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Some Parameters of Non-Zero Zero Divisor Graphs

Varta Teotia[1,*] and Sapna Shrimali[1]

1 Department of Mathematics, Pacific Academy of Higher Education & Research University, Udaipur, Rajasthan, India.

Abstract: Let S be a commutative semigroup including zero. The zero divisor graph associated to S, denoted by $\Gamma(Z)$ is the graph whose vertices are the nonzero zero divisors of S and two vertices x and y are adjacent in case their product in the semigroup is zero. There are many known results on the possible structure of such graphs. We study the converse problem. In this paper, we investigate diameter, radius and cut vertex parameters of some non-zero zero divisor graphs.

Keywords: Commutative semigroup, zero-divisor,graphs, diameter, radius, cut-vertex.

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1. Introduction and Preliminaries

Graph theory is most interesting branches of mathematics. The great mathematician Leonard Euler firstly introduced the graph theory in 1735 with the problem of Konigsberg bridge problem.Given the concept of the graph $G(R)$ to a commutative ring R introduce by I. Beck. He considered the case $xy = 0$ where x and y are two vertices connected by an edge. In the graph zero element is adjacent to every vertex. The coloring number and clique number are related to each other studied by I. Beck. He supposed that the coloring and clique number were equivalent for all graphs associated to rings. D.D. Anderson and M. Naseer [2] provided a counter example for this assumption. Modified definition of the zero divisor graph of a commutative ring is given by D.F. Anderson and P. Livingston [3]. They associated a simple graph $G(R)$ to R with vertices $Z(R)^* = Z(R) - \{0\}$, the set of nonzero zero-divisors of R, and $x, y \in Z(R)^*$ where x and y are distinct. The developing graph, which they denote $G(R)$, is an investigated subgraph of the graph established by Beck which they show better represent the zero divisor structure of the ring. The study of zero divisor graphs has been extended by Mulay [4], Redmond [5] DeMeyer-Schneider [6] and many others.

Definition 1.1 (Eccentricity of a vertex)**.** *The eccentricity of a vertex $v \in V(G)$ is $e(v) = \max\{d(u, v)|u \in V(G)\}$.*

Example 1.2.

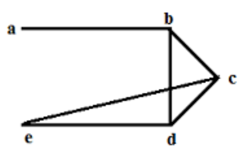

Figure 1.

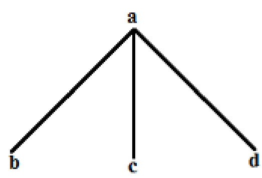

Figure 2.

* *E-mail: vartateotia@gmail.com (Research Scholar)*

In Figure 1, eccentricity is $e(a) = 3$*,* $e(b) = 2$*,* $e(c) = 2$*,* $e(d) = 2$*,* $e(e) = 3$*. In Figure 2, eccentricity is* $e(a) = 1$*,* $e(b) = 2$*,* $e(c) = 2$*,* $e(d) = 2$*.*

Note: $e(a) = 1$ *if and only if* v *is adjacent to all other vertices.*

Definition 1.3 (Diameter of graph)**.** *The diameter is defined for the whole graph is equal to the maximum to all of the eccentricities.*

$$Diam(G) = \max\{e(v)|v \in V(G)\}$$

In Figure 1, $Diam = 3$*, in Figure 1,* $Diam = 2$*.*

Definition 1.4 (Radius of graph)**.** *The radius is defined for the whole graph is equal to the minimum to all of the eccentricities.*

$$Rad(G) = \min\{e(v)|v \in V(G)\}$$

In Figure 1, $Rad = 2$*, in Figure 2,* $Rad = 1$*.*

Definition 1.5 (Peripheral vertex)**.** *If* $e(v) = diam(G)$ *then* v *is a peripheral vertex. The set of all such vertices make the periphery of* G*.*

Definition 1.6 (Central vertex)**.** *If* $e(v) = rad(G)$ *then* v *is a central vertex. The set of all such vertices make the center of* G*.*

Fact 1.7. $rad(G) \leq diam(G) \leq 2rad(G)$*.*

Proof. $rad(G) \leq diam(G)$ is obviously by definition. Now we see, $diam(G) \leq 2rad(G)$. Let $u, v \in V(G)$ such that $d(u, v) = diam(G)$. Let w be the central vertex of G then

$$\begin{aligned} d(u,v) &\leq d(u,w) + d(w,v) \\ &\leq 2e(w) \\ &= 2rad(G) \end{aligned}$$

□

Definition 1.8 (Cut-Vertex)**.** *A vertex* $v \in V(G)$ *is a cut vertex of* G *if and only if* $\exists\, u, w \in V(G)$*.* $u, w \neq v$ *such that* v *is on every* $u - w$ *path of* G*.*

2. Some Parameters of Zero Divisor Graphs

In this section we study of the diameter, radius and cut-vertex of zero divisor graphs.

Theorem 2.1. *If* $n = 2p$ *where zero divisor graph of modulo n and p is an odd prime, then the non-zero zero divisor graph is star graph and* $K_{1,p-1}$ *is complete bipartite graph.*

Proof. Let $n = 2p$, where zero divisor graph of modulo n and p is an odd prime. Then the vertex set of non-zero zero divisor graph is $Z(R)^* = V = \{(a, b) : ab = 0 \text{ multiplication modulo n for all } ab \in V\}$. Let a, b be two distinct even integers which is in V. The product of ab will become an even integer which is not divisible by n where n is the number of vertex $n = 2p$, p is an odd prime number. The product ab is divisible by $n = 2p$ if one vertex is 2 or multiples of 2. Therefore two distinct even integers a and b are not adjacent.

