



Heronian Mean Labeling of Double Triangular and Double Quadrilateral Snake Graphs

Research Article

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Abstract: In this paper, We contribute some new results for Heronian Mean labeling of graphs. We prove that Double Triangular snake, Alternate Double Triangular snake, Double Quadrilateral Snake and Alternate Double Quadrilateral Snake graphs are Heronian mean graphs.

MSC: 05C78.

Keywords: Heronian Mean labeling, Triangular snake, Double Triangular snake, Quadrilateral snake, Double Quadrilateral Snake.

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1. Introduction

The graph considered here will be simple, finite and undirected graph $G = (V, E)$ with p vertices and q edges without loops or parallel edges. For all detailed survey of graph labeling, we refer to J.A.Gallian [1]. For all other terminology and notations we follow Harary [2]. A Double Triangular snake $D(T_n)$ is obtained from a path $u_1u_2\dots u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i for $1 \leq i \leq n-1$. That is, a Double Triangular snake $D(T_n)$ consists of two Triangular snakes that have a common path. An Alternate Double Triangular snake $A[D(T_n)]$ consists of two Alternate Triangular snake that have a common path. A Double Quadrilateral snake $D(Q_n)$ is obtained from a path $u_1u_2\dots u_n$ by joining u_i and u_{i+1} to new vertices v_i, w_i and x_i, y_i respectively and then joining v_i and w_i, x_i and $y_i, 1 \leq i \leq n-1$. That is, A Double Quadrilateral snake $D(Q_n)$ consists of two Quadrilateral snakes that have a common path. An Alternate Double Quadrilateral snake $A[D(Q_n)]$ consists of two Alternate Quadrilateral snakes that have a common path.

Definition 1.1. A graph $G = (V, E)$ with p vertices and q edges is said to be a Heronian Mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, \dots, q+1$ in such a way that when each edge $e = uv$ is labeled with,

$$f(e=uv) = \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil \quad (OR) \quad \left\lfloor \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rfloor$$

then the edge labels are distinct. In this case f is called a **Heronian Mean labeling** of G .

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The concept of Heronian Mean labeling was introduced by S.S.Sandhya, E.Ebin Raja Merly and S.D.Deepa in [5], [6], [7], [8] and [9]. In this paper we contribute four results for Heronian Mean labeling of graphs. We shall make frequent references to the following results.

Theorem 1.2. Any Triangular snake T_n is a Heronian mean graph.

Theorem 1.3. Alternate Triangular snakes $A(T_n)$ are Heronian mean graphs.

Theorem 1.4. Any Quadrilateral snake Q_n is a Heronian mean graph.

Theorem 1.5. Alternate Quadrilateral snake $A(Q_n)$ are Heronian mean graphs.

2. Main Results

Theorem 2.1. Any Double Triangular snake $D(T_n)$ is a Heronian mean graph.

Proof. Let $D(T_n)$ be a Double Triangular snake. Consider a path u_1, u_2, \dots, u_n . Join u_i, u_{i+1} with two new vertices $v_i, w_i, 1 \leq i \leq n - 1$. Define a function, $f : V(A(T_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned} f(u_i) &= 5i - 4, \forall 1 \leq i \leq n, \\ f(v_i) &= 5i - 3, \forall 1 \leq i \leq n - 1, \\ f(w_i) &= 5i, \forall 1 \leq i \leq n - 1. \end{aligned}$$

Edges are labeled by,

$$\begin{aligned} f(u_i u_{i+1}) &= 5i - 2, \forall 1 \leq i \leq n - 1, \\ f(u_i v_i) &= 5i - 4, \forall 1 \leq i \leq n - 1, \\ f(u_{i+1} v_i) &= 5i - 1, \forall 1 \leq i \leq n - 1, \\ f(u_i w_i) &= 5i - 3, \forall 1 \leq i \leq n - 1, \\ f(u_{i+1} w_i) &= 5i, \forall 1 \leq i \leq n - 1. \end{aligned}$$

Obviously f is a Heronian mean labeling and $D(T_n)$ is a Heronian mean graph. □

Example 2.2. A Heronian mean labeling of $D(T_5)$ is a given below.



