 2. An $n-$ star S_n, is the graph consisting of one vertex of degree n and n vertices of degree 1. But S_n consist of $n+1$ vertices and n edges. A tree is a graph contains no cycle. Path and stars are example of trees. For notations and terminology we refer to Bondy and Murthy [1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [8]. Two integers a and b are said to be relatively prime if their greatest common divisor is 1 denoted by $(a, b) = 1$. Every path P_n, cycle C_n and star S_n are prime [2] . Every wheel W_n if and only if n is even [2], all helm H_n, crown C_n^* and gear graph G_n are prime [2]. We refer Gallian's dynamic survey [2] for a comprehensive listing of the families of graphs that are known to have or known not to have prime vertex labeling. S.K.Vaidya [5,6,7] investigated about the existence of prime labeling for some path, cycle and wheel related graphs in the context of some graph operations namely duplication, fusion and vertex switching etc. Let G and H be two graphs. The corona product $G \odot H$ is obtained by taking one copy of G and $|V(G)|$ copies of H and by joining each vertex of the i-th copy of H to the i-th vertex of G where $1 \leq i \leq |V(G)|$ [3]. For all $m, n \in N$ with $n \geq 3$, an m-hairy $n-$cycle , denoted by $C_n * S_m$, is the cycle C_n with m pendants attached to each cycle vertex [4]. In other words this graph is corona product $C_n \odot \overline{K}_m$. The crown graph C_n^* is obtained from a cycle C_n by attaching a pendent edge at

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each vertex of the n-cycle. We consider the graph obtained by, attaching the centre vertex of a copy of the star S_m to each vertex of Wheel W_n and crown C_n^*. The resulting graph will have m pendants at each vertex and it is denoted by $W_n * S_m$ and $C_n^* * S_m$. The star notation indicates that we attach a copy of the star S_m at its vertex of the degree m to all the vertices of W_n and C_n^*. In other words $W_n * S_m$ is the corona product $W_n \odot \overline{K}_m$ and $C_n^* * S_m$ is the corona product $C_n^* \odot \overline{K}_m$. In this paper, we proved that the graphs obtained by attaching m-star to each vertex of Wheel W_n and crown C_n^* are all prime graphs.

2. Main Results

Theorem 2.1. *The graph $W_n * S_2$ admits prime labeling where W_n is the wheel graph if $n \not\equiv 1\,(mod\ 5)$.*

Proof. Let c_0 be the centre of the wheel graph W_n. And let $c_1, c_2, \ldots, c_n$ be the rim vertices of the wheel graph W_n. let p_0^1, p_0^2 be the pendant vertices adjacent to the central vertex c_0 and let p_i^j, $1 \leq j \leq 2$ be the pendant vertices attached at c_i for $1 \leq i \leq n$. The graph $W_n * S_2$ has $3n+3$ vertices. Define a labeling $f : V \to \{1, 2, 3, \ldots, 3n+3\}$ as follows. Let

$$
\begin{aligned}
f\left(c_0\right) = 1, \quad f\left(c_i\right) &= 3i+2, && \text{for } 1 = i = n, \\
f\left(p_0^j\right) &= j+1, && \text{for } 1 \leq j \leq 2 \\
f\left(p_i^1\right) &= 3i+1, && \text{for } 1 \leq i \leq n \\
f\left(p_i^2\right) &= 3i+3, && \text{for } 1 \leq i \leq n \\
gcd\left(f\left(c_0\right), f\left(p_0^j\right)\right) &= gcd\left(1, j+1\right) = 1, && \text{for } 1 \leq j \leq 2.
\end{aligned}
$$

For $1 \leq i \leq n$,

$$
\begin{aligned}
gcd\left(f\left(c_0\right), f\left(c_i\right)\right) &= gcd\left(1, 3i+2\right) = 1, \\
gcd\left(f\left(c_i\right), f\left(c_{i+1}\right)\right) &= gcd\left(3i+2, 3\left(i+1\right)+2\right) \\
&= gcd\left(3i+2, \left(3i+2\right)+3\right) = 1.
\end{aligned}
$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3

$$
\begin{aligned}
gcd\left(f\left(c_1\right), f\left(c_n\right)\right) &= gcd\left(5, 3n+2\right) = 1, \quad \text{for } 3n \not\equiv 3\,(mod\ 5), 3n+2 \not\equiv 0\,(mod\ 5) \\
gcd\left(f\left(c_i\right), f\left(p_i^1\right)\right) &= gcd\left(3i+2, 3i+1\right) = 1 \\
gcd\left(f\left(c_i\right), f\left(p_i^2\right)\right) &= gcd\left(3i+2, 3i+3\right) = 1
\end{aligned}
$$

as these two number are consecutive integers. Thus f is a prime labeling. Hence $W_n * S_2$ is a prime graph. □

Theorem 2.2. *The graph $W_n * S_3$ admits prime labeling where W_n is the wheel graph if $n \not\equiv 1\,(mod\ 7)$.*

Proof. Let c_0 be the centre of the wheel graph W_n. And let $c_1, c_2, \ldots c_n$ be the rim vertices of the wheel graph W_n and let p_i^j, $1 \leq j \leq 3$ be the pendant vertices attached at c_i for $0 \leq i \leq n$ respectively. The graph $W_n * S_3$ has $4n+4$ vertices and $5n+3$ edges. Define a labeling $f : V \to \{1, 2, 3, \ldots, 4n+4\}$ as follows. Let

$$
\begin{aligned}
f\left(c_0\right) = 1, \quad f\left(c_i\right) &= 4i+3, && \text{for } 1 \leq i \leq n, \\
f\left(p_0^j\right) &= j+1, && \text{for } 1 \leq j \leq 3
\end{aligned}
$$

For $1 \leq i \leq n$

$$f\left(p_i^1\right) = 4i+1, \quad f\left(p_i^2\right) = 4i+2, \quad f\left(p_i^3\right) = 4i+4.$$

$$gcd\left(f\left(c_0\right), f\left(p_0^j\right)\right) = gcd\left(1,\ j+1\right) = 1,$$

