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Multiplicative Connectivity Indices of $TUC_4C_8(R)$ Nanotube

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Abstract: Chemical graph theory is an important branch of mathematical chemistry which has wide range applications. In chemical graph theory, a molecular graph or chemical graph is a representation of the structural formula of a chemical compound in terms of graph theory. Degree based topological indices have vital role in chemical graph theory. In this paper we compute the degree based topological indices like Multiplicative first and second Zagreb, Multiplicative first and second Hyper-Zagreb, General first and second Multiplicative Zagreb, Multiplicative Sum connectivity, Multiplicative product connectivity, Multiplicative ABC and Multiplicative GA indices of TUC_4C_8 Rhomboidal Nanotubes.

MSC: 05C05, 05C07, 05C90.

Keywords: Topological index, molecular graphs, degree based topological index, multiplicative indices, TUC_4C_8 nanotubes.

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1. Introduction

In this paper, we consider finite simple undirected graphs without loops and multiple edges. A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds (hydrogen atoms are often omitted). A numerical quantity which recognize the molecular graph in chemical graph theory. This is called a Topological Index. In chemical science, the physico-chemical properties of chemical compounds are often modeled by means of a molecular graph based structure descriptors, which are referred to as topological indices. A C_4C_8 net is a trivalent decoration made by alternating rhombs C_4 and octagons C_8. It can cover either a cylinder or a torus. Such a covering can be derived from a square net by the leapfrog operation. Let $TUC_4C_8(R)$ denote the C_4C_8 rhomboidal nanotube. An example is shown in Figure 1.

Figure 1. Three-dimensional perception of a $TUC_4C_8(R)$ nanotube

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In this paper, we determine some topological indices for a family of linear [n] $TUC_4C_8(R)$, lattice of $TUC_4C_8(R)$ nanotube and nanotori. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. In [1] Todeshine et.al introduced the First and Second multiplicative Zagreb indices. These indices are defined as

$$II_1(G) = \prod_{u \in V(G)} d_G(u)^2,$$
$$II_2(G) = \prod_{uv \in E(G)} d_G(u)d_G(v).$$

In [2], Eliasi et.al introduced a new multiplicative version of the first Zagreb index as

$$II_1^*(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)].$$

In [3], Kulli proposed the first and second multiplicative hyper-Zagreb indices as

$$HII_1(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)]^2,$$
$$HII_2(G) = \prod_{uv \in E(G)} [d_G(u)d_G(v)]^2.$$

In [4], Kulli, Stone, Wang and Wei introduced the general first and second multiplicative Zagreb indices. These indices are defined as

$$MZ_1^a(G) = \prod_{uv \in E(G)} [d_G(u) + d_G(v)]^a,$$
$$MZ_2^a(G) = \prod_{uv \in E(G)} [d_G(u)d_G(v)]^a.$$

In [6], one of the best known and widely used topological index is the product connectivity index or Randic index introduced by Randic. These are defined as

$$\chi(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}.$$

From this randic index, In [6], Kulli introduced the multiplicative sum connectivity index, multiplicative product connectivity index, multiplicative atom bond connectivity index and multiplicative geometric-arithmetic index. The multiplicative sum connectivity index of a graph G is defined as

$$XII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}.$$

The multiplicative product connectivity index of a graph G is defined as

$$\chi II(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}.$$

The multiplicative atom bond connectivity index of a graph G is defined as

$$ABCII(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u)d_G(v)}}.$$

The multiplicative geometric-arithmetic index of a graph G is defined as follows:

$$GAII(G) = \prod_{uv \in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u) + d_G(v)}.$$

Recently many other multiplicative indices were studied in [7–13]. Also various topological indices like Wiener index, eccentric connectivity index and Schultz molecular topological index for $TUC_4C_8(R)$ nanotube in [14–16]. In this paper, we compute the multiplicative Zagreb, multiplicative hyper-Zagreb, multiplicative sum connectivity, multiplicative product connectivity, general multiplicative Zagreb, multiplicative ABC and multiplicative GA indices for linear [n] $TUC_4C_8(R)$, lattice of $TUC_4C_8(R)$ nanotube and nanotori.

2. Results and Discussion

Throughout this paper, we using the following Notations.

Result 2.1.

