Int. J. Math. And Appl., 6(2–B)(2018), 409–415
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Edge-Odd Graceful for Cartesian Product of a Wheel With n Vertices and a Path with Two Vertices

G. Balasubramanian[1,*] and B. Ambika[2]

1 Department of Mathematics, Government Arts College for Men, Krishnagiri, Tamilnadu, India.

Abstract: Abhyankar [1] investigated direct methods of gracefully labeling graphs. Bahl [2] got gracefulness labeling for few families of spiders in few terms of merging such graphs. Barrientos [3] obtained graceful labelings for chain graphs. Edwards and Howard [4] analyzed a survey of some classes of graceful trees. Kaneria [6] obtained graceful labeling for graphs related to cycle. Kaneria [7] made new graceful graphs by merging stars. Kaneria [8] received graceful labeling by attaching cycle to cycles and cycle with a complete bipartite graph. Mishra and Panigrahi [10] investigated new classes of graceful lobsters obtained from diameter four trees. Ramachandran and Sekar [11] got graceful labelling of super subdivision of ladder. $A(p,q)$ connected graph is edge-odd graceful graph if there exists an injective map $f: E(G) \to \{1,3,\dots,2q-1\}$ so that induced map $f_+: V(G) \to \{0,1,2,3,\dots,(2k-1)\}$ defined by $f_+(x) \equiv \sum f(xy)\ (mod\ 2k)$, where the vertex x is incident with other vertex y and $k = \max\{p,q\}$ makes all the edges distinct and odd. In this article, the edge-odd gracefulness of cartesian product of P_2 and W_n is obtained.

Keywords: Graceful graphs, edge-odd graceful labeling, edge-odd graceful graph.

 Accepted on: 26.02.2018

1. Introduction

Liu [9] analyzed gracefulness of cartesian product graphs like as path, cycle, and star. Sudha and Kanniga [13] found graceful labeling on the combination of path and star. Wang and Li [18] showed result about gracefulness of few special graphs by identifying cycle and standard trees. Wang [19] verified result about gracefulness of graphs related to wheels. Gao [5] found odd graceful labeling for the union of standard graphs like as path, wheel, and circuit. Seoud and Abdel-Aal [12] analyzed odd graceful labelings for graphs merging two paths or a path and a circuit. Vaidya and Lekha [14] obtained odd labeling for some new graphs adding wheel and star or path. They [15] also found new families of odd graceful trees. They [16] further got some new graceful graphs. Vaidya, and Shah [17] got few graceful graphs from identifying a vertex in path or star with a pendent vertex of a tree lie as rooted tree and a symmetric tree.

2. Edge-Odd Graceful Labeling of Cartesian Product of P_2 and W_n

The following definitions are first given.

Definition 2.1 (Graceful graph)**.** *A function f of a graph G is called a graceful labeling with m edges, if f is an injection from the vertex set of G to the set $\{0,1,2,\dots,m\}$ such that when each edge uv is assigned the label $|f(u)-f(v)|$ and the resulting edge labels are distinct. Then the graph G is graceful.*

* *E-mail: solaijmc@gmail.com*

Definition 2.2. *Edge-odd graceful graph] $A(p,q)$ connected graph is edge-odd graceful graph if there exists an injective map $f : E(G) \to \{1, 3, \dots, 2q-1\}$ so that induced map $f^+ : V(G) \to \{0, 1, 2, \dots, (2k-1)\}$ defined by $f_+(x) \equiv \sum f(x,y)\ (mod\ 2k)$, where the vertex x is incident with other vertex y and $k = \max\{p,q\}$ makes all the edges distinct and odd. Hence the graph G is edge- odd graceful.*

Theorem 2.3. *The cartesian product of P_2 and W_n is edge-odd graceful for any positive even integer n.*

Proof. The cartesian product of a path P_2 with 2 vertices and a wheel W_n with n vertices is given as follows. The arbitrary labelings for vertices and edges for $P_2 \times W_n$ are mentioned below.

