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Sum Divisor Cordial Labeling On Some Special Graphs

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Abstract: A sum divisor cordial labeling of a graph G with vertex set V is a bijection $f : V(G) \to \{1, 2, ..., |V(G)|\}$ such that each edge uv assigned the label 1 if 2 divides $f(u) + f(v)$ and 0 otherwise. Further, the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that the plus graph, umbrella graph, path union of odd cycles, kite and complete binary tree are sum divisor cordial graphs.

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Keywords: Divisor cordial labeling, Sum divisor cordial labeling.

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1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory, we refer to Harary [2]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc.
Varatharajan [10] introduced the concept of divisor cordial labeling. For dynamic survey of various graph labeling, we refer to Gallian [1]. Lourdusamy and Patrick [5] introduced the concept of sum divisor cordial labeling. Sugumaran and Rajesh [6] proved that Swastik graph Sw_n, path union of finite copies of Swastik graph Sw_n, cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n, n)$ and Petersen graph are sum divisor cordial graphs. Sugumaran and Rajesh [7] proved that the Theta graph and some graph operations in Theta graph are sum divisor cordial graphs. Sugumaran and Rajesh [8] proved that the Herschel graph and some graph operations in Herschel graph are sum divisor cordial graphs. Sugumaran and Rajesh [9] proved that H_n (n is odd), $C_3@K_{1,n}$, $< F_n^1 \Delta F_n^2 >$, open star of Swastik graph $S(t.Sw_n)$, when t is odd are sum divisor cordial graphs. In this paper we investigate sum divisor cordial labeling of graphs such as plus graph, umbrella graph, path union of odd cycles, kite and full binary tree.

Definition 1.1 ([10]). *Let $G = (V(G), E(G))$ be a simple graph and let $f : V(G) \to \{1, 2,..., |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u)|f(v)$ or $f(v)|f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph.*

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Definition 1.2 ([5]). *Let $G = (V(G), E(G))$ be a simple graph and let $f : V(G) \to \{1, 2, ..., |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $2|(f(u) + f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.*

Definition 1.3. *An (n, t)-tadpole (also known as kite or dragon) graph formed by joining the end point of a path P_t to a cycle C_n. The path and the cycle are called the tail and the body of the tadpole, respectively.*

Definition 1.4. *The path union of a graph G is the graph obtained from a path P_n $(n \geq 3)$ by replacing each vertex of the path by G and it is denoted by $P(n.G)$.*

Definition 1.5. *A connected acyclic graph is called a tree. A binary tree is a tree in which only one vertex of degree two and each of the remaining vertices is of degree one or three. A vertex of degree two in a binary tree is called its root vertex. In a binary tree a vertex v is said to be at level i if v is at a distance i from the root vertex.*

Definition 1.6. *A binary tree with level n is said to be complete if each level i of the binary tree contains exactly 2^i vertices, where $0 \leq i \leq n$. A complete binary tree with level n is denoted by BT_n.*

Note that the complete binary tree BT_n contains $|V| = 1+2+2^2+\cdots+2^n = 2^{n+1}-1$ vertices and $|E| = |V|-1 = 2^{n+1}-2$ edges.

Definition 1.7. *For any integer $m > 2$ and $n > 1$, an umbrella graph $U(m, n)$ is the graph obtained by appending a path P_n to the central vertex of a fan $F_m = P_m + K_1$. Let $V(P_n) = \{v_i : 1 \leq i \leq n\}$ and let $V(F_m) = \{u, u_i : 1 \leq i \leq m\}$. Let $V(U(m, n)) = V(P_n) \cup V(F_m)$ where $u = v_1$. The edge set $E(U(m, n)) = \{u_i u_{i+1} : 1 \leq i \leq m-1\} \cup \{u_i v_1 : 1 \leq i \leq m\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. Thus $|V(U(m, n))| = m + n$ and $|E(U(m, n))| = 2m + n - 2$.*

Definition 1.8 ([4]). *Take $P_2, P_4, ..., P_{n-2}, P_n, P_n, P_{n-2}, ..., P_4, P_2$ paths on $2, 4, ..., n-2, n, n, n-2, ..., 4, 2$ vertices respectively and arrange them centrally horizontal, where $n = 0(mod2)$, $n \neq 2$. A graph obtained by joining vertical vertices of given successive paths is known as a plus graph of size n and it is denoted by Pl_n. Obviously $|V(Pl_n)| = \frac{n^2}{2} + n$ and $|E(Pl_n)| = n^2$.*

For any positive integer n, we fix the position of the vertices in the plus graph Pl_n in the same manner as mentioned in the graph given in Figure 1, unless or otherwise specified. The position of the vertices of the plus graph Pl_8 is mentioned in the Figure 1.