$|V|$: *Number of vertices in the Molecular graph.*

$|E|$: *Number of edges in the Molecular graph.*

$|E_1|$: *Number of edges having endpoints (2,2).*

$|E_2|$: *Number of edges having endpoints (2,3).*

$|E_3|$: *Number of edges having endpoints (3,3).*

C : *Number of Columns.*

R : *Number of Rows.*

2.1. Results for linear [n] $TUC_4C_8(R)$

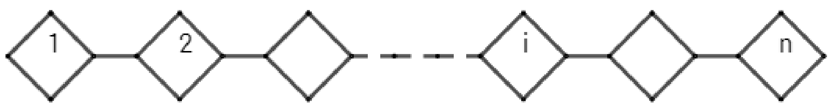

Figure 2. **The molecular graph of a linear [n]** $TUC_4C_8(R)$

Lemma 2.2. *It holds that*

Nanostructure	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert E_1\rvert$	$\lvert E_2\rvert$	$\lvert E_3\rvert$
T	$4C$	$5C-1$	4	$4C-4$	$C-1$

Table 1. **Computing the number of vertices and edges for linear [n]** $TUC_4C_8(R)$

Proof. In this case we take $R = 1$, $C = 2$. Here there exist three types of edges, namely $[E_1] = uv$, $[E_2] = xy$ and $[E_3] = ab$. Also $d(u) = d(v) = 2$; $d(a) = d(b) = 3$; $d(x) = 3$, $d(y) = 2$. Therefore, Number of edges of type 1, type 2 and type 3 are 4, 4, 1. Now, it is easy to see that $T = T[n]$ has 4C vertices and $4C - 1$ edges.

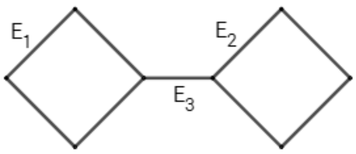

Figure 3. **Basic Structure of a** $TUC_4C_8(R)$

R	C	$\|V\|$	$\|E\|$	$\|E_1\| = (2,2)$	$\|E_2\| = (2,3)$	$\|E_3\| = (3,3)$
1	1	4	4	4	0	0
1	2	8	9	4	4	1
1	3	12	14	4	8	2

Table 2. **Computing numbers edges and vertices for linear [n]** $TUC_4C_8(R)$ **from Figure 3.**

From the table, by using an algebraic method we obtain $|E_1| = 4$, $|E_2| = 4C - 4$ and $|E_3| = C - 1$, $|V| = 4C$ and $|E| = 5C - 1$. $\square$

Theorem 2.3. *Let T be a linear [n] $TUC_4C_8(R)$. Then*

(1). $II_1^*(T) = 2^{C+7} \times 3^{C-1} \times 5^{4C-4}$.

(2). $II_2(T) = 2^{4C+4} \times 3^{6C-6}$.

(3). $HII_1(T) = 2^{2C+14} \times 3^{2C-2} \times 5^{8C-8}$.

(4). $HII_2(T) = 2^{8C+8} \times 3^{12C-12}$.

(5). $XII(T) = 2^{\frac{-(7+C)}{2}} \times 3^{\frac{(1-C)}{2}} \times 5^{\frac{(4-4C)}{2}}$.

(6). $\chi II(T) = 2^{-2(C+1)} \times 3^{3-3C}$.

(7). $MZ_1^a(T) = 2^{aC+7a} \times 3^{aC-a} \times 5^{4aC-4a}$.

(8). $MZ_2^a(T) = 2^{a(4C+4)} \times 3^{a(6C-6)}$.

(9). $ABCII(T) = 2^{-C} \times 3^{1-C}$.

(10). $GAII(T) = \left(\frac{2\sqrt{6}}{5}\right)^{4C-4}$.

Proof. From the definitions of multiplicative indices and partition of edges described in Table 1 of Lemma 2.2, we can see that

$$
\begin{aligned}
(1).\ II_1^*(T) &= \prod_{uv \in E(T)} [d_T(u) + d_T(v)] \\
&= \prod_{uv \in E_1} 4 \times \prod_{uv \in E_2} 5 \times \prod_{uv \in E_3} 6 \\
&= 4^4 \times 5^{4C-4} \times 6^{C-1} \\
&= 2^{C+7} \times 3^{C-1} \times 5^{4C-4}.
\end{aligned}
$$