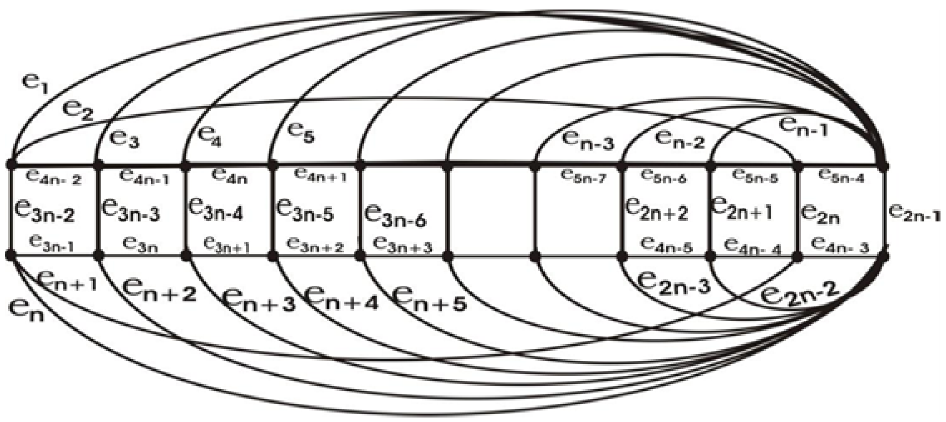

Figure 1. **Edge-odd graceful graph** $P_2 \times W_n$

Let n be an even positive integer. To find edge-odd graceful, define $f : E(P_2 \times W_n) \to \{1, 3, \dots, 2q-1\}$ by

Case (1): For $n \equiv 0\ (mod\ 8)$

$$\left.\begin{array}{l} f(e_1) = 7,\ f(e_4) = 1, \\ f(e_i) = 2i-1,\ i = 2, 3, \dots, 5, 6, (n-1), (n+2), (n+3), \dots, (5n-4) \\ f(e_n) = 2n+1,\ f(e_{n+1}) = 2n-1, \end{array}\right\} Rule\ (1)$$

Case (2): For $n \equiv 2\ (mod\ 8)$

$$\left.\begin{array}{l} f(e_i) = 2i-1, \quad i = 1, 2, 3, \dots, (2n-2), (2n), (2n+1), \dots, (3n-2) \\ f(e_{3n-1}) = f(e_{3n-2}) + 4(n-1) \\ f(e_{3n}) = f(e_{3n-2}) + 2(n-1) \\ f(e_{3n-1+i}) = f(e_{3n-1}) - i, \quad i = 2, 4, 6, \dots, (2n-6) \\ f(e_{3n+i}) = f(e_{3n}) - i, \quad i = 2, 4, 6, \dots, (2n-4) \\ f(e_{5n-4}) = f(e_{2n-2}) + 2;\ f(e_{2n-1}) = f(e_{5n-6}) - 2. \end{array}\right\} Rule\ (2)$$

Case (3): For $n \equiv 4\ (mod\ 8)$

$$\left.\begin{array}{l} f(e_1) = 5,\ f(e_2) = 3,\ f(e_3) = 1 \\ f(e_i) = 2i-1,\ i = 4, 5, \dots, n, (n+2), \dots, (2n-3), (2n-1), (2n+1), \dots, (4n-4), (4n-2), \dots, (5n-3), (5n-5) \\ f(e_{n+1}) = 4n-5,\ f(e_{5n-4}) = 8n-7 \\ f(e_{2n-2}) = 2n+1,\ f(e_{4n-3}) = 10n-11 \end{array}\right\} Rule\ (3)$$

For $n \equiv 6\ (mod\ 8)$, the arbitrary labeling for vertices and edges for $P_2 \times W_n$ are mentioned below