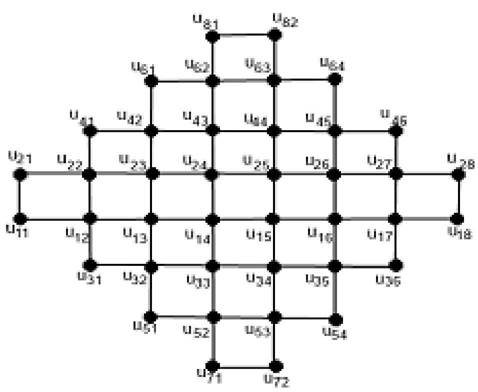

Figure 1.

2. Main Results

The following results are the results investigated in this paper.

Theorem 2.1. *A plus graph Pl_n is a sum divisor cordial labeling, where n is even.*

Proof. Let $G = Pl_n$ be any plus graph. Then $|V(G)| = \frac{n^2}{2} + n$ and $|E(G)| = n^2$. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

Case (1): $j = 1(mod4)$

$$u_{ji} = (j-1)\left\{n - \frac{(j-3)}{2}\right\} + i, \quad where \quad i = 1, 2, ..., n - (j-1)$$

Case (2): $j = 2(mod4)$

$$u_{ji} = (j-1)\left\{n - \frac{(j-2)}{2}\right\} + \frac{(j-2)}{2} + i, \quad where \quad i = 1, 2, ..., n - (j-2)$$

Case(3): $j = 3(mod4)$

$$u_{ji} = (j-1)\left\{n - \frac{(j-3)}{2}\right\} + 2i, \quad where \quad i = 1, 2, ..., n - (j-1)$$

Case(4): $j = 0(mod4)$

$$u_{ji} = (j-2)\left\{n - \frac{(j-4)}{2}\right\} + 2i - 1, \quad where \quad i = 1, 2, ..., n - (j-2)$$

From the above labeling pattern, we have $e_f(0) = e_f(1) = \frac{n^2}{2}$. Hence $|e_f(0) - e_f(1)| \leq 1$. Thus Pl_n is a Sum divisor cordial graph. □

Example 2.2. *The sum divisor cordial labeling of Pl_8 is shown in Figure 2.*

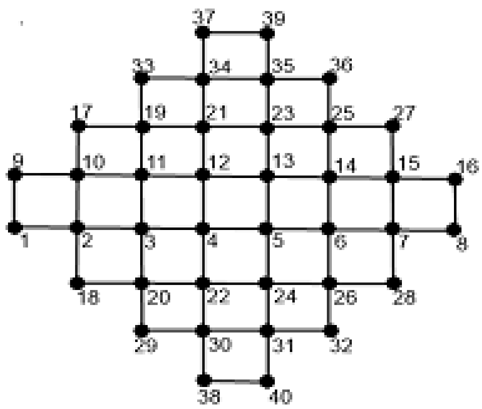

Figure 2.

Theorem 2.3. *The umbrella graph $U(n, n)$ is a sum divisor cordial labeling for $n > 2$, where n is odd.*