(2). $II_2(T) = \prod_{uv\in E(T)} d_T(u)d_T(v)$

$$= \prod_{uv\in E_1} 4 \times \prod_{uv\in E_2} 6 \times \prod_{uv\in E_3} 9$$

$$= 4^4 \times 6^{4C-4} \times 9^{C-1}$$

$$= 2^{4C+4} \times 3^{6C-6}.$$

(3). $HII_1(T) = \prod_{uv\in E(T)} [d_T(u) + d_T(v)]^2$

$$= \prod_{uv\in E_1} 4^2 \times \prod_{uv\in E_2} 5^2 \times \prod_{uv\in E_3} 6^2$$

$$= 4^8 \times 5^{2(4C-4)} \times 6^{2(C-1)}$$

$$= 2^{2C+14} \times 3^{2C-2} \times 5^{8C-8}.$$

(4). $HII_2(T) = \prod_{uv\in E(T)} [d_T(u)d_T(v)]^2$

$$= \prod_{uv\in E_1} 4^2 \times \prod_{uv\in E_2} 6^2 \times \prod_{uv\in E_3} 9^2$$

$$= 4^8 \times 6^{2(4C-4)} \times 9^{2(C-1)}$$

$$= 2^{8C+8} \times 3^{12C-12}.$$

(5). $XII(T) = \prod_{uv\in E(T)} \frac{1}{\sqrt{d_T(u) + d_T(v)}}$

$$= \prod_{uv\in E_1} \frac{1}{\sqrt{4}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{5}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{6}}$$

$$= \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{\sqrt{5}}\right)^{4C-4} \times \left(\frac{1}{\sqrt{6}}\right)^{C-1}$$

$$= 2^{\frac{-(7+C)}{2}} \times 3^{\frac{(1-C)}{2}} \times 5^{\frac{(4-4C)}{2}}.$$

(6). $\chi II(T) = \prod_{uv\in E(T)} \frac{1}{\sqrt{d_T(u)d_T(v)}}$

$$= \prod_{uv\in E_1} \frac{1}{\sqrt{4}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{6}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{9}}$$

$$= \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{\sqrt{6}}\right)^{4C-4} \times \left(\frac{1}{\sqrt{9}}\right)^{C-1}$$

$$= 2^{-2(C+1)} \times 3^{3-3C}.$$

(7). $MZ_1^a(T) = \prod_{uv\in E(T)} [d_T(u) + d_T(v)]^a$

$$= \prod_{uv\in E_1} 4^a \times \prod_{uv\in E_2} 5^a \times \prod_{uv\in E_3} 6^a$$

$$= 4^{4a} \times 5^{a(4C-4)} \times 6^{a(C-1)}$$

$$= 2^{aC+7a} \times 3^{aC-a} \times 5^{4aC-4a}.$$

$$
\begin{aligned}
(8).\ MZ_2^a(T) &= \prod_{uv\in E(T)} [d_T(u)d_T(v)]^a \\
&= \prod_{uv\in E_1} 4^a \times \prod_{uv\in E_2} 6^a \times \prod_{uv\in E_3} 9^a \\
&= 4^{4a} \times 6^{a(4C-4)} \times 9^{a(C-1)} \\
&= 2^{a(4C+4)} \times 3^{a(6C-6)}.
\end{aligned}
$$

$$
\begin{aligned}
(9).\ ABCII(T) &= \prod_{uv\in E(T)} \sqrt{\frac{d_T(u)+d_T(v)-2}{d_T(u)d_T(v)}} \\
&= \prod_{uv\in E_1} \sqrt{\frac{2+2-2}{2\times 2}} \times \prod_{uv\in E_2} \sqrt{\frac{3+2-2}{3\times 2}} \times \prod_{uv\in E_3} \sqrt{\frac{3+3-2}{3\times 3}} \\
&= 2^{-C} \times 3^{1-C}
\end{aligned}
$$

$$
\begin{aligned}
(10).\ GAII(T) &= \prod_{uv\in E(T)} \frac{2\sqrt{d_T(u)d_T(v)}}{d_T(u)+d_T(v)} \\
&= \prod_{uv\in E_1} \frac{2\sqrt{2\times 2}}{2+2} \times \prod_{uv\in E_2} \frac{2\sqrt{3\times 2}}{3+2} \times \prod_{uv\in E_3} \frac{2\sqrt{3\times 3}}{3+3} = \left(\frac{2\sqrt{6}}{5}\right)^{4C-4}.
\end{aligned}
$$