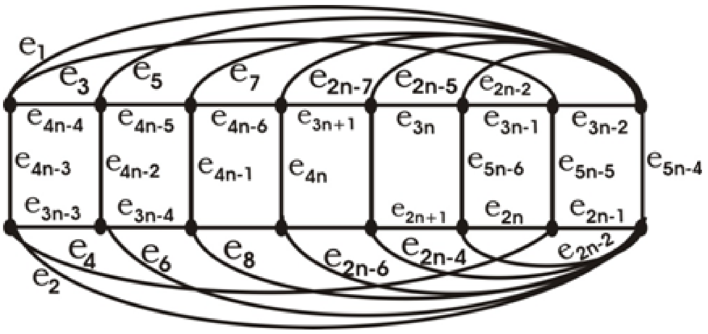

Figure 2. **Edge-odd graceful graph** $P_2 \times W_n$ **for** $n \equiv 6\ (mod\ 8)$

Case (4): For $n \equiv 6\ (mod\ 8)$

$$\left.\begin{aligned} &f(e_i) = 2i-1,\ i = 1,3,4,5,6,\dots,(2n-3),(2n-1),(2n),\dots,(5n-4) \\ &f(e_2) = 4n-5,\ f(e_{2n-2}) = 3 \end{aligned}\right\} Rule\ (4)$$

Define $f^+ : V(G) \to \{0,1,2,\dots,(2k-1)\}$ by $f^+(v) \equiv \sum f(uv)\ mod\ (2q)$, where this sum run over all edges through $v\dots$ [*Rule* (9)]. Hence the induced map f^+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the cartesian product graph P_2 and W_n is edge-odd graceful. □

Theorem 2.4. *The cartesian product of P_2 and W_n is edge-odd graceful for any positive odd integer n.*

Proof. The cartesian product of a path P_2 with 2 vertices and a wheel W_n with n vertices is given in figure 1 in which the arbitrary labeling for vertices and edges for $P_2 \times W_n$ are mentioned. Let n be odd positive integer.

Case (5): For $n \equiv 1\ (mod\ 8)$

$$\left.\begin{aligned} &f(e_1) = 7,\ f(e_4) = 1 \\ &f(e_i) = 2i-1,\ i = 2,3,\dots,(5n-4) \end{aligned}\right\} Rule\ (5)$$

Case (6): For $n \equiv 3\ (mod\ 8)$

$$\left.\begin{aligned} &f(e_1) = 3,\ f(e_2) = 1, \\ &f(e_i) = 2i-1,\ i = 34,\dots,(n-1),(2n-1),(2n),\dots,(5n-4) \\ &f(e_{2n-2-i}) = f(e_{n-1}) + 2i + 2,\ i = 1,2,3,\dots,(n-4) \\ &f(e_n) = f(e_{n+2}) + 2,\ f(e_{n+1}) = f(e_{n-1}) + 2,\ f(e_{2n-2}) = f(e_n) + 2 \end{aligned}\right\} Rule\ (6)$$

Case (7): For $n \equiv 5\ (mod\ 8)$

$$\left.\begin{aligned} &f(e_1) = 7,\ f(e_4) = 1 \\ &f(e_i) = 2i-1,\ i = 2,3,5,6,\dots,(5n-4) \end{aligned}\right\} Rule\ (7)$$

Case (8): For $n \equiv 7\ (mod\ 8)$

$$\left.\begin{aligned} &f(e_i) = 2i-1,\ i = 1,2,34,\dots,(n-1),(2n),(2n+1),\dots,(5n-5) \\ &f(e_{2n-1-i}) = f(e_{n-1}) + 2i,\ i = 1,2,3,\dots,(n-3) \\ &f(e_{2n-1}) = 10n-9,\ f(e_{5n-4}) = 10n-9, f(e_{n+1}) = f(e_{n+2}) + 4 \\ &f(e_n) = f(e_{n+2}) + 2,\ f(e_{5n-4}) = 4n-3 \end{aligned}\right\} Rule\ (8)$$

Define $f^+ : V(G) \to \{0,1,2,\dots,(2q-1)\}$ by $f^+(v) \equiv \sum f(uv)\ mod\ (2q)$, where this sum run over all edges through $v\dots$ [*Rule* (9)]. Hence the induced map f_+ provides the distinct labels for vertices and also the edge labeling is distinct. Hence the cartesian product graph $P_2 \times W_n$ is edge-odd graceful. □

Example 2.5. *The cartesian product graph $P_2 \times W_8$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_8$ is a connected graph with 16 vertices and 36 edges, where $n \equiv 0 \ (mod\ 8)$. Due to the rules (1) & (9) in Theorem 2.1, edge-odd graceful labeling of the required graph is obtained as follows.