Figure 1.

Theorem 2.3. Alternate Double Triangular Snakes $A(D(T_n))$ are Heronian mean graphs.

Proof. Let G be the graph $A(D(T_n))$. Consider a path u_1, u_2, \dots, u_n . To construct G , Join u_i, u_{i+1} (alternatively) with two new vertices $v_i, w_i, 1 \leq i \leq n - 1$. Here we consider two different cases.

Case (i): If the Double Triangular Snake $A(D(T_n))$ starts from u_1 , then we need to consider two subcases

Subcase(i)(a): If n is even, then

Define a function, $f : V(A(T_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned} f(u_i) &= 3i - 1, \forall 1 \leq i \leq n, \\ f(v_i) &= 6i - 5, \forall 1 \leq i \leq \frac{n}{2}, \\ f(w_i) &= 6i - 4, \forall 1 \leq i \leq \frac{n}{2}. \end{aligned}$$

Edges are labeled by,

$$\begin{aligned} f(u_i u_{i+1}) &= 3i + 1, \forall i = 1, 3, \dots, n - 1 \\ f(u_i u_{i+1}) &= 3i, \forall i = 2, 4, \dots, n \\ f(u_{2i-1} v_i) &= 6i - 5, \forall 1 \leq i \leq \frac{n}{2} \\ f(u_{2i} v_i) &= 6i - 4, \forall 1 \leq i \leq \frac{n}{2} \\ f(u_{2i-1} w_i) &= 6i - 3, \forall 1 \leq i \leq \frac{n}{2} \\ f(u_{2i} w_i) &= 6i - 1, \forall 1 \leq i \leq \frac{n}{2} \end{aligned}$$

In this case f is a Heronian mean labeling. The labeling pattern is given below.

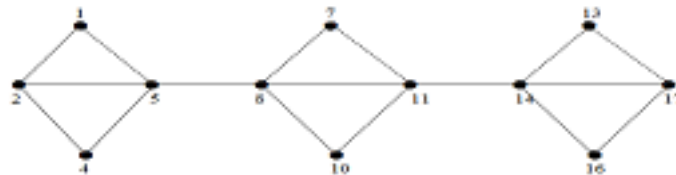


Figure 2.

Subcase(i)(b): If n is odd, then

Define a function, $f : V(A(T_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned} f(u_i) &= 3i - 1, \forall 1 \leq i \leq n, \\ f(v_i) &= 6i - 5, \forall 1 \leq i \leq \frac{(n-1)}{2}, \\ f(w_i) &= 6i - 4, \forall 1 \leq i \leq \frac{(n-1)}{2}. \end{aligned}$$

Edges are labeled by,

$$\begin{aligned} f(u_i u_{i+1}) &= 3i + 1, \forall i = 1, 3, \dots, n \\ f(u_i u_{i+1}) &= 3i, \forall i = 2, 4, \dots, n - 1 \\ f(u_{2i-1} v_i) &= 6i - 5, \forall 1 \leq i \leq \frac{(n-1)}{2} \\ f(u_{2i} v_i) &= 6i - 4, \forall 1 \leq i \leq \frac{(n-1)}{2} \\ f(u_{2i-1} w_i) &= 6i - 3, \forall 1 \leq i \leq \frac{(n-1)}{2} \\ f(u_{2i} w_i) &= 6i - 1, \forall 1 \leq i \leq \frac{(n-1)}{2} \end{aligned}$$

In this case f is a Heronian mean labeling. The labeling pattern is shown below.

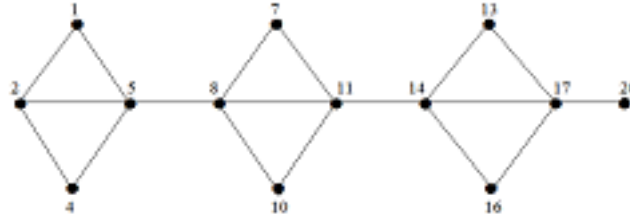


Figure 3.

Case:(ii): If a Triangular Snake $A(T_