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Correlation and Ranking Analysis Using Neutrosophic Sets in a Multi-criteria Single-valued Decision Making Problems

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Abstract: As a generalization of intuitionistic fuzzy sets, neutrosophic sets (NSs) can be better handle the incomplete, indeterminate and inconsistent information, which have attracted the widespread concerns for researchers. In this paper, some new correlation coefficient and ranking method are introduced using neutrosophic fuzzy operators and fuzzy average operator in a single-valued neutrosophic environment. Firstly, the definition and operational laws of single-valued neutrosophic numbers (SVNNs) are introduced. Then, the single-valued neutrosophic average operator and the single-valued ranking techniques in neutrosophic are developed, and few properties of on these operators are also analyzed. Furthermore, a method for solving multi-criteria decision-making (MCDM) problems is explored based on the correlation coefficient and raking technique. Finally, an illustrative example is shown to verify the effectiveness and practicality of the proposed method.

1. Introduction

In the real world, the decision-making problems with incomplete or inaccurate information are difficult to be precisely expressed by decision-makers. Under these circumstances, Zadeh [1] firstly proposed the theory of fuzzy sets (FSs), where the membership degree is presented using a crisp value between zero and one, have been applied successfully in many different fields. However, FSs only have a membership and lack non-membership degree. In order to solve the problem, Atanassov [1] proposed the intuitionistic fuzzy sets (IFSs), which is an extension of Zadeh's FSs. IFSs have been widely extended and got more attention in solving MCDM problems [3]. Although the theories of FSs and IFSs have been generalized, it can not handle all kinds of uncertainties in many cases. The indeterminate information and inconsistent information existing commonly in the real world can not be deal with by FSs and IFSs. For example, during a voting process, forty percent vote "yes", thirty percent vote "no", twenty percent are not sure, and ten percent give up. This issue is beyond the scope of IFSs, which cannot distinguish the information between unsure and giving up. Therefore, on the basis of IFSs, Smarandache introduced neutrosophic logic and neutrosophic sets (NSs) by adding an independent indeterminacy-membership. Then, the aforementioned example can be expressed as $x(0.4, 0.2, 0.3)$ with respect to NSs. Moreover, true-membership, indeterminacy-membership and false-membership in NSs are completely independent, whereas the uncertainty

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is dependent on the true-membership and false-membership in IFSs. So the notion of NSs is more general and overcomes the aforementioned issues.

From scientific or engineering point of view, the neutrosophic set and set-theoretic operators will be difficult to apply in the real application without specific description. Therefore, a single-valued netrosophic set (SVNS) is proposed [8], which is an extension of NSs, and some properties of SVNS are also provided. Ye [9] proposed the correlation coefficient and weighted correlation coefficient of SVNSs, and proved that the cosine similarity degree is a special case of the correlation coefficient in SVNS. Majumdar [11] defined similarity measures between two SVNSs and introduced a measure of entropy of SVNSs. Ye [12] proposed the cross-entropy of SVNSs. Furthermore, Ye [13] introduced the concept of simplified neutrosophic sets (SNSs), and proposed a MCDM method using a simplified neutrosophic weighted arithmetic average operator and a simplified neutrosophic weighted geometric average operator. Liu [14] proposed a multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Wang [15] proposed the concept of interval neutrosophic set (INS) and gave the set-theoretic operators of INS. Zhang [16] defined the operations for INSs, and developed two interval neutrosophic number aggregation operators. Ye [17] defined the Hamming and Euclidean distances between INSs, and proposed the similarity measures between INSs based on the relationship between similarity measures and distances. Liu [18] proposed some Hamacher aggregation operators for the interval-valued intuitionistic fuzzy numbers. Peng [19] defined multi-valued NSs, and discussed operations based on Einstein. Liu [20] proposed the concept of the interval neutrosophic hesitant fuzzy set, presented the operations and developed generalized hybrid weighted aggregation operators.