Proof. Let $G = U(n, n)$ where $\{v_1, v_2, ..., v_n\}$ be the vertices of path P_n which is attached to the central vertex of the fan F_n with vertex set $\{u, u_1, u_2, ..., u_n\}$. $V(G) = \{u_1, u_2, .., u_n, v_1(= u), v_2, ..., v_n\}$ and $E(G) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_1 : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n-1\}$. Then G has $2n$ vertices and $3n - 2$ edges. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

$$f(u_i) = i, \qquad i = 1(mod\ 4).$$

$$f(u_i) = i + 1, \quad i = 2(mod\ 4).$$

$$f(u_i) = i - 1, \quad i = 3(mod\ 4).$$

$$f(u_i) = i, \quad i = 0(mod\ 4).$$

$$f(v_i) = n + i, \quad i = 1(mod\ 4).$$

$$f(v_i) = n + i + 1, \quad i = 2(mod\ 4).$$

$$f(v_i) = n + i - 1, \quad i = 3(mod\ 4).$$

$$f(v_i) = n + i, \quad i = 0(mod\ 4).$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$. Hence G is a sum divisor cordial graph. □

Example 2.4. *The sum divisor cordial labeling of $U(7,7)$ is shown in Figure 3.*

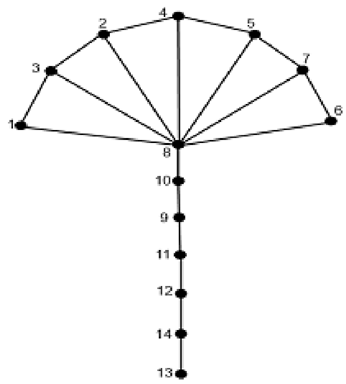

Figure 3.

Theorem 2.5. *The path union of r copies of C_n is a sum divisor cordial graph, where n is odd.*

Proof. Let $G = P(r.C_n)$ be the path union of r copies of cycle C_n. In graph G, $|V(G)| = nr$ and $|E(G)| = nr + r - 1$. We denote v_i^k is the i^{th} vertex in the k^{th} copy of cycle C_n, where $i = 1, 2, ..., n$ and $k = 1, 2, ..., r$. Notice that the vertices v_1^k and v_1^{k+1} are connected by an edge in G, where $k = 1, 2, ..., r - 1$. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

Case (1): When $n = 5, 9, 13, ...$

$$f(v_i^k) = (r - 1)n + i, \quad i = 1(mod\ 4).$$

$$f(v_i^k) = (r - 1)n + i + 1, \quad i = 2(mod\ 4).$$

$$f(v_i^k) = (r - 1)n + i - 1, \quad i = 3(mod\ 4).$$

$$f(v_i^k) = (r - 1)n + i, \quad i = 0(mod\ 4).$$

Case (2): When $n = 3, 7, 11, ...$

Sub Case 2(a): For $r = 2k - 1$

$$f(v_i^k) = (r - 1)n + i + 1, \quad i = 1(mod\ 4).$$

$$f(v_i^k) = (r - 1)n + i - 1, \quad i = 2(mod\ 4).$$

$$f(v_i^k) = (r-1)n + i, \qquad i = 3(mod\ 4).$$

$$f(v_i^k) = (r-1)n + i, \qquad i = 0(mod\ 4).$$

Sub Case 2(b): For $r = 2k$

$$f(v_i^k) = (r-1)n + i, \qquad i = 1(mod\ 4).$$

$$f(v_i^k) = (r-1)n + i + 1, \quad i = 2(mod\ 4).$$

$$f(v_i^k) = (r-1)n + i - 1, \quad i = 3(mod\ 4).$$

$$f(v_i^k) = (r-1)n + i, \qquad i = 0(mod\ 4).$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$. Hence G is a sum divisor cordial graph. □

Example 2.6. *The sum divisor cordial labeling of $P(3.C_7)$ and $P(3.C_5)$ are shown in Figure 4 and Figure 5 respectively.*

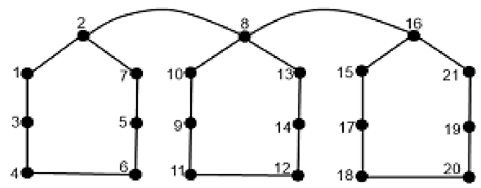

Figure 4.

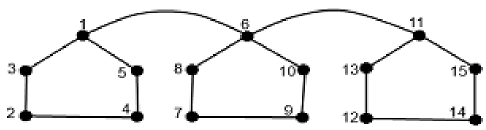

Figure 5.