For $1 \leq i \leq n$ and $1 \leq j \leq 3$

$$gcd\left(f\left(c_0\right), f\left(c_i\right)\right) = gcd\left(1, 4i+3\right) = 1.$$

$$gcd\left(f\left(c_i\right), f\left(c_{i+1}\right)\right) = gcd\left(4i+3, 4\left(i+1\right)+3\right)$$
$$= gcd\left(4i+3, (4i+3)+4\right) = 1.$$

as these two numbers are odd and their difference is 4 both are not multiples of 3

$$gcd\left(f\left(c_1\right), f\left(c_n\right)\right) = gcd\left(7, 3n+2\right) = 1,$$

Since $n \not\equiv 1\,(mod\ 7)$, $4n \not\equiv 4\,(mod\ 7)$, $4n+3 \not\equiv 0\,(mod\ 7)$

$$gcd\left(f\left(c_i\right), f\left(p_i^1\right)\right) = gcd\left(4i+3, 4i+1\right) = 1$$

as these two number are consecutive odd integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^2\right)\right) = gcd\left(4i+3, 4i+2\right) = 1$$
$$gcd\left(f\left(c_i\right), f\left(p_i^3\right)\right) = gcd\left(4i+3, 4i+4\right) = 1$$

as these two number are consecutive integers. Thus f is a prime labeling. Hence $W_n * S_3$ is a prime graph. □

Illustration 2.3.

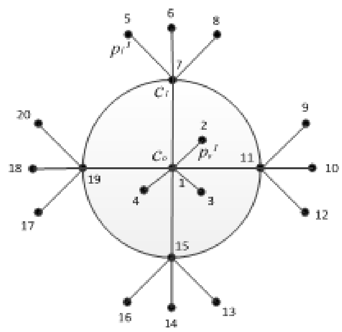

Figure 1. **Prime labeling of $W_4 * S_3$**

Theorem 2.4. *The graph $W_n * S_5$ admits prime labeling where W_n is the wheel graph if $n \not\equiv 1\,(mod\ 11)$.*

Proof. Let c_0 be the centre of the wheel graph W_n. And let $c_1, c_2, \ldots c_n$ be the rim vertices of the wheel graph W_n and let p_i^j be the pendant vertices adjacent to c_i, for $0 \leq i \leq n$ and $1 \leq j \leq 5$ respectively. The graph $W_n * S_5$ has $6n+6$ vertices and $7n+5$ edges. Define a labeling $f : V \to \{1, 2, 3, \ldots, 6n+6\}$ as follows. Let

$$f\left(c_0\right) = 1, \quad f\left(c_i\right) = 6i+5, \quad \text{for } 1 \leq i \leq n,$$

$$f\left(p_0^j\right) = j+1, \qquad\qquad \text{for } 1 \le j \le 5$$

For $1 \le i \le n$

$$f\left(p_i^j\right) = 6i+j, \quad for\ 1 \le j \le 5, \quad f\left(p_i^5\right) = 6i+6.$$

For $1 \le i \le n$ and $1 \le j \le 5$

$$\begin{aligned}
gcd\left(f\left(c_0\right), f\left(p_0^j\right)\right) &= gcd\left(1, j+1\right) = 1,\\
gcd\left(f\left(c_0\right), f\left(c_i\right)\right) &= gcd\left(1, 6i+5\right) = 1.\\
gcd\left(f\left(c_i\right), f\left(c_{i+1}\right)\right) &= gcd\left(6i+5, 6\left(i+1\right)+5\right)\\
&= gcd\left(6i+5, \left(6i+5\right)+6\right) = 1.
\end{aligned}$$

as these two numbers are odd and their difference is 6 and both are not multiples of 3, $gcd\left(f\left(c_1\right), f\left(c_n\right)\right) = gcd\left(11, 6n+5\right) = 1$, since $n \not\equiv 1\left(mod\ 11\right)$.

For $1 \le i \le n$

$$gcd\left(f\left(c_i\right), f\left(p_i^1\right)\right) = gcd\left(6i+5, 6i+1\right) = 1$$

as these two number are odd and their difference is 4

$$gcd\left(f\left(c_i\right), f\left(p_i^2\right)\right) = gcd\left(6i+5, 6i+2\right) = 1$$

as among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3

$$gcd\left(f\left(c_i\right), f\left(p_i^3\right)\right) = gcd\left(6i+5, 6i+3\right) = 1$$

as these two numbers are consecutive odd integers.

$$\begin{aligned}
gcd\left(f\left(c_i\right), f\left(p_i^4\right)\right) &= gcd\left(6i+5, 6i+4\right) = 1\\
gcd\left(f\left(c_i\right), f\left(p_i^5\right)\right) &= gcd\left(6i+5, 6i+6\right) = 1, \quad for \quad 1 \le i \le n,
\end{aligned}$$

as these two numbers are consecutive integers. Thus f is a prime labeling. Hence $W_n * S_5$ is a prime graph. □

Illustration 2.5.

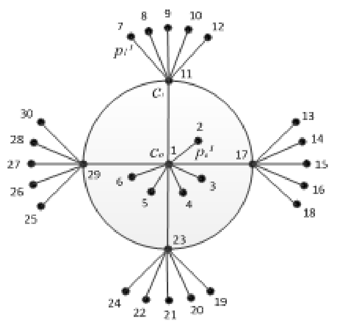

Figure 2. **Prime labeling of $W_4 * S_5$**

Theorem 2.6. *The graph $W_n * S_7$ admits prime labeling where W_n is the wheel graph if $n \not\equiv 11\,(mod\ 39)$, $n \not\equiv 27\,(mod\ 39)$ and $n \not\equiv 40\,(mod\ 39)$.*