Information aggregation is very important in MCDM and MCGDM problems, so various aggregation operators have been proposed and developed in the past years. Yager [21] and Xu [22] proposed weighted arithmetic average operator and weighted geometric average operator, which are two of the most common operators. Zhao [23] developed generalized aggregation operators for intuitionistic fuzzy sets (IFSs). The NSs is an extension of IFSs, so it is significant meaningful to research the aggregation operators for NSs. However, until to now, there are a few researches on aggregation operators for SVNSs, and apply them to decision-making problems. Many traditional aggregation operators do not consider the relationship of different input arguments in the decision process. Yager [18] firstly defined the power average operator which makes the arguments support each other. Xu [25] introduced the power geometric operator. Zhou [26] developed a generalized power average operator. Liu [27] defined intuitionistic trapezoidal fuzzy power generalized aggregation operator. However, power average operators have not been applied to handle MCDM problems under single-valued neutrosophic environment. Therefore, the aim of the paper is to develop single-valued neutrosophic power average aggregation operators. Meanwhile, we will discuss its properties, such as idempotency, commutativity.

The rest of paper is organized as follows. In Section 2, we introduce some concepts and operations of SVNS. New power aggregation operators for SVNN are defined, and some properties are discussed in Section 3. Section 4 establishes the detail decision method for multi-criteria decision making based on the proposed operators under single-valued neutrosophic fuzzy information environment. Section 5 presents an illustrative example according to our method. Finally, the main conclusions of this paper are summarized in Section 6.

2. Preliminaries

In this Section, some concepts and definitions with respect to SVNs are introduced, which will be utilized in the remainder of the paper.

Definition 2.1 ([8])**.** *Let X be a universe set with generating element x. A neutrosophic set (NS) $\boldsymbol{A}$ in X is $\{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ where $T_A(x)$ is the truth-membership function, $I_A(x)$ is the indeterminacy-membership,*

& $F_A(x)$ is the falsity-membership function, and all are real standard or non-standard subsets of $]0^-, 1^+[$. There is no restriction on these three membership functions, but $0^- \leq \sup(T_A(x) + I_A(x) + F_A(x)) \leq 3^+$.

Definition 2.2 ([9]). *Let X be a universe set with generating element x. A single-valued neutrosophic set (SVNS) in X is* $\{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$. *Here* $0 \leq \sup(T_A(x) + I_A(x) + F_A(x)) \leq 3$, *and each of memberships* $T_A(x)$, $I_A(x)$, $F_A(x)$ *is in* $[0, 1]$ *for all x in X. Note that A SVNS or simplified neutrosophic (SNS) is a subclass of NS, implies that SVNS is also an special case of SNS.*

Further a SVNS is an example of NS, and SNS is a subclass of NS, so that SVNS is also an special case of SNS. $x = (Tx, Ix, Fx)$ *is used to represent an element in SVNSs, and called it as a single-valued neutrosophic number (SVNN). The set of all single-valued neutrosophic numbers is noted as SVNNS.*

Definition 2.3 ([13]). *Let and be two SVNNs, then the operational relations are defined as follows:*

(1). $x \oplus y = (T_1 + T_2 - T_1 T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2)$.

(2). $x \otimes y = (T_1 T_2, I_1 I_2, F_1 F_2)$.

(3). $\lambda x = (1 - (1 - T_1)^\lambda, 1 - (1 - I_1)^\lambda, 1 - (1 - F_1)^\lambda), \lambda > 0$.

(4). $x^\lambda = (T_1^\lambda, I_1^\lambda, F_1^\lambda), \lambda > 0$.

There are some limitations related to definition 3[16], and some novel operations are defined.

Definition 2.4. *Let* $x = (T_1, I_1, F_1)$ *and* $y = (T_2, I_2, F_2)$ *be two SVNNs, then operational relations are defined as follows:*

(1). $x \oplus y = (T_1 + T_2 - T_1 T_2, I_1 I_2, F_1 F_2)$.

(2). $x \otimes y = (T_1 T_2, I_1 + I_2 - I_1 I_2, F_1 + F_2 - F_1 F_2)$.