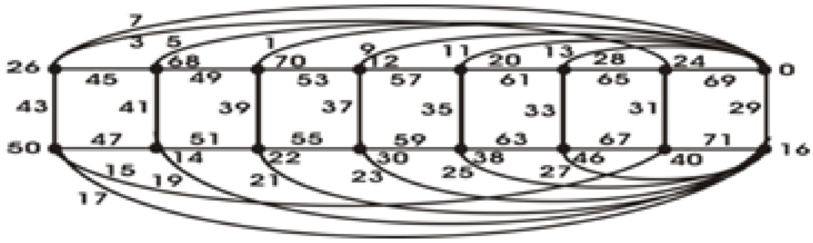

Figure 3. **Edge-odd graceful graph** $P_2 \times W_8$

∎

Example 2.6. *The cartesian product graph $P_2 \times W_{10}$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_{10}$ is a connected graph with 20 vertices and 46 edges, where $n \equiv 2 \ (mod\ 8)$. Due to the rules (2) & (9) in Theorem 2.1, edge-odd graceful labelings of the required graph is obtained as follows.

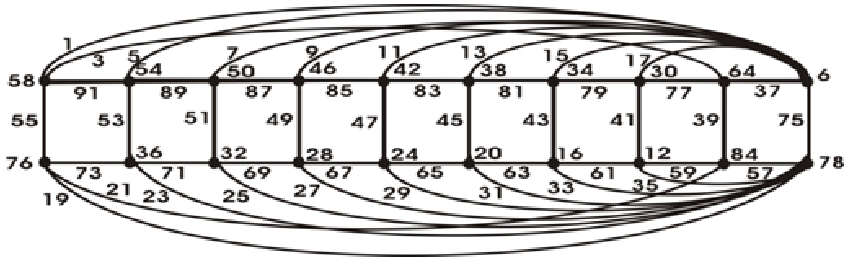

Figure 4. **Edge-odd graceful graph** $P_2 \times W_{10}$

∎

Example 2.7. *The cartesian product graph $P_2 \times W_{12}$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_{12}$ is a connected graph with 24 vertices and 56 edges, where $n \equiv 4 \ (mod\ 8)$. Due to the rules (3) & (9) in Theorem 2.1, edge-odd graceful labelings of the required graph is obtained as follows.

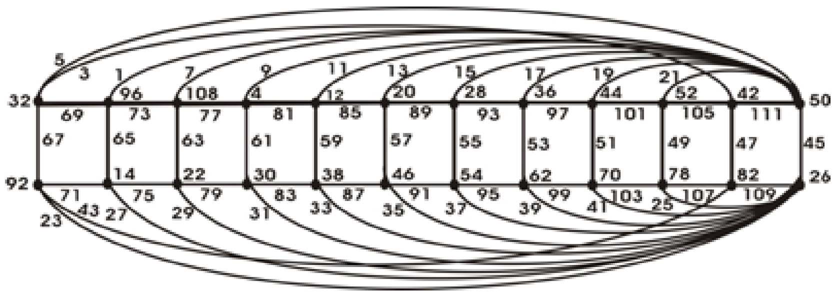

Figure 5. **Edge-odd graceful graph** $P_2 \times W_{12}$

∎

Example 2.8. *The cartesian product graph $P_2 \times W_9$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_9$ is a connected graph with 18 vertices and 41 edges, where $n \equiv 1 \ (mod\ 8)$. Due to the rules (5) & (9) in Theorem 2.2, edge-odd graceful labelings of the required graph is obtained as follows.