Proof. Let c_0 be the centre of the wheel graph W_n. And let $c_1, c_2, \ldots, c_n$ be the rim vertices of the wheel graph W_n and let p_i^j be the pendant vertices adjacent to c_i, for $0 \le i \le n$ and $1 \le j \le 7$ respectively. The graph $W_n * S_7$ has $8n+8$ vertices and $9n+7$ edges. Define a labeling $f : V \to \{1, 2, 3, \ldots, 8n+8\}$ as follows. Let $f(c_0) = 1$, $f\left(p_0^j\right) = j+1$, for $1 \le j \le 7$. For $1 \le i \le n$

$$
\begin{aligned}
f(c_i) &= 8i+5, && \text{for } i \not\equiv 2\,(mod\ 3)\\
f(c_i) &= 8i+3, && \text{for } i \equiv 2\,(mod\ 3) \text{ and } i+1 \not\equiv 0\,(mod\ 15)\\
f(c_i) &= 8i+1, && \text{for } i+1 \equiv 0\,(mod\ 15)\\
f\left(p_i^1\right) &= 8i+1, && \text{for } i+1 \not\equiv 0\,(mod\ 15)\\
f\left(p_i^1\right) &= 8i+3, && \text{for } i+1 \equiv 0\,(mod\ 15)\\
f\left(p_i^2\right) &= 8i+2, &&\\
f\left(p_i^3\right) &= 8i+3, && \text{for } i \not\equiv 2\,(mod\ 3)\\
f\left(p_i^3\right) &= 8i+5, && \text{for } i \equiv 2\,(mod\ 3)\\
f\left(p_i^4\right) &= 8i+4, &&\\
f\left(p_i^j\right) &= 8i+(j+1), && \text{for } 5 \le j \le 7\\
gcd\left(f(c_0), f\left(p_0^j\right)\right) &= gcd(1, j+1) = 1, && \text{for } 1 \le j \le 7
\end{aligned}
$$

For $1 \le i \le n$ and $i \not\equiv 2\,(mod\ 3)$

$$
\begin{aligned}
gcd(f(c_0), f(c_i)) &= gcd(1, 8i+5) = 1,\\
gcd(f(c_i), f(c_{i+1})) &= gcd(8i+5, 8(i+1)+5)\\
&= gcd(8i+5, (8i+5)+8) = 1.
\end{aligned}
$$

as these two numbers are odd and their difference is 8 and both are not multiples of 3 or 5.

$$gcd\left(f(c_i), f\left(p_i^2\right)\right) = gcd(8i+5, 8i+2) = 1,$$

among these two numbers one is odd and other is even and their difference is 3 and both are not multiples of 2 or 3or 4 or 5.

$$gcd\left(f(c_i), f\left(p_i^3\right)\right) = gcd(8i+5,\ 8i+3) = 1,$$

as these two numbers are consecutive odd integers.

$$
\begin{aligned}
gcd\left(f(c_i), f\left(p_i^4\right)\right) &= gcd(8i+5, 8i+4) = 1,\\
gcd\left(f(c_i), f\left(p_i^5\right)\right) &= gcd(8i+5, 8i+6) = 1,
\end{aligned}
$$

as these two numbers are consecutive integers.

$$gcd\left(f(c_i), f\left(p_i^6\right)\right) = gcd(8i+5, 8i+7) = 1,$$

as these two numbers are consecutive odd integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^7\right)\right) = gcd\left(8i+5,\ 8i+8\right) = 1,$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 2 or 3 or 5.
For $1 \leq i \leq n$, $i \equiv 2\,(mod\ 3)$ and $i+1 \not\equiv 0\,(mod\ 15)$

$$\begin{aligned} gcd\left(f\left(c_0\right), f\left(c_i\right)\right) &= gcd\left(1, 8i+3\right) = 1, \\ gcd\left(f\left(c_i\right), f\left(c_{i+1}\right)\right) &= gcd\left(8i+3, 8\left(i+1\right)+5\right) \\ &= gcd\left(8i+3, \left(8i+3\right)+10\right) = 1, \end{aligned}$$

as these two numbers are odd and their difference is 10 and both are not multiples of 3 or 5.

$$\begin{aligned} gcd\left(f\left(c_{i-1}\right), f\left(c_i\right)\right) &= gcd\left(8\left(i-1\right)+5, 8i+3\right) \\ &= gcd\left(8i+3\right)-6, 8i+3) = 1, \end{aligned}$$

as these two numbers are odd and their difference is 6 and both are not multiples of 3 or 5.

$$\begin{aligned} gcd\left(f\left(c_i\right), f\left(p_i^2\right)\right) &= gcd\left(8i+3, 8i+2\right) = 1, \\ gcd\left(f\left(c_i\right), f\left(p_i^4\right)\right) &= gcd\left(8i+3, 8i+4\right) = 1, \end{aligned}$$

these two numbers are consecutive integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^5\right)\right) = gcd\left(8i+3, \left(8i+3\right)+3\right) = 1,$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 2 or 3 or 5.

$$gcd\left(f\left(c_i\right), f\left(p_i^6\right)\right) = gcd\left(8i+3,\ 8i+7\right) = 1,$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 or 5.

$$gcd\left(f\left(c_i\right), f\left(p_i^7\right)\right) = gcd\left(8i+3,\ 8i+8\right) = 1,$$

among these two numbers one is odd and other is even and their difference is 5 and they are not multiples of 3 or 5.

$$\begin{aligned} gcd\left(f\left(c_i\right), f\left(p_i^1\right)\right) &= gcd\left(8i+3, 8i+1\right) = 1, \\ gcd\left(f\left(c_i\right), f\left(p_i^3\right)\right) &= gcd\left(8i+3,\ 8i+5\right) = 1. \end{aligned}$$

as these two numbers are consecutive odd integers.
For $1 \leq i \leq n$ and $i+1 \equiv 0\,(mod\ 15)$

$$\begin{aligned} gcd\left(f\left(c_0\right), f\left(c_i\right)\right) &= gcd\left(1,\ 8i+1\right) = 1, \\ gcd\left(f\left(c_i\right), f\left(c_{i+1}\right)\right) &= gcd\left(8i+1, 8\left(i+1\right)+5\right) \end{aligned}$$

$$= gcd\,(8i+1,(8i+1)+12) = 1.$$

as these two numbers are odd and their difference is 12 and both are not multiples of 3 or 5.

$$gcd\,(f\,(c_{i-1})\,,f\,(c_i)) = gcd\,(8\,(i-1)+5, 8i+1)$$
$$= gcd\,(8i+1)-4, 8i+1) = 1,$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 or 5.

$$gcd\left(f\,(c_i)\,,f\left(p_i^1\right)\right) = gcd\,(8i+1, 8i+3) = 1,$$

as these two numbers are consecutive odd integers

$$gcd\left(f\,(c_i)\,,f\left(p_i^2\right)\right) = gcd\,(8i+1, 8i+2) = 1,$$

as these two numbers are consecutive integers

$$gcd\left(f\,(c_i)\,,f\left(p_i^4\right)\right) = gcd\,(8i+1, 8i+4) = 1,$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3 or 5.