(3). $\lambda x = (1 - (1 - T)_1^\lambda, I_1^\lambda, F_1^\lambda)$.

(4). $x^\lambda = (T_1^\lambda, 1 - (1 - I)_1^\lambda, 1 - (1 - F)_1^\lambda), \lambda > 0$

Definition 2.5. *Let* $x = (T_1, I_1, F_1)$ *and* $y = (T_2, I_2, F_2)$ *be any two SVNNs, then the Hamming distance between x and y can be defined as follows:*

$$d(x, y) = |T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2| \tag{1}$$

Definition 2.6. *Let* $x(T, I, F)$ *be a SVNN, and the cosine similarity measure* $S(x)$ *between SVNN x and the ideal alternative* $(1, 0, 0)$ *can be defined as follows:*

$$S(x) = \frac{T}{\sqrt{T^2 + I^2 + F^2}} \tag{2}$$

3.   Rank Techniques in Multi-Criteria Decision-Making Method

Definition 3.1. *Let* $\tilde{a}_j = (t_j, 1 - f_j)$, $j = 1, 2, \ldots, n$ *be a collection of vague values, and let the vague fuzzy weighted averaging operator VWA is defined as* $VWA : Q_n \to Q$ *if* $VMA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = \sum_{j=1}^{n} w_j \tilde{a}_j = (1 - \prod_{j=1}^{n} (1 - t_j)^{w_j}, \prod_{j=1}^{n} (1 - f_j)^{w_j}$ *where the weight vector* $w = (w_1, w_2, \ldots, w_m)^T$ *of the attributes can be determined in advance. Note that* $w_i > 0$ *for each* $i = 1$ *to n, and* $\sum_{j=1}^{n} w_j = 1$.

Definition 3.2. *Let $\widetilde{a}_j = (t_j, f_j)$, $j = 1, 2, ..., n$ be a collection of vague values, and let the vague fuzzy hybrid weighting average operator VHA is defined as $VHA : Q_n \to Q$ if $VHA_w(\tilde{a}_1, \tilde{a}_2, \ldots, \tilde{a}_n) = (1 - \prod_{j=1}^{n}(1 - t_j)^{w_j}, 1 - \prod_{j=1}^{n}(1 - f_j)^{w_j}$ where the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ of the attributes can be determined in advance. Note that $w_i > 0$ for each $i = 1$ to n, and $\sum_{j=1}^{n} w_j = 1$.*

3.1. Model Assumptions and procedures

Let $A = \{A_1, A_2, ..., A_n\}$ be a set of alternatives, $G = \{G_1, G_2, ..., G_n\}$ be the set of alternatives, $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is the weighting vector of the attribute G_j, $j = 1, 2, \ldots, n$, where $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$. Let $D = \{D_1, D_2, ..., D_t\}$ be the set of decision makers, $V = (V_1, V_2, ..., V_n)$ be the weighting vector of the decision makers, with $V_k \in [0, 1]$, $\sum_{k=1}^{t} V_k = 1$. Let $\widetilde{R}_k = \left(\widetilde{r}_{ij}^{(k)}\right)_{m \times n} = \left(t_{ij}^{(k)}, f_{ij}^{(k)}\right)_{m \times n}$ be the vague decision matrix, where $t_{ij}^{(k)}$ is the degree of the truth membership value that the alternative A_i satisfies the attribute G_j given by the decision maker D_k and $f_{ij}^{(k)}$ is the degree of false membership value that the alternative for the alternative A_i, where $t_{ij}^{(k)}, f_{ij}^{(k)} \subset [0, 1]$ and, $t_{ij}^{(k)} + f_{ij}^{(k)} \leq 1$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, t$.

3.2. An algorithm for a developed model of MAGDM

Here the steps mentioned below are studied for a model of MAGDM.