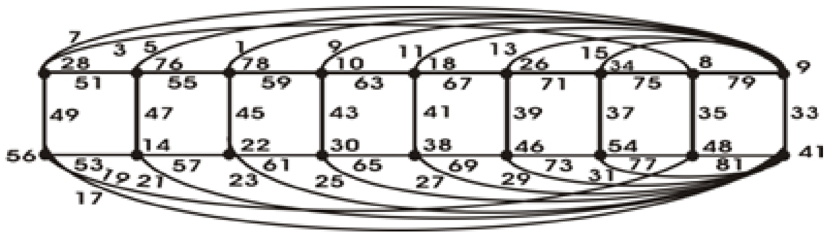

Figure 6. **Edge-odd graceful graph** $P_2 \times W_9$

□

Example 2.9. *The cartesian product graph $P_2 \times W_{11}$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_{11}$ is a connected graph with 22 vertices and 51 edges, where $n \equiv 3\ (mod\ 8)$. Due to the rules (6) & (9) in Theorem 2.2, edge-odd graceful labeling of the required graph is obtained as follows.

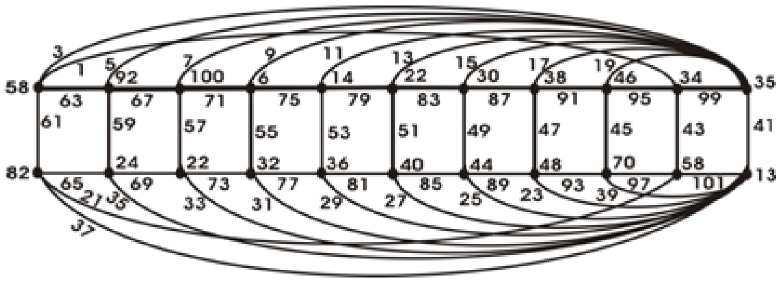

Figure 7. **Edge-odd graceful graph** $P_2 \times W_{11}$

□

Example 2.10. *The cartesian product graph $P_2 \times W_5$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_5$ is a connected graph with 10 vertices and 21 edges, where $n \equiv 5\ (mod\ 8)$. Due to the rules (7) & (9) in Theorem 2.2, edge-odd graceful labeling of the required graph is obtained as follows.

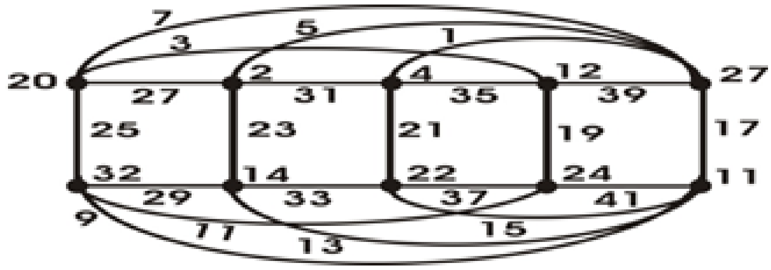

Figure 8. **Edge-odd graceful graph** $P_2 \times W_5$

□

Example 2.11. *The cartesian product graph $P_2 \times W_7$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_7$ is a connected graph with 14 vertices and 31 edges, where $n \equiv 5\ (mod\ 8)$. Due to the rules (7) & (9) in Theorem 2.2, edge-odd graceful labeling of the required graph is obtained as follows.

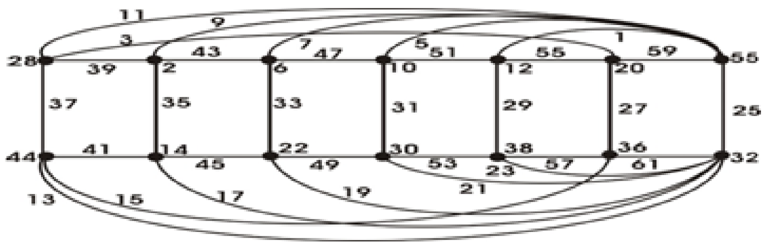

Figure 9. **Edge-odd graceful graph** $P_2 \times W_7$

□

Example 2.12. *The cartesian product graph $P_2 \times W_6$ is edge-odd graceful.*

Proof. The cartesian product graph $P_2 \times W_7$ is a connected graph with 12 vertices and 26 edges, where $n \equiv 6$ $(mod\ 8)$. Due to the rules (4) & (9) in Theorem 2.2, edge-odd graceful labeling of the required graph is obtained as follows.