□

2.2. Results for $TUC_4C_8(R)$ Nanotube

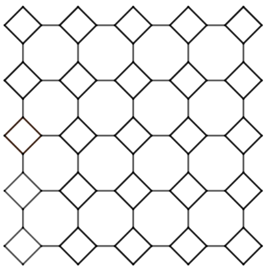

Figure 4. **2-dimensional lattice of $TUC_4C_8(R)$ nanotube**

Lemma 2.4. *It holds that*

Nanostructure	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert E_1\rvert$	$\lvert E_2\rvert$	$\lvert E_3\rvert$
G	$4RC$	$6RC-C-R$	4	$4C+4R-8$	$6RC-5C-5R+4$

Table 3. **Computing the number of vertices and edges for $TUC_4C_8(R)$ nanotube**

Proof.

R	C	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert E_1\rvert=(2,2)$	$\lvert E_2\rvert=(2,3)$	$\lvert E_3\rvert=(3,3)$	No. of Square	No. of Octagon
Case 1: $R=C$								
2	2	16	20	4	8	8	4	1
3	3	36	48	4	16	28	9	4
4	4	64	88	4	24	60	16	9
Case 2: $R<C$								
2	3	24	31	4	12	15	6	2
3	4	48	65	4	20	41	12	6
4	5	80	111	4	28	79	20	12

R	C	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert E_1\rvert = (2,2)$	$\lvert E_2\rvert = (2,3)$	$\lvert E_3\rvert = (3,3)$	No. of Square	No. of Octagon
Case 3: $R > C$								
3		24	31	4	12	15	6	2
4	3	48	65	4	20	41	12	6
5	4	80	111	4	28	79	20	12

Table 4. **Computing numbers edges and vertices for $TUC_4C_8(R)$ nanotube from Figure 4**

From the table, by using an algebraic method we obtain $|E_1| = 4$, $|E_2| = 4C+4R-8$, $|E_3| = 6RC-5C-5R+4$, $|V| = 4RC$ and $|E| = 6RC - C - R$. Also we find Number of Square= RC and Number of Octagon= $(R-1)(C-1)$. $\square$

Theorem 2.5. *Let G be a 2-dimensional lattice of $TUC_4C_8(R)$ nanotube. Then*

(1). $II_1^*(G) = 2^{12-5C-5R+6RC} \times 3^{4-5C-5R+6RC} \times 5^{-8+4C+4R}$.

(2). $II_2(G) = 2^{4C+4R} \times 3^{12RC-6C-6R}$.

(3). $HII_1(G) = 2^{12RC-10C-10R+24} \times 3^{12RC-10C-10R+8} \times 5^{8C+8R-16}$.

(4). $HII_2(G) = 2^{8C+8R} \times 3^{24RC-12C-12R}$.

(5). $XII(G) = 2^{\frac{-6RC+5C+5R-12}{2}} \times 3^{\frac{(-6RC+5C+5R-4)}{2}} \times 5^{\frac{(-4C-4R+8)}{2}}$.

(6). $\chi II(G) = 2^{-2(R+C)} \times 3^{3(-2RC+R+C)}$.

(7). $MZ_1^a(G) = 2^{C(6R-5)a+(12-5r)a} \times 3^{C(6R-5)a+(4-5R)a} \times 5^{a(4C+4R-8)}$.

(8). $MZ_2^a(G) = 2^{4a(R+C)} \times 3^{6a(2RC-C-R)}$.

(9). $ABCII(G) = 2^{6RC-7C-7R+6} \times 3^{-6RC+5C+5R-4}$.

(10). $GAII(G) = \left(\frac{2\sqrt{6}}{5}\right)^{4C+4R-8}$.

Proof. From the definitions of multiplicative indices and partition of edges described in Table 3 of Lemma 2.3, we can see that

(1).
$$\begin{aligned} II_1^*(G) &= \prod_{uv\in E(G)} [d_G(u) + d_G(v)] \\ &= \prod_{uv\in E_1} 4 \times \prod_{uv\in E_2} 5 \times \prod_{uv\in E_3} 6 \\ &= 4^4 \times 5^{4C+4R-8} \times 6^{6RC-5C-5R+4} \\ &= 2^{12-5C-5R+6RC} \times 3^{4-5C-5R+6RC} \times 5^{-8+4C+4R}. \end{aligned}$$