Theorem 2.7. *The (n, n)-kite graph is a sum divisor cordial labeling.*

Proof. Let $G = (n, n)$ where $\{v_1, v_2, ..., v_n\}$ be the vertices of path P_n which is attached to the vertex u_1 of the cycle C_n with vertex set $\{u_1, u_2, ..., u_n\}$ i.e., $u_1 = v_1$. Then G has $2n-1$ vertices and $2n-1$ edges. We define the vertex labeling $f : V(G) \to \{1, 2, ..., |V(G)|\}$ as follows.

Case (1): When n is odd.

$$f(u_i) = i, \qquad i = 1(mod\ 4).$$

$$f(u_i) = i + 1, \qquad i = 2(mod\ 4).$$

$$f(u_i) = i - 1, \qquad i = 3(mod\ 4).$$

$$f(u_i) = i, \qquad i = 0(mod\ 4).$$

$$f(v_i) = n + i, \qquad i = 2(mod\ 4).$$

$$f(v_i) = n + i - 2, \quad i = 3(mod\ 4).$$

$$f(v_i) = n + i - 1, \quad i = 0(mod\ 4).$$

$$f(v_i) = n + i - 1, \quad i = 1(mod\ 4) \ \ but \ \ i \neq 1$$

Case (2): when n is even

$$
\begin{aligned}
f(u_i) &= i, & i &= 1(mod\ 4).\\
f(u_i) &= i, & i &= 2(mod\ 4).\\
f(u_i) &= i+1, & i &= 3(mod\ 4).\\
f(u_i) &= i-1, & i &= 0(mod\ 4).\\
f(v_i) &= n+i-1, & i &= 2(mod\ 4).\\
f(v_i) &= n+i-1, & i &= 1(mod\ 4)\ \ but\ \imath \neq 1\\
f(v_i) &= n+i, & i &= 3(mod\ 4).\\
f(v_i) &= n+i-2, & i &= 0(mod\ 4).
\end{aligned}
$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$. Hence G is a sum divisor cordial graph. □

Example 2.8. *The sum divisor cordial labeling of* $(5,5)$*-kite and* $(4,4)$*-kite are shown in Figure 6 and Figure 7 respectively.*

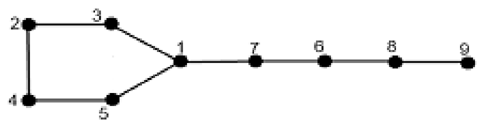

Figure 6.

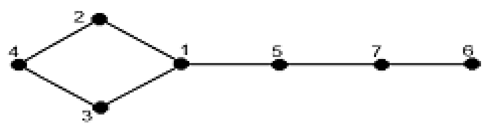

Figure 7.

Theorem 2.9. *Every complete binary tree* BT_n *is a sum divisor cordial labeling.*

Proof. Let $G = BT_n$ be a complete binary tree with level n. Let v be a root of BT_n, which is called a zero level vertex. Clearly, the i^{th} level of BT_n has 2^i vertices. The number of vertices of BT_n is $2^{n+1} - 1$ and the number of edges is $2^{n+1} - 2$. Now assign the label 1 to the root v. Next, we assign the labels $2^i, 2^i + 1, 2^i + 2, ..., 2^{i+1} - 1$ to the i^{th} level vertices, where $1 \leq i \leq n$. Notice that $e_f(0) = e_f(1)$. Hence $|e_f(0) - e_f(1)| \leq 1$. Thus BT_n is a sum divisor cordial graph. □

Example 2.10. *The sum divisor cordial labeling of complete binary tree* BT_3 *is shown in Figure 8.*

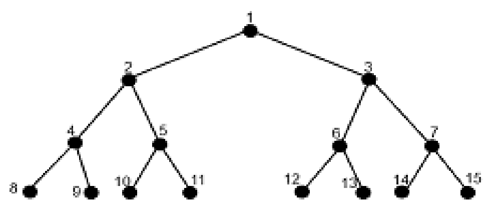

Figure 8.

3. Conclusion

In this paper, we have proved that the some of the special graphs such as plus graph Pl_n, umbrella graph $U(n,n)$ (n is odd), Path union of r copies of C_n (n is odd), (n,n)-kite graph and complete binary tree BT_n are sum divisor cordial graphs.

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