$$gcd\left(f\,(c_i)\,,f\left(p_i^5\right)\right) = gcd\,(8i+1, 8i+6) = 1,$$

among these two numbers one is odd and other is even and their difference is 5 and they are not multiples of 3.

$$gcd\left(f\,(c_i)\,,f\left(p_i^6\right)\right) = gcd\,(8i+1, 8i+7) = 1,$$

as these two numbers are odd and their difference is 6 and both are not multiples of 3 or 5.

$$gcd\left(f\,(c_i)\,,f\left(p_i^7\right)\right) = gcd\,(8i+1, 8i+8) = 1,$$

among these two numbers one is odd and other is even and their difference is 7 and they are not multiples of 7.

For $1 \leq i \leq n$, $i \not\equiv 2\,(mod\ 3)$ and $i+1 \not\equiv 0\,(mod\ 15)$

$$gcd\left(f\,(c_i)\,,f\left(p_i^1\right)\right) = gcd\,(8i+5, 8i+1) = 1,$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 or 5.

For $1 \leq i \leq n$, $i \equiv 2\,(mod\ 3)$ and $i+1 \equiv 0\,(mod\ 15)$

$$gcd\left(f\,(c_i)\,,f\left(p_i^3\right)\right) = gcd\,(8i+1, 8i+5) = 1.$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 or 5. Thus f is a prime labeling. Hence $W_n * S_7$ is a prime graph. □

Illustration 2.7.

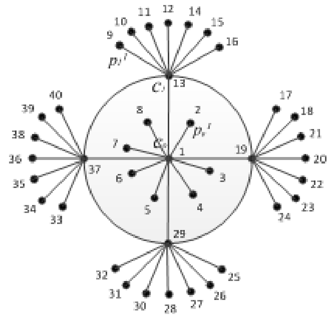

Figure 3. **Prime labeling of $W_4 * S_7$**

Theorem 2.8. *The graph $C_n^* * S_2$ admits prime labeling where C_n^* is the crown graph.*

Proof. Let $c_1, c_2, \ldots, c_n$ be the rim vertices of the crown graph C_n^* and let $p_1, p_2, \ldots p_n$ be the pendant vertices adjacent to $c_1, c_2, \ldots c_n$ respectively. Let p_i^j and l_i^j be the pendant vertices adjacent to c_i and p_i respectively for $1 \le i \le n$ and $1 \le j \le 2$. The graph $C_n^* * S_2$ has $6n$ vertices and edges. Define a labeling $f : V \to \{1, 2, 3, \ldots, 6n\}$ as follows.

For $1 \le i \le n$,

$$f(c_i) = 6i - 5, \quad f(p_i) = 6i - 1, \quad f\left(p_i^1\right) = 6i - 4,$$
$$f\left(p_i^2\right) = 6i - 3, \quad f\left(l_i^1\right) = 6i - 2, \quad f\left(l_i^2\right) = 6i.$$

For $1 \le i \le n$,

$$\begin{aligned} gcd\left(f(c_i), f(c_{i+1})\right) &= gcd\left(6i - 5, 6(i+1) - 5\right) \\ &= gcd\left(6i - 5, (6i - 5) + 6\right) = 1. \end{aligned}$$

as these two numbers are odd and their difference is 6 and both are not multiples of 3

$$gcd\left(f(c_1), f(c_n)\right) = gcd\left(1, 6n - 5\right) = 1,$$
$$gcd\left(f(c_i), f(p_i)\right) = gcd\left(6i - 5, 6i - 1\right) = 1$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3

$$gcd\left(f(c_i), f\left(p_i^1\right)\right) = gcd\left(6i - 5, 6i - 4\right) = 1$$
$$gcd\left(f(p_i), f\left(l_i^1\right)\right) = gcd\left(6i - 1, 6i - 2\right) = 1$$
$$gcd\left(f(p_i), f\left(l_i^2\right)\right) = gcd\left(6i - 1, 6i\right) = 1$$

as these two numbers are consecutive integers

$$gcd\left(f(c_i), f\left(p_i^2\right)\right) = gcd\left(6i - 5, 6i - 3\right) = 1$$

as these two numbers are consecutive odd integers. Thus f is a prime labeling. Hence $C_n^* * S_2$ is a prime graph. □

Theorem 2.9. *The graph $C_n^* * S_3$ admits prime labeling where C_n^* is the crown graph.*

Proof. Let $c_1, c_2, \ldots, c_n$ be the rim vertices of the crown graph C_n^* and let $p_1, p_2, \ldots, p_n$ be the pendant vertices attached at $c_1, c_2, \ldots, c_n$ respectively. Let p_i^j and l_i^j be the pendant vertices attached at c_i and p_i respectively for $1 \le i \le n$ and $1 \le j \le 3$ The graph $C_n^* * S_3$ has $8n$ vertices and edges. Define a labeling $f : V \to \{1, 2, 3, \ldots, 8n\}$ as follows.

$$f(c_1) = 1, \quad f\left(p_1^1\right) = 2, \quad f\left(p_1^2\right) = 3, \quad f\left(p_1^3\right) = 4$$

For $2 \le i \le n$,

$$\begin{aligned} f(c_i) = 8i - 5, \quad f(p_i) = 8i - 1, \quad f\left(p_i^1\right) = 8i - 7, \quad f\left(p_i^2\right) = 8i - 6, \\ f\left(p_i^3\right) = 8i - 4, \quad f\left(l_i^1\right) = 8i - 3, \quad f\left(l_i^2\right) = 8i - 2, \quad f\left(l_i^3\right) = 8i \end{aligned}$$

For $1 \le i \le n$,

$$\begin{aligned} gcd\left(f(c_i), f(c_{i+1})\right) &= gcd\left(8i - 5, 8(i+1) - 5\right) \\ &= gcd\left(8i - 5, (8i - 5) + 8\right) = 1. \end{aligned}$$

as these two numbers are odd and their difference is 8 and both are not multiples of 3 or 5.