(2).
$$\begin{aligned} II_2(G) &= \prod_{uv\in E(G)} d_G(u)d_G(v) \\ &= \prod_{uv\in E_1} 4 \times \prod_{uv\in E_2} 6 \times \prod_{uv\in E_3} 9 \\ &= 4^4 \times 6^{4C+4R-8} \times 9^{6RC-5C-5R+4} \\ &= 2^{4C+4R} \times 3^{12RC-6C-6R} \end{aligned}$$

(3). $HII_1(G) = \prod_{uv\in E(G)} [d_G(u) + d_G(v)]^2$

$$= \prod_{uv\in E_1} 4^2 \times \prod_{uv\in E_2} 5^2 \times \prod_{uv\in E_3} 6^2$$

$$= 4^8 \times 5^{2(4C+4R-8)} \times 6^{2(6RC-5C-5R+4)}$$

$$= 2^{12RC-10C-10R+24} \times 3^{12RC-10C-10R+8} \times 5^{8C+8R-16}$$

(4). $HII_2(G) = \prod_{uv\in E(G)} [d_G(u)d_G(v)]^2$

$$= \prod_{uv\in E_1} 4^2 \times \prod_{uv\in E_2} 6^2 \times \prod_{uv\in E_3} 9^2$$

$$= 4^8 \times 6^{2(4C+4R-8)} \times 9^{2(6RC-5C-5R+4)}$$

$$= 2^{8C+8R} \times 3^{24RC-12C-12R}$$

(5). $XII(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}}$

$$= \prod_{uv\in E_1} \frac{1}{\sqrt{4}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{5}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{6}}$$

$$= \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{\sqrt{5}}\right)^{4C+4R-8} \times \left(\frac{1}{\sqrt{6}}\right)^{6RC-5C-5R+4}$$

$$= 2^{\frac{-6RC+5C+5R-12}{2}} \times 3^{\frac{(-6RC+5C+5R-4)}{2}} \times 5^{\frac{(-4C-4R+8)}{2}}$$

(6). $\chi II(G) = \prod_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$

$$= \prod_{uv\in E_1} \frac{1}{\sqrt{4}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{6}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{9}}$$

$$= \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{\sqrt{6}}\right)^{4C+4R-8} \times \left(\frac{1}{\sqrt{9}}\right)^{6RC-5C-5R+4}$$

$$= 2^{-2(R+C)} \times 3^{3(-2RC+R+C)}$$

(7). $MZ_1^a(G) = \prod_{uv\in E(G)} [d_G(u) + d_G(v)]^a$

$$= \prod_{uv\in E_1} 4^a \times \prod_{uv\in E_2} 5^a \times \prod_{uv\in E_3} 6^a$$

$$= 4^{4a} \times 5^{a(4C+4R-8)} \times 6^{a(6RC-5C-5R+4)}$$

$$= 2^{C(6R-5)a+(12-5r)a} \times 3^{C(6R-5)a+(4-5R)a} \times 5^{a(4C+4R-8)}.$$

(8). $MZ_2^a(G) = \prod_{uv\in E(G)} [d_G(u)d_G(v)]^a$

$$= \prod_{uv\in E_1} 4^a \times \prod_{uv\in E_2} 6^a \times \prod_{uv\in E_3} 9^a$$

$$= 4^{4a} \times 6^{a(4C+4R-8)} \times 9^{a(6RC-5C-5R+4)}$$

$$= 2^{4a(R+C)} \times 3^{6a(2RC-C-R)}$$

$$\begin{aligned}
(9).\ ABCII(G) &= \prod_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\
&= \prod_{uv\in E_1} \sqrt{\frac{2+2-2}{2\times 2}} \times \prod_{uv\in E_2} \sqrt{\frac{3+2-2}{3\times 2}} \times \prod_{uv\in E_3} \sqrt{\frac{3+3-2}{3\times 3}} \\
&= \left(\frac{1}{\sqrt{2}}\right)^4 \times \left(\frac{1}{\sqrt{2}}\right)^{4C+4R-8} \times \left(\frac{2}{3}\right)^{6RC-5C-5R+4} \\
&= 2^{6RC-7C-7R+6} \times 3^{-6RC+5C+5R-4}
\end{aligned}$$

$$\begin{aligned}
(10).\ GAII(G) &= \prod_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} \\
&= \prod_{uv\in E_1} \frac{2\sqrt{2\times 2}}{2+2} \times \prod_{uv\in E_2} \frac{2\sqrt{3\times 2}}{3+2} \times \prod_{uv\in E_3} \frac{2\sqrt{3\times 3}}{3+3} \\
&= (1)^4 \times \left(\frac{2\sqrt{6}}{5}\right)^{4C+4R-8} \times (1)^{6RC-5C-5R+4} \\
&= \left(\frac{2\sqrt{6}}{5}\right)^{4C+4R-8}
\end{aligned}$$