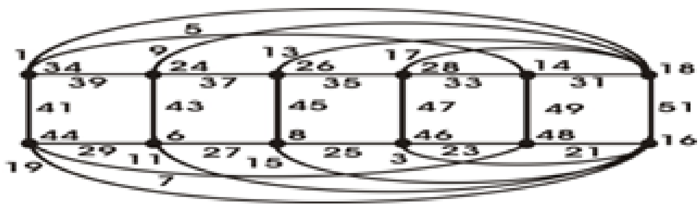

Figure 10. **Edge-odd graceful graph** $P_2 \times W_6$

□

References

[1] V. J. Abhyankar, *Direct Methods of Gracefully Labeling Graphs*, Ph. D. Thesis, University of Mumbai, (2002).

[2] P. Bahl, S. Lake and A. Wertheim, *Gracefulness of families of spiders*, Involve, 3(2010), 241-247.

[3] C. Barrientos, *On graceful chain graphs*, Util. Math., 78(2009), 55-64.

[4] M. Edwards and L. Howard, *A survey of graceful trees*, Atlantic Electronic Journal of Mathematics, 1(2006), 5-30.

[5] Z. Gao,*Odd graceful labelings of some union graphs*, J. Nat. Sci. Heilongjiang Univ., 24(2007), 35-39.

[6] V. J. Kaneria, H. M. Makadia and M. M. Jariya, *Graceful labeling for cycle of graphs*, Int. J. Math. Res., 6(2)(2014), 173-178.

[7] V. J. Kaneria, H. M. Makadia and M. Meghpara, *Some graceful graphs*, Int. J. Math. Soft Comp., 4(2)(2014), 165-172.

[8] V. J. Kaneria, M. Meghpara and H. M. Makadia, *Gracefulness of cycle of cycle sand cycle of complete bipartite graphs*, Int. J. Math. Trend Tech., 12(1)(2014), 19-26.

[9] J. B. Liu, T. Zou and Y. Lu, *Gracefulness of Cartesian product graphs*, Pure Appl. Math. (Xi'an), 28(3)(2012), 329-332.

[10] D. Mishra and P. Panigrahi, *Some new classes of graceful lobsters obtained from diameter four trees*, Math. Bohem., 135(2010), 257-278.

[11] V. Ramachandran and C. Sekar, *Graceful labelling of supersubdivision of ladder*, Internat. J. Math. Appl., 2(2)(2014), 29-36.

[12] M. A. Seoud and M. E. Abdel-Aal, *On odd graceful graphs*, Ars Comb., 108(2013), 161-185.

[13] S. Sudha and V. Kanniga, *Graceful labeling on the combination of some graphs*, Math. Sci. Internat. Research J., 2(2)(2013), 630-633.

[14] S. K. Vaidya and B. Lekha, *Odd graceful labeling of some new graphs*, Modern Appl. Sci., 4(10)(2010), 65-70.

[15] S. K. Vaidya and B. Lekha, *New families of odd graceful graphs*, Internat. J. Open Problems Comp. Sci. Math., 3(5)(2010), 166-171.

[16] S. K. Vaidya and B. Lekha, *Some new graceful graphs*, Internat. J. Math. Soft Comp., 1(1)(2011), 37-45.

[17] S. K. Vaidya and N. H. Shah, *Graceful and odd graceful labeling of some graphs*, Internat. J. of Math. Soft Computing, 3(1)(2013), 61-68.

[18] T. Wang and D. M. Li, *Gracefulness of some special graph classes*, J. Wuhan Univ.Natur. Sci. Ed., 58(5)(2012), 437-440.

[19] T. Wang, H. S. Liu and D. M. Li, *Gracefulness of graphs related to wheels*, Acta Sci. Natur. Univ. Sunyatseni, 50(6)(2011), 16-19.