$$\begin{aligned} gcd\left(f(c_1), f(c_n)\right) &= gcd\left(1, 8n - 5\right) = 1, \\ gcd\left(f(c_1), f(p_1)\right) &= gcd(1, 7) = 1 \\ gcd\left(f(c_1), f\left(p_1^1\right)\right) &= gcd(1, 2) = 1, \\ gcd\left(f(c_1), f\left(p_1^2\right)\right) &= gcd(1, 3) = 1 \\ gcd\left(f(c_1), f\left(p_1^3\right)\right) &= gcd(1, 4) = 1. \end{aligned}$$

For $1 \le i \le n$,

$$gcd\left(f(c_i), f\left(p_i^1\right)\right) = gcd\left(8i - 5, 8i - 7\right) = 1$$

as these two numbers are consecutive odd integers.

$$\begin{aligned} gcd\left(f(c_i), f\left(p_i^2\right)\right) &= gcd\left(8i - 5, 8i - 6\right) = 1 \\ gcd\left(f(c_i), f\left(p_i^3\right)\right) &= gcd\left(8i - 5, 8i - 4\right) = 1 \\ gcd\left(f(p_i), f\left(l_i^2\right)\right) &= gcd\left(8i - 1, 8i - 2\right) = 1 \\ gcd\left(f(p_i), f\left(l_i^3\right)\right) &= gcd\left(8i - 1, 8i\right) = 1 \end{aligned}$$

as these two numbers are consecutive integers

$$gcd\left(f(p_i), f\left(l_i^1\right)\right) = gcd\left(8i - 1, 8i - 3\right) = 1$$

as these two numbers are consecutive odd integers

$$gcd\left(f(c_i), f(p_i)\right) = gcd\left(8i - 5, 8i - 1\right) = 1$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 or 5. Thus f is a prime labeling. Hence $C_n^* * S_3$ is a prime graph. □

Theorem 2.10. *The graph $C_n^* * S_5$ admits prime labeling where C_n^* is the crown graph.*

Proof. Let $c_1, c_2, \ldots c_n$ be the rim vertices of the crown graph C_n^* and let $p_1, p_2, \ldots, p_n$ be the pendant vertices attached at $c_1, c_2, \ldots, c_n$ respectively. Let p_i^j and l_i^j be the pendant vertices attached at c_i and p_i respectively for $1 \le i \le n$ and $1 \le j \le 5$. The graph $C_n^* * S_5$ has $12n$ vertices and edges. Define a labeling $f : V \to \{1, 2, 3, \ldots, 12n\}$ as follows.

$$f(c_1) = 1, \quad f\left(p_1^j\right) = j + 1, \quad for \quad 1 \le j \le 5,$$

For $2 \le i \le n$,

$$\begin{aligned}
f(c_i) = 12i - 7, \quad f(p_i) &= 12i - 5, && \text{for } i \not\equiv 0\,(mod\ 5)\\
f\left(p_i^j\right) &= 12i - (12 - j), && \text{for } 1 \le j \le 4\\
f\left(l_i^j\right) &= 12i - (5 - j), && \text{for } 1 \le j \le 4\\
f\left(l_i^5\right) &= 12i, && \text{for } i \not\equiv 0\,(mod\ 5)\\
f\left(l_i^4\right) &= 12i - 5, && \text{for } i \equiv 0\,(mod\ 5)\\
f(p_i) &= 12i - 1, && \text{for } i \equiv 0\,(mod\ 5)\\
gcd\left(f(c_1), f(p_1)\right) &= gcd(1, 7) = 1, &&\\
gcd\left(f(c_1), f\left(p_1^j\right)\right) &= gcd(1, j + 1) = 1, && \text{for } 1 \le j \le 5.\\
gcd\left(f(c_1), f(c_n)\right) &= gcd(1, 12n - 7) = 1, &&
\end{aligned}$$

For $2 \le i \le n$,

$$\begin{aligned}
gcd\left(f(c_i), f(c_{i+1})\right) &= gcd\left(12i - 7,\ 12(i + 1) - 7\right)\\
&= gcd\left(12i - 7,\ (12i - 7) + 12\right) = 1,
\end{aligned}$$

as these two numbers are odd and their difference is 12 and both are not multiples of 3.

$$\begin{aligned}
gcd\left(f(c_i), f(p_i)\right) &= gcd(12i - 7, 12i - 5) = 1, \quad for \quad i \not\equiv 0\,(mod\ 5)\\
gcd\left(f(c_i), f\left(p_i^1\right)\right) &= gcd(12i - 7, 12i - 11) = 1
\end{aligned}$$

as these two numbers are odd and their difference is 4 and both are not multiples of 4.

$$gcd\left(f(c_i), f\left(p_i^2\right)\right) = gcd(12i - 7, 12i - 10) = 1$$

as among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3.

$$gcd\left(f(c_i), f\left(p_i^3\right)\right) = gcd(12i - 7, 12i - 9) = 1$$

as these two numbers are consecutive odd integers.

$$\begin{aligned}
gcd\left(f(c_i), f\left(p_i^4\right)\right) &= gcd(12i - 7, 12i - 8) = 1\\
gcd\left(f(c_i), f\left(p_i^5\right)\right) &= gcd(12i - 7, 12i - 6) = 1
\end{aligned}$$

as these two numbers are consecutive integers.

For $1 \leq i \leq n$ and $i \not\equiv 0\,(mod\ 5)$,

$$gcd\left(f\left(p_i\right), f\left(l_i^1\right)\right) = gcd\left(12i-5, 12i-4\right) = 1, \quad for \ \ i \not\equiv 0\,(mod\ 5)$$

as these two numbers are consecutive integers

$$gcd\left(f\left(p_i\right), f\left(l_i^2\right)\right) = gcd\left(12i-5,\ 12i-3\right) = 1$$

as these two numbers are consecutive odd integers.

$$gcd\left(f\left(p_i\right), f\left(l_i^3\right)\right) = gcd\left(12i-5,\ 12i-2\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3

$$gcd\left(f\left(p_i\right), f\left(l_i^4\right)\right) = gcd\left(12i-5,\ 12i-1\right) = 1,$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 or 5.

$$gcd\left(f\left(p_i\right), f\left(l_i^5\right)\right) = gcd\left(12i-5,\ 12i\right) = 1,$$

as among these two numbers one is odd and other is even and their difference is 5 and they are not multiples of 5.