□

2.3. Results for $TUC_4C_8(R)$ nanotori

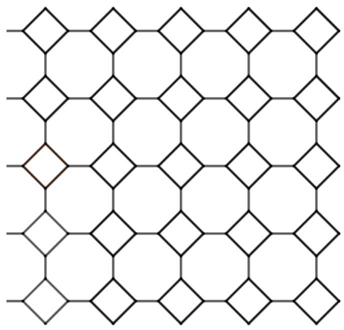

Figure 5. **2-dimensional lattices of $TUC_4C_8(R)$ nanotori**

Lemma 2.6. *It holds that*

Nanostructure	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert E_2\rvert$	$\lvert E_3\rvert$
K	$4RC$	$6RC-C$	$4C$	$6RC-5C$

Table 5. **Computing the number of vertices and edges for $TUC_4C_8(R)$ nanotori**

Proof.

R	C	$\lvert V\rvert$	$\lvert E\rvert$	$\lvert E_2\rvert=(2,3)$	$\lvert E_3\rvert=(3,3)$	No. of Square	No. of Octagon
Case 1: $R=C$							
2	2	16	22	8	14	4	1
3	3	36	51	12	39	9	4
4	4	64	92	16	76	16	9
Case 2: $R<C$							
2	3	24	33	12	21	6	2
3	4	48	68	16	52	12	6
4	5	80	115	20	95	20	12

R	C	$\|V\|$	$\|E\|$	$\|E_2\| = (2,3)$	$\|E_3\| = (3,3)$	No. of Square	No. of Octagon
Case 3: $R > C$							
3	2	24	34	8	26	6	2
4	3	48	69	12	57	12	6
5	4	80	116	16	100	20	12

Table 6. **Computing numbers edges and vertices for $TUC_4C_8(R)$ nanotori from Figure 5.**

From the table, by using an algebraic method we obtain, $|E_2| = 4C$, $|E_3| = 6RC - 5C$, $|V| = 4RC$ and $|E| = 6RC - C$. Also we find Number of Square= RC and Number of Octagon= $(R-1)(C-1)$. $\square$

Theorem 2.7. *Let K be a 2-dimensional lattice of $TUC_4C_8(R)$ nanotori. Then*

(1). $II_1^*(K) = 5^{4C} \times 6^{6RC-5C}$.

(2). $II_2(K) = 2^{4C} \times 3^{C(12R-6)}$.

(3). $HII_1(K) = 5^{8C} \times 6^{12RC-10C}$.

(4). $HII_2(K) = 2^{8C} \times 3^{C(24R-12)}$.

(5). $XII(K) = \left(\frac{1}{5}\right)^{2C} \times \left(\frac{1}{\sqrt{6}}\right)^{6RC-5C}$.

(6). $\chi II(K) = 2^{-2C} \times 3^{C(3-6R)}$.

(7). $MZ_1^a(K) = 5^{4Ca} \times 6^{a(6RC-5C)}$.

(8). $MZ_2^a(K) = 2^{4Ca} \times 3^{C(12R-6)a}$.

(9). $ABCII(K) = 2^{C(6R-7)} \times 3^{C(5-6R)}$.

(10). $GAII(K) = \left(\frac{2\sqrt{6}}{5}\right)^{4C}$.