$p \geq 3$, then let us take b is a prime number and a is an even integer in V. The product of ab is an even integer and it is divisible by n where $n = 2p$, p is a prime and every even integer and prime number is adjacent. Therefore p is adjacent to $2, 4, 6, \ldots, 2(-1)$. Therefore K_p is a star graph. The star graph K_p is classified into two non-empty subsets A and B Where $A = \{p\}$ and $B = \{2, 4, 6, \ldots, 2(-1)\}$. All the vertices of B is adjacent to the vertex A. Therefore $K_{1,p-1}$ is a complete bipartite graph. Therefore $n = 2p$ where p is a prime number and $p \geq 3$ is a star graph and $K_{1,p-1}$ is a complete bipartite graph. □

Theorem 2.2. *If $n = 2p$ where p is an odd prime, then the non-zero zero divisor graph having a cut vertex and the diameter is 2, radius is 1.*

Proof. $\Gamma(Z_n)$ is a star graph by previous theorem. Diameter is a maximum eccentricity of the graph which is 2, radius is the minimum eccentricity of the graph which is 1 and Cut-vertex denoted by C_v, when $n = 2p$, where p is an odd prime. Therefore $Diam(\Gamma(Z_n)) = 2$. Therefore $Rad(\Gamma(Z_n)) = 1$. Therefore $C_v(\Gamma(Z_n)) =$ having a cut vertex. □

Theorem 2.3. *$\Gamma(Z_n)$ is complete bipartite graph if $n = 3p$ where p is an odd prime, $p > 3$ then the diameter of non-zero zero divisor graph is 2, radius is 2 and there is no cut vertex.*

Proof. $\Gamma(Z_n)$ complete bipartite graph is the Diameter is a maximum eccentricity of the graph which is 2, radius is the minimum eccentricity of the graph which is 2 and Cut-vertex denoted by C_v. when $n = 3p$, where p is an odd prime. Therefore $Diam(G(Z_n)) = 2$. Therefore $Rad(\Gamma(Z_n)) = 2$. Therefore $C_v(\Gamma(Z_n)) =$ no cut vertex. □

Theorem 2.4. *If $n = 5p$ where p is an odd prime, $p > 5$ then the non-zero zero divisor graph is $K_{4,p-1}$ complete bipartite graph. The diameter of non-zero zero divisor graph is 2, radius is 2 but there is no cut vertex.*

Proof. $\Gamma(Z_n)$ is complete bipartite graph. the Diameter is a maximum eccentricity of the graph which is 2, radius is the minimum eccentricity of the graph which is 1 and Cut-vertex denoted by C_v. When $n = 5p$, where p is an odd prime. Therefore $Diam(\Gamma(Z_n)) = 2$. Therefore $Rad(\Gamma(Z_n)) = 2$. Therefore $C_v(\Gamma(Z_n)) =$ no cut vertex. □

Theorem 2.5. *If $n = pq$ where p and q are prime, $p > q$ then the diameter, radius and cut vertex of non-zero zero divisor graph is $- - --?$.*

Proof. Diameter is a maximum eccentricity of the graph, radius is the minimum eccentricity of the graph and Cut-vertex denoted by C_v. When $n = pq$, where p and q are primes. Therefore $Diam(\Gamma(Z_n)) = 2$. Therefore

$$Rad(\Gamma(Z_n)) = \begin{cases} 1, & \text{when q is even prime} \\ 2, & \text{when q is odd prime} \end{cases}$$

Therefore

$$C_v(\Gamma(Z_n)) = \begin{cases} having\ a\ cut\ vertex, & \text{when q is even prime} \\ no\ cut\ vertex, & \text{when q is odd prime} \end{cases}$$

□

Theorem 2.6. *If $n = p^2$ where an odd prime, then the diameter is 1, radius is 1 and there is no cut vertex.*

Proof. Let $n = p^2$ where p is an odd prime. Then the vertex set of non-zero zero divisor graph is $V = Z(R)^* = \{p, 2p, 3p, \ldots, p(p-1)\}$. The edge set $E(G)$ defined by $E(G) = \{(a, b)/ab = 0 \text{ (multiplication modulo n) for all } ab \in Z(R)^*\}$. When we draw the graph we get a complete graph on $(p - 1)$ vertices. $\Gamma(Z_n)$ is complete bipartite graph. the Diameter is a maximum eccentricity of the graph which is 1, radius is the minimum eccentricity of the graph which is 1 and there is no cut vertex. When $n = p^2$, where p is an odd prime. Therefore $Diam(\Gamma(Z_n)) = 1$. Therefore $Rad(\Gamma(Z_n)) = 1$. Therefore $C_v(\Gamma(Z_n)) =$ no cut vertex. □

Theorem 2.7. *If $n = p^\alpha$ where p is an odd prime, $\alpha > 2$ then the diameter, radius and cut vertex of non-zero zero divisor graph is $- - --?$.*

Proof. Let $n = p^\alpha$ where p is an odd prime and $\alpha > 2$. Then the vertex set $V = Z(R)^* = \{p, 2p, 3p, \dots, \; p^{\alpha-1}, \dots, (p^\alpha - p)\}$. If $n = P^\alpha$ then is $p^{\alpha-1}$ adjacent to every other vertex of (Z_n). Hence the Diameter is a maximum eccentricity of the graph which is 2, radius is the minimum eccentricity of the graph which is 1 and Cut-vertex denoted by C_v, when $n = 5p$, where p is an odd prime. Therefore $Diam(\Gamma(Z_n)) = 2$. Therefore $Rad(\Gamma(Z_n)) = 2$. Therefore $C_v(\Gamma(Z_n)) =$ no cut vertex. □

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