Algorithm: The following steps are now given:

Step 1: Utilize the vague decision matrix $R^k = (\tilde{r}_{ij})_{m \times n}^{(k)} = ((t_{ij})^k, 1 - (f_{ij})^k)_{m \times n}$, and the FWA operator which has the associated weighting vector $w = (w_1, w_2, \ldots, w_m)^T$ generated from the Definition 3.3. Let $(\tilde{r}_{ij})^k = (t_{ij}^k, 1 - f_{ij}^k)$, $i = 1, 2, \ldots, m$; $j = 1, 2, ..., n$ be a matrix of vague values for each $k = 1$ to t. Let $R^k = ((\tilde{r}_{ij})^{(k)})$ be the collection of t number of $m \times n$ matrices of each the form $R^k = ((\tilde{r}_{ij})^{(k)})$ where $k = 1, 2, \ldots, t$. Then the operator $FWA : [(M_{m \times n})^k \to (M_{m \times n})]$, $R^1, R^2, \ldots, R^k) \to R(r_{ij})$ is defined by $VWA((\tilde{r}_{ij})^{(1)}, (\tilde{r}_{ij})^{(2)}, \ldots, (\tilde{r}_{ij})^{(k)})$ which is found due to the Definition 3.1. Here $V = (V_1, V_2, ..., V_t)$ be the weighting vector of the decision maker or generated from the Definition 3.3.

Step 2: Utilizing the information from the collective decision matrix $R = (C_{ij}, D_{ij})_{m \times n}$ found in the Step 1. Then NFHWA operator $R = \widetilde{r}_i = (t_i, 1 - f_i)$ is defined by $(1 - (\prod_{j=1}^{n}(1 - c_{ij})^{w_j}), 1 - (\prod_{j=1}^{n}(1 - c_{ij})^{w_j})$, $i = 1, 2, \ldots, m$ derive the collective overall preference values of the alternative A_i, which have weight w_i in such a way that the weighting vector as $w = (w_1, w_2, \ldots, w_m)^T$ generated from the Definition 3.3.

Step 3: Calculate the distance between the collective overall preference values $\widetilde{r}_i$ and the positive ideal vague value $\widetilde{r}^+$, or the negative ideal vague value $\widetilde{r}^-$, where $\widetilde{r}^+ = (1, 0)$ and $\widetilde{r}^- = (0, 1)$. Using the Euclidean distance function we can find the distances between the collective overall preference values $\widetilde{r}_i$ and the positive ideal vague value $\widetilde{r}^+$ as follows:

$$d(\widetilde{r}_i, \widetilde{r}^+) = \sqrt{\frac{1}{2} \sum_{i=1}^{n} [(t_{\widetilde{r}_i}(x_i) - t_{\widetilde{r}^+}(x_i))^2 + ((1 - f_{\widetilde{r}_i}(x_i)) - (1 - f_{\widetilde{r}^+}(x_i)))^2]}$$

Step 4: Rank all the alternatives A_i, where $i = 1, 2, \ldots, m$ and select the best one in accordance with the distance obtained in Step 3.

3.3. Numerical illustration

Suppose an investment company, wanting to invest a sum of money in the best option, and there is a panel with five possible alternatives to invest the money; A_1 is an IT company; A_2 is a multinational company; A_3 is a tools company, A_4 is an airlines company and A_5 is an automobile company. The investment company must take a decision according to the four

following attributes; G_1 is the risk analysis, G_2 is the growth analysis, G_3 is the socio-political impact analysis and G_4 is the environmental impact analysis. The five possible alternatives A_i, where $i = 1, 2, \ldots, m$, are to be evaluated by three decision makers whose weighting vector is $V = (0.12, 0.16, 0.20, 0.24, 0.28)^T$ under the method in Definition 3.3 with $r = 1$, $\alpha = 0.4$, and $n = 5$, and above said four attributes whose weighting vector is $w = (0.16, 0.22, 0.28, 0.34)^T$, which is generated from the method in 3.3 with $r = 1$, $\alpha = 0.4$, and $n = 4$:

$$R_1 = \begin{bmatrix}
(0.4873, 0.7256) & (0.5221, 0.7222) & (0.6286, 0.8312) & (0.4427, 0.9986) \\
(0.3271, 0.9001) & (0.6676, 0.5413) & (0.4261, 0.8126) & (0.7710, 0.9442) \\
(0.5238, 0.8011) & (0.4278, 0.5261) & (0.5527, 0.6216) & (0.5687, 0.7981) \\
(0.7218, 0.6283) & (0.7213, 0.8912) & (0.8311, 0.9219) & (0.6626, 0.8215) \\
(0.6257, 0.7983) & (0.8321, 0.9426) & (0.6256, 0.7119) & (0.4136, 0.6295)
\end{bmatrix}$$

$$R_2 = \begin{bmatrix}
(0.4351, 0.7846) & (0.5121, 0.7221) & (0.1009, 0.6221) & (0.2217, 0.7184) \\
(0.6321, 0.8221) & (0.6226, 0.8108) & (0.3009, 0.5129) & (0.6225, 0.9105) \\
(0.5387, 0.9105) & (0.4124, 0.7216) & (0.5010, 0.7101) & (0.4491, 0.5426) \\
(0.7317, 0.8119) & (0.5221, 0.8001) & (0.2091, 0.4104) & (0.2101, 0.4110) \\
(0.5273, 0.6217) & (0.3125, 0.7278) & (0.4728, 0.7182) & (0.6210, 0.8109)
\end{bmatrix}$$

$$R_3 = \begin{bmatrix}
(0.3198, 0.8279) & (0.4419, 0.9816) & (0.2211, 0.5221) & (0.6661, 0.7027) \\
(0.7726, 0.8901) & (0.6245, 0.7815) & (0.6216, 0.8225) & (0.7101, 0.9005) \\
(0.5201, 0.7287) & (0.5821, 0.6286) & (0.7117, 0.9211) & (0.6105, 0.9117) \\
(0.3247, 0.4821) & (0.7139, 0.8148) & (0.4212, 0.5334) & (0.5529, 0.7217) \\
(0.7351, 0.9113) & (0.8001, 0.9112) & (0.2221, 0.6121) & (0.4214, 0.5005)
\end{bmatrix}$$

$$R_4 = \begin{bmatrix}
(0.3269, 0.9111) & (0.4575, 0.8222) & (0.5527, 0.8686) & (0.8421, 0.9526) \\
(0.6321, 0.8108) & (0.5010, 0.8126) & (0.7317, 0.9191) & (0.7351, 0.9005) \\
(0.3247, 0.4821) & (0.7227, 0.9444) & (0.5529, 0.8287) & (0.1553, 0.9891) \\
(0.8112, 0.9238) & (0.2091, 0.9545) & (0.4494, 0.8898) & (0.6522, 0.7377) \\
(0.1983, 0.9916) & (0.3125, 0.8278) & (0.7428, 0.8482) & (0.6217, 0.9197)
\end{bmatrix}$$

$$R_5 = \begin{bmatrix}
(0.2686, 0.8812) & (0.4427, 0.8986) & (0.6676, 0.8126) & (0.7717, 0.9552) \\
(0.5218, 0.7918) & (0.4278, 0.7176) & (0.8311, 0.9519) & (0.4163, 0.7295) \\
(0.5122, 0.9100) & (0.2091, 0.5515) & (0.2101, 0.8126) & (0.3125, 0.8287) \\
(0.3198, 0.9728) & (0.4491, 0.9861) & (0.6261, 0.8522) & (0.7101, 0.9552) \\
(0.5210, 0.6268) & (0.7711, 0.9211) & (0.6195, 0.7119) & (0.7513, 0.9311)
\end{bmatrix}$$