Proof. From the definitions of multiplicative indices and partition of edges described in Table 5 of Lemma 2.6, we can see that

$$\begin{aligned}(1).\ II_1^*(K) &= \prod_{uv\in E(K)} [d_K(u) + d_K(v)] \\ &= \prod_{uv\in E_2} 5 \times \prod_{uv\in E_3} 6 \\ &= 5^{4C} \times 6^{6RC-5C}\end{aligned}$$

$$\begin{aligned}(2).\ II_2(K) &= \prod_{uv\in E(K)} d_K(u)d_K(v) \\ &= \prod_{uv\in E_2} 6 \times \prod_{uv\in E_3} 9 \\ &= 6^{4C} \times 9^{6RC-5C} \\ &= 2^{4C} \times 3^{C(12R-6)}\end{aligned}$$

$$\begin{aligned}(3).\ HII_1(K) &= \prod_{uv\in E(K)} [d_K(u) + d_K(v)]^2 \\ &= \prod_{uv\in E_2} 5^2 \times \prod_{uv\in E_3} 6^2 \\ &= 5^{2(4C)} \times 6^{2(6RC-5C)} \\ &= 5^{8C} \times 6^{12RC-10C}\end{aligned}$$

(4). $HII_2(K) = \prod\limits_{uv\in E(K)} [d_K(u)d_K(v)]^2$

$$= \prod_{uv\in E_2} 6^2 \times \prod_{uv\in E_3} 9^2$$

$$= 6^{2(4C)} \times 9^{2(6RC-5C)}$$

$$= 2^{8C} \times 3^{C(24R-12)}$$

(5). $XII(K) = \prod\limits_{uv\in E(K)} \frac{1}{\sqrt{d_K(u)+d_K(v)}}$

$$= \prod_{uv\in E_1} \frac{1}{\sqrt{4}} \times \prod_{uv\in E_2} \frac{1}{\sqrt{5}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{6}}$$

$$= \left(\frac{1}{\sqrt{5}}\right)^{4C} \times \left(\frac{1}{\sqrt{6}}\right)^{6RC-5C}$$

$$= \left(\frac{1}{5}\right)^{2C} \times \left(\frac{1}{\sqrt{6}}\right)^{6RC-5C}$$

(6). $\chi II(K) = \prod\limits_{uv\in E(K)} \frac{1}{\sqrt{d_K(u)d_K(v)}}$

$$= \prod_{uv\in E_2} \frac{1}{\sqrt{6}} \times \prod_{uv\in E_3} \frac{1}{\sqrt{9}}$$

$$= \left(\frac{1}{\sqrt{6}}\right)^{4C} \times \left(\frac{1}{\sqrt{9}}\right)^{6RC-5C}$$

$$= 2^{-2C} \times 3^{C(3-6R)}$$

(7). $MZ_1^a(K) = \prod\limits_{uv\in E(K)} [d_K(u)+d_K(v)]^a$

$$= \prod_{uv\in E_2} 5^a \times \prod_{uv\in E_3} 6^a$$

$$= 5^{a(4C)} \times 6^{a(6RC-5C)}$$

(8). $MZ_2^a(K) = \prod\limits_{uv\in E(K)} [d_K(u)d_K(v)]^a$

$$= \prod_{uv\in E_2} 6^a \times \prod_{uv\in E_3} 9^a$$

$$= 6^{a(4C)} \times 9^{a(6RC-5C)}$$

$$= 2^{4Ca} \times 3^{C(12R-6)a}$$

(9). $ABCII(K) = \prod\limits_{uv\in E(K)} \sqrt{\frac{d_K(u)+d_K(v)-2}{d_K(u)d_K(v)}}$

$$= \prod_{uv\in E_2} \sqrt{\frac{3+2-2}{3\times 2}} \times \prod_{uv\in E_3} \sqrt{\frac{3+3-2}{3\times 3}}$$

$$= \left(\frac{1}{\sqrt{2}}\right)^{4C} \times \left(\frac{2}{3}\right)^{6RC-5C}$$

$$= 2^{C(6R-7)} \times 3^{C(5-6R)}$$

(10). $$\begin{aligned} GAII(K) &= \prod_{uv\in E(K)} \frac{2\sqrt{d_K(u)d_K(v)}}{d_K(u)+d_K(v)} \\ &= \prod_{uv\in E_2} \frac{2\sqrt{3\times 2}}{3+2} \times \prod_{uv\in E_3} \frac{2\sqrt{3\times 3}}{3+3} \\ &= \left(\frac{2\sqrt{6}}{5}\right)^{4C} \times (1)^{6RC-5C} \\ &= \left(\frac{2\sqrt{6}}{5}\right)^{4C} \end{aligned}$$

□

3. Conclusion

Chemical graph theory is an important tool for studying molecular structures and has an important effect on the development of chemical sciences. The study of topological indices is currently one of the most active research fields in chemical graph theory. We have presented here some multiplicative connectivity indices of $TUC_4C_8(R)$ Rhomboidal nanotube.

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