For $1 \leq i \leq n$ and $i \equiv 0\,(mod\ 5)$,

$$gcd\left(f\left(p_i\right), f\left(l_i^4\right)\right) = gcd\left(12i-1,\ 12i-5\right) = 1,$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 or 5.

$$gcd\left(f\left(p_i\right), f\left(l_i^5\right)\right) = gcd\left(12i-1, 12i\right) = 1$$

as these two numbers are consecutive integers. Thus f is a prime labeling. Hence $C_n^* * S_5$ is a prime graph. □

Illustration 2.11.

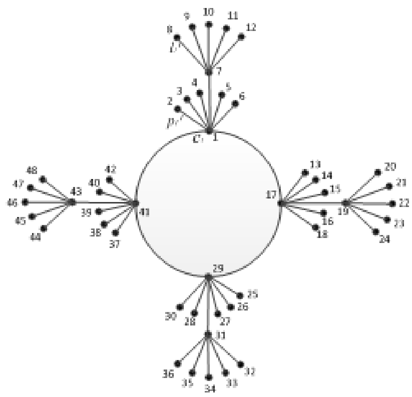

Figure 4. **Prime labeling of $C_4^* * S_5$**

Theorem 2.12. *The graph $C_n^* * S_7$ admits prime labeling where C_n^* is the crown graph.*

Proof. Let $c_1, c_2, \ldots, c_n$ be the rim vertices of the crown graph C_n^* and let $p_1, p_2, \ldots, p_n$ be the pendant vertices attached at $c_1, c_2, \ldots, c_n$ respectively. Let p_i^j and l_i^j be the pendant vertices attached at c_i and p_i respectively for $1 \leq i \leq n$ and $1 \leq j \leq 7$. The graph $C_n^* * S_7$ has $16n$ vertices and edges. Define a labeling $f : V \to \{1, 2, 3, \ldots, 16n\}$ as follows.

$$f(c_1) = 1, \quad f\left(p_1^j\right) = j + 1, \quad for \quad 1 \leq j \leq 7,$$

For $2 \leq i \leq n$,

$$\begin{aligned}
f(c_i) &= 16i - 11, && \text{for } i \not\equiv 2\,(mod\ 3) \\
f(c_i) &= 16i - 9, && \text{for } i \equiv 2\,(mod\ 3),\ i \not\equiv 8\,(mod\ 63) \text{ and } i \not\equiv 14\,(mod\ 15) \\
f(c_i) &= 16i - 15, && \text{for } i \equiv 8\,(mod\ 63), \text{ (or) } i \equiv 14\,(mod\ 15),\ i \equiv 2\,(mod\ 3) \\
f(p_i) &= 16i - 3, && \text{for } i \not\equiv 0\,(mod\ 3) \\
f(p_i) &= 16i - 5, && \text{for } i \equiv 0\,(mod\ 3),\ i \not\equiv 0\,(mod\ 15) \\
f(p_i) &= 16i - 7, && \text{for } i \equiv 0\,(mod\ 15),\ i \equiv 0\,(mod\ 3)
\end{aligned}$$

For $2 \leq i \leq n$,

$$\begin{aligned}
f\left(p_i^1\right) &= 16i - 15, && \text{for } i \not\equiv 8\,(mod\ 63),\ i \not\equiv 14\,(mod\ 15) \\
f\left(p_i^1\right) &= 16i - 9, && \text{for } i \equiv 8\,(mod\ 63),\ i \equiv 14\,(mod\ 15) \\
f\left(p_i^2\right) &= 16i - 14, && f\left(p_i^3\right) = 16i - 13, \quad f\left(p_i^4\right) = 16i - 12, \quad f\left(p_i^5\right) = 16i - 10 \\
f\left(p_i^6\right) &= 16i - 9, && \text{for } i \not\equiv 2\,(mod\ 3) \\
f\left(p_i^6\right) &= 16i - 11, && \text{for } i \equiv 2\,(mod\ 3) \\
f\left(p_i^7\right) &= 16i - 8
\end{aligned}$$

For $1 \leq i \leq n$,

$$\begin{aligned}
& f\left(l_i^1\right) = 16i - 7, \ \text{for } i \not\equiv 0\,(mod\ 15),\ f\left(l_i^1\right) = 16i - 5, \text{ for } i \equiv 0\,(mod\ 15) \\
& f\left(l_i^2\right) = 16i - 6, \\
& f\left(l_i^3\right) = 16i - 5, \ \text{for } i \not\equiv 0\,(mod\ 3),\ f\left(l_i^3\right) = 16i - 3, \text{ for } i \equiv 0\,(mod\ 3) \\
& f\left(l_i^4\right) = 16i - 4, \quad f\left(l_i^5\right) = 16i - 2, \quad f\left(l_i^6\right) = 16i - 1, \quad f\left(l_i^7\right) = 16i.
\end{aligned}$$

For $2 \leq i \leq n$,

$$\begin{aligned}
gcd\left(f(c_i), f(c_{i+1})\right) &= gcd\left(16i - 11, 16(i+1) - 11\right) \\
&= gcd\left(16i - 11, (16i - 11) + 16\right) = 1, \quad \text{for } i \not\equiv 2\,(mod\ 3)
\end{aligned}$$

as these two numbers are odd and their difference is 16 and both are not multiples of 3,5,7,11 or 13.

$$gcd\left(f(c_i), f(c_{i+1})\right) = gcd\left(16i - 9, 16(i+1) - 11\right)$$

$$= gcd\left(16i-9,(16i-9)+14\right)=1,$$

Since $i \equiv 2\,(mod\ 3)$, $i \not\equiv 8\,(mod\ 63)$, $i \not\equiv 14\,(mod\ 15)$ as these two numbers are odd and their difference is 14 and both are not multiples of 3,5,7,11 or 13.

$$gcd\left(f\left(c_i\right), f\left(c_{i+1}\right)\right) = gcd\left(16i-15, 16\left(i+1\right)-11\right)$$
$$= gcd\left(16i-15,(16i-15)+20\right)=1,$$

Since $i \equiv 8\,(mod\ 63)$, $i \equiv 14\,(mod\ 15)$ as these two numbers are odd and their difference is 20 and both are not multiples of 3,5,7,11,13 or 17.

$$gcd\left(f\left(c_1\right), f\left(p_1^j\right)\right) = gcd\left(1, j+1\right) = 1, \quad for \quad 1 \le j \le 7,$$
$$gcd\left(f\left(c_1\right), f\left(c_2\right)\right) = gcd\left(1, 23\right) = 1,$$
$$gcd\left(f\left(c_1\right), f\left(p_1\right)\right) = gcd\left(1, 13\right) = 1,$$
$$gcd\left(f\left(c_1\right), f\left(c_n\right)\right) = gcd\left(1, 16n-11\right) = 1, \quad for \quad n \not\equiv 2\left(mod\ 3\right).$$
$$gcd\left(f\left(c_1\right), f\left(c_n\right)\right) = gcd\left(1, 16n-9\right) = 1,$$

Since $n \equiv 2\,(mod\ 3)$, $n \not\equiv 8\,(mod\ 63)$, $n \not\equiv 14\,(mod\ 15)$.

$$gcd\left(f\left(c_1\right), f\left(c_n\right)\right) = gcd\left(1, 16n-15\right) = 1,$$

Since $n \equiv 8\,(mod\ 63)$, $n \equiv 14\,(mod\ 15)$.

For $2 \le i \le n$,

$$gcd\left(f\left(c_i\right), f\left(p_i\right)\right) = gcd\left(16i-11, 16i-3\right) = 1,$$

Since $i \not\equiv 2\,(mod\ 3)$, $i \not\equiv 0\,(mod\ 3)$ as these two numbers are odd and their difference is 8 and both are not multiples of 3 or 5 or 7.

$$gcd\left(f\left(c_i\right), f\left(p_i\right)\right) = gcd\left(16i-9, 16i-3\right) = 1,$$

Since $i \equiv 2\,(mod\ 3)$, $i \not\equiv 8\,(mod\ 63)$, $i \not\equiv 14\,(mod\ 15)$.

$$gcd\left(f\left(c_i\right), f\left(p_i\right)\right) = gcd\left(16i-11, 16i-5\right) = 1,$$

Since $i \equiv 0\,(mod\ 3)$, $i \not\equiv 0\,(mod\ 15)$ as these two numbers are odd and their difference is 6 and both are not multiples of 3 or 5.

$$gcd\left(f\left(c_i\right), f\left(p_i\right)\right) = gcd\left(16i-11, 16i-7\right) = 1, \quad for \quad i \equiv 0\left(mod\ 15\right)$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3.

$$gcd\left(f\left(c_i\right), f\left(p_i\right)\right) = gcd\left(16i-15, 16i-3\right),$$
$$= 1, \quad for\ i \equiv 8\left(mod\ 63\right), \quad i \equiv 14\left(mod\ 15\right)$$

as these two numbers are odd and their difference is 12 and both are not multiples of 3 or 5 or 7 or 11.

For $2 \leq i \leq n$ and $i \not\equiv 2\,(mod\ 3)$,

$$gcd\left(f\left(c_i\right), f\left(p_i^1\right)\right) = gcd\left(16i - 11, 16i - 15\right) = 1$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3.

$$gcd\left(f\left(c_i\right), f\left(p_i^2\right)\right) = gcd\left(16i - 11, 16i - 14\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3.

$$gcd\left(f\left(c_i\right), f\left(p_i^3\right)\right) = gcd\left(16i - 11, 16i - 13\right) = 1$$

as these two numbers are consecutive odd integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^4\right)\right) = gcd\left(16i - 11, 16i - 12\right) = 1$$
$$gcd\left(f\left(c_i\right), f\left(p_i^5\right)\right) = gcd\left(16i - 11, 16i - 10\right) = 1$$

as these two numbers are consecutive integers

$$gcd\left(f\left(c_i\right), f\left(p_i^6\right)\right) = gcd\left(16i - 11, 16i - 9\right) = 1$$

as these two numbers are consecutive odd integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^7\right)\right) = gcd\left(16i - 11, 16i - 8\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3.
For $1 \leq i \leq n$ and $i \not\equiv 0\,(mod\ 3)$,

$$gcd\left(f\left(p_i\right), f\left(l_i^1\right)\right) = gcd\left(16i - 3, 16i - 7\right) = 1,$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3.

$$gcd\left(f\left(p_i\right), f\left(l_i^2\right)\right) = gcd\left(16i - 3, 16i - 6\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3

$$gcd\left(f\left(p_i\right), f\left(l_i^3\right)\right) = gcd\left(16i - 3, 16i - 5\right) = 1$$

as these two numbers are consecutive odd integers

$$gcd\left(f\left(p_i\right), f\left(l_i^4\right)\right) = gcd\left(16i - 3, 16i - 4\right) = 1,$$
$$gcd\left(f\left(p_i\right), f\left(l_i^5\right)\right) = gcd\left(16i - 3, 16i - 2\right) = 1$$

as these two numbers are consecutive integers

$$gcd\left(f\left(p_i\right), f\left(l_i^6\right)\right) = gcd\left(16i - 3, 16i - 1\right) = 1$$

as these two numbers are consecutive odd integers

$$gcd\left(f\left(p_i\right), f\left(l_i^7\right)\right) = gcd\left(16i-3, 16i\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3.

For $2 \leq i \leq n$ and $i \equiv 2\,(mod\ 3)$, $i \not\equiv 8\,(mod\ 63)$, $i \not\equiv 14\,(mod\ 15)$,

$$gcd\left(f\left(c_i\right), f\left(p_i^1\right)\right) = gcd\left(16i-9, 16i-15\right) = 1$$

as these two numbers are odd and their difference is 6 and both are not multiples of 3 or 5.

$$gcd\left(f\left(c_i\right), f\left(p_i^2\right)\right) = gcd\left(16i-9, 16i-14\right) = 1$$

among these two numbers one is odd and other is even and their difference is 5 and they are not multiples of 5.

$$gcd\left(f\left(c_i\right), f\left(p_i^3\right)\right) = gcd\left(16i-9, 16i-13\right) = 1$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3.

$$gcd\left(f\left(c_i\right), f\left(p_i^4\right)\right) = gcd\left(16i-9, 16i-12\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3.

$$gcd\left(f\left(c_i\right), f\left(p_i^5\right)\right) = gcd\left(16i-9, 16i-10\right) = 1$$

as these two numbers are consecutive integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^6\right)\right) = gcd\left(16i-9, 16i-11\right) = 1$$

as these two numbers are consecutive odd integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^7\right)\right) = gcd\left(16i-9, 16i-8\right) = 1$$

as these two numbers are consecutive integers.