3.4.   Explanation: The steps for the given algorithm are as follows

Step 1: Utilizing the decision information given in the matrix $\widetilde{R}_k = \left(\widetilde{r}_{ij}^{(k)} \right)_{5 \times 4}$, $k = 1, 2, 3, 4, 5$ and the VWA operator which has the associated weighting vector $w = (0.28, 0.24, 0.2, 0.16, 0.12)^T$ a collective decision matrix $\widetilde{R}_k = \left(\widetilde{r}_{ij}^{(k)} \right)_{5 \times 4}$ is obtained as follows:

$$R = \begin{bmatrix}
(0.3948, 0.8059) & (0.4850, 0.8048) & (0.4587, 0.7095) & (0.5999, 0.8490) \\
(0.5917, 0.8510) & (0.6001, 0.7085) & (0.5777, 0.7582) & (0.6901, 0.8921) \\
(0.4980, 0.7588) & (0.4994, 0.6887) & (0.5497, 0.7507) & (0.4723, 0.7767) \\
(0.6554, 0.7103) & (0.5392, 0.8729) & (0.5841, 0.6702) & (0.5680, 0.6785) \\
(0.5702, 0.7763) & (0.6829, 0.8594) & (0.5562, 0.7118) & (0.5570, 0.7115)
\end{bmatrix}$$

Step 2: Utilizing the VFHWA operator, the collective overall preference values of the alternatives A_i, $j = 1, 2, \ldots, 5$ are found mentioned below. Using the weighting vector $w = (0.34, 0.28, 0.22, 0.16)$,

$$\widetilde{r_1} = (0.4718, 0.7960);$$

$$\widetilde{r_2} = (0.6087, 0.8101);$$

$$\widetilde{r_3} = (0.5064, 0.7423);$$

$$\widetilde{r_4} = (0.5961, 0.7594);$$

$$\widetilde{r_5} = (0.6006, 0.7837);$$

Step 3: Calculating the distances between the collective overall preference values $\widetilde{r}_i$ and the positive ideal vague value $\tilde{r} = (1, 0, 0)$. The distances calculated from the following distance function:

$$d(\tilde{r}_i, \tilde{r}) = \sqrt{\frac{1}{2} \sum_{i=1}^{n} \left[(t_{\widetilde{r}_i} - t_{\tilde{r}})^2 + ((1 - f_{\widetilde{r}_i}) - (1 - f_{\tilde{r}}))^2 \right]}$$

Thus

$$d(\tilde{r}_1, \tilde{r}) = 0.6755;$$

$$d(\tilde{r}_2, \tilde{r}) = 0.6361;$$

$$d(\tilde{r}_3, \tilde{r}) = 0.3937;$$

$$d(\tilde{r}_4, \tilde{r}) = 0.3324;$$

$$d(\tilde{r}_5, \tilde{r}) = 0.6219;$$

Step 4: Rank the alternatives based on the shortest distance: $A_1 > A_2 > A_5 > A_3 > A_4$. Thus A_1 is the best alternative. Let us consider the replacing of Step 3 with the correlation coefficient proposed Robinson & Amirtharaj [?]. Then the ranking order of the alternatives is obtained as follows. Thus $A_1 > A_2 > A_5 > A_3 > A_4$. Thus A_1 is the best alternative. From the comparison, it can be observed that there is a change in the ranking of the best alternatives. In the proposed method with a distance function, A_1 is the best alternative, and with the replacement of Step 3 in the algorithm with methods as proposed by Robinson & Amirtharaj [?], it can be seen that A_1 is the best alternative.

4. Second Model-Correlation Between Neotrosopic Fuzzy Set

4.1. Introduction and formulae

Robinson & Amirtharaj [?] proposed a correlation coefficient for vague sets which took into account the truth membership degree, false membership degree and the hesitation or vague degree and derived it in the interval $[0, 1]$. Let $X = \{x_1, x_2, ..., x_n\}$ be the finite universal set and $A, B \in NFS(X)$ be given by $A = \{\langle x, [t_A (x), 1 - f_A (x)]\rangle / x \in X\}$, $B = \{\langle x, [t_B (x), 1 - f_B (x)]\rangle / x \in X\}$.