For $2 \leq i \leq n$ and $i \equiv 8\,(mod\ 63)$, $i \equiv 14\,(mod\ 15)$,

$$gcd\left(f\left(c_i\right), f\left(p_i^1\right)\right) = gcd\left(16i-15, 16i-9\right) = 1$$

as these two numbers are odd and their difference is 6 and both are not multiples of 3or 5.

$$gcd\left(f\left(c_i\right), f\left(p_i^2\right)\right) = gcd\left(16i-15, 16i-14\right) = 1$$

as these two numbers are consecutive integers

$$gcd\left(f\left(c_i\right), f\left(p_i^3\right)\right) = gcd\left(16i-15, 16i-13\right) = 1$$

as these two numbers are consecutive odd integers.

$$gcd\left(f\left(c_i\right), f\left(p_i^4\right)\right) = gcd\left(16i - 15, 16i - 12\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3.

$$gcd\left(f\left(c_i\right), f\left(p_i^5\right)\right) = gcd\left(16i - 15, 16i - 10\right) = 1$$

among these two numbers one is odd and other is even and their difference is 5 and they are not multiples of 5.

$$gcd\left(f\left(c_i\right), f\left(p_i^6\right)\right) = gcd\left(16i - 15, 16i - 11\right) = 1, \quad for \quad i \equiv 2\left(mod\ 3\right)$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3.

$$gcd\left(f\left(c_i\right), f\left(p_i^7\right)\right) = gcd\left(16i - 15, 16i - 8\right) = 1$$

among these two numbers one is odd and other is even and their difference is 7 and they are not multiples of 7.

For $1 \leq i \leq n$ and $i \equiv 0\left(mod\ 3\right)$, $i \not\equiv 0\left(mod\ 15\right)$,

$$gcd\left(f\left(p_i\right), f\left(l_i^1\right)\right) = gcd\left(16i - 5, 16i - 7\right) = 1,$$

as these two numbers are consecutive odd integers

$$gcd\left(f\left(p_i\right), f\left(l_i^2\right)\right) = gcd\left(16i - 5, 16i - 6\right) = 1$$

as these two numbers are consecutive integers.

$$gcd\left(f\left(p_i\right), f\left(l_i^3\right)\right) = gcd\left(16i - 5, 16i - 3\right) = 1$$

as these two numbers are consecutive odd integers

$$gcd\left(f\left(p_i\right), f\left(l_i^4\right)\right) = gcd\left(16i - 5,\ 16i - 4\right) = 1,$$

as these two numbers are consecutive integers.

$$gcd\left(f\left(p_i\right), f\left(l_i^5\right)\right) = gcd\left(16i - 5, 16i - 2\right) = 1$$

among these two numbers one is odd and other is even and their difference is 3 and they are not multiples of 3.

$$gcd\left(f\left(p_i\right), f\left(l_i^6\right)\right) = gcd\left(16i - 5, 16i - 1\right) = 1$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3 .

$$gcd\left(f\left(p_i\right), f\left(l_i^7\right)\right) = gcd\left(16i - 5, 16i\right) = 1$$

among these two numbers one is odd and other is even and their difference is 6 and both are not multiples of 6.

For $1 \leq i \leq n$ and $i \equiv 0\,(mod\ 15)$,

$$gcd\left(f\left(p_i\right), f\left(l_i^1\right)\right) = gcd\left(16i-7, 16i-5\right) = 1,$$

as these two numbers are consecutive odd integers

$$gcd\left(f\left(p_i\right), f\left(l_i^2\right)\right) = gcd\left(16i-7, 16i-6\right) = 1$$

as these two numbers are consecutive integers.

$$gcd\left(f\left(p_i\right), f\left(l_i^3\right)\right) = gcd\left(16i-7, 16i-3\right) = 1,$$

as these two numbers are odd and their difference is 4 and both are not multiples of 3.

$$gcd\left(f\left(p_i\right), f\left(l_i^4\right)\right) = gcd\left(16i-7, 16i-4\right) = 1,$$

among these two numbers one is odd and other is even and their difference is 3 and both are not multiples of 3

$$gcd\left(f\left(p_i\right), f\left(l_i^5\right)\right) = gcd\left(16i-7, 16i-2\right) = 1,$$

among these two numbers one is odd and other is even and their difference is 5 and they are not multiples of 5

$$gcd\left(f\left(p_i\right), f\left(l_i^6\right)\right) = gcd\left(16i-7, 16i-1\right) = 1,$$

as these two numbers are odd and their difference is 6 and both are not multiples of 3 or 5.

$$gcd\left(f\left(p_i\right), f\left(l_i^7\right)\right) = gcd\left(16i-7, 16i\right) = 1.$$

among these two numbers one is odd and other is even and their difference is 7 and they are not multiples of 7. Thus f is a prime labeling. Hence $C_n^* * S_7$ is a prime graph. □

Illustration 2.13.

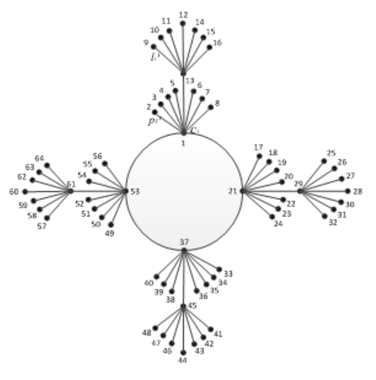

Figure 5. **Prime labeling of $C_4^* * S_7$**

3. Conclusion

Labeled graph is the topic of current due to its diversified application. We investigate eight new results on prime labeling of graphs. It is an effort to relate the prime labeling and some graph operations. This approach is novel as it provides prime labeling for the larger graph resulted due to certain graph operations on a given graph.. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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