4.2. Formulae

The length of the vague values are given by $\pi_A(x) = 1 - t_A(x) - f_A(x)$, and $\pi_B(x) = 1 - t_B(x) - f_B(x)$.

(a). Now for each $A \in VS(X)$, the informational vague energy of A is defined as follows

$$E_{VS}(A) = \frac{1}{n} \sum_{i=1}^{n} \left[t_A^2 (x_i) + (1 - f_A (x_i))^2 + \pi_A^2 (x_i) \right]. \tag{3}$$

(b). And for each $B \in VS(X)$, the informational vague energy of B is defined as follows

$$E_{VS}(B) = \frac{1}{n} \sum_{i=1}^{n} \left[t_B^2(x_i) + (1 - f_B(x_i))^2 + \pi_B^2(x_i) \right]. \tag{4}$$

(c). The correlation of A and B is given by the formula

$$C_{VS}(A, B) = \frac{1}{n} \sum_{i=1}^{n} \left[t_A(x_i) t_B(x_i) + (1 - f_A(x_i))(1 - f_B(x_i)) + \pi_A(x_i) \pi_B(x_i) \right]. \tag{5}$$

(d). Furthermore, the correlation coefficient of A and B is defined by the formula:

$$K_{VS}(A, B) = \frac{C_{VS}(A, B)}{\sqrt{E_{VS}(A).E_{VS}(B)}}, \quad where \;\; 0 \leq K_{VS}(A, B) \leq 1. \tag{6}$$

MODEL II: Consider $A = \{(T_A(x), I_A(x), F_A(x)) : x \in X\}$, and $B = \{(T_B(x), I_B(x), F_B(x)) : x \in X\}$ be two given neutrosophic fuzzy sets as follows.

$$A = \begin{bmatrix} \langle 0.5, 0.6, 0.3 \rangle & \langle 0.25, 0.3, 0.4 \rangle & \langle 0.17, 0.8, 0.1 \rangle & \langle 0.8, 0.6, 0.1 \rangle \\ \langle 0.6, 0.2, 0.3 \rangle & \langle 0.1, 0.4, 0.8 \rangle & \langle 0.3, 0.5, 0.2 \rangle & \langle 0.6, 0.25, 0.3 \rangle \\ \langle 0.25, 0.45, 0.5 \rangle & \langle 0.4, 0.75, 0.1 \rangle & \langle 0.35, 0.75, 0.15 \rangle & \langle 0.37, 0.68, 0.16 \rangle \\ \langle 0.45, 0.38, 0.27 \rangle & \langle 0.07, 0.8, 0.6 \rangle & \langle 0.36, 0.74, 0.18 \rangle & \langle 0.64, 0.24, 0.28 \rangle \end{bmatrix}$$

$$B = \begin{bmatrix} \langle 0.28, 0.7, 0.6 \rangle & \langle 0.16, 0.3, 0.6 \rangle & \langle 0.37, 0.5, 0.3 \rangle & \langle 0.17, 0.6, 0.65 \rangle \\ \langle 0.3, 0.1, 0.5 \rangle & \langle 0.2, 0.6, 0.7 \rangle & \langle 0.4, 0.4, 0.3 \rangle & \langle 0.7, 0.35, 0.2 \rangle \\ \langle 0.35, 0.55, 0.6 \rangle & \langle 0.23, 0.1, 0.6 \rangle & \langle 0.45, 0.65, 0.25 \rangle & \langle 0.47, 0.89, 0.27 \rangle \\ \langle 0.6, 0.2, 0.2 \rangle & \langle 0.27, 0.6, 0.10 \rangle & \langle 0.63, 0.47, 0.21 \rangle & \langle 0.41, 03, 0.28 \rangle \end{bmatrix}$$

By the formulae, it obtains that $E_A = 0.9871$ by using calculations in (3). $E_B = 0.9073$ by using calculations in (4). $C_{A,B} = 0.8614$ by using calculations in (5). Correlation $= 0.9102$ by using calculations in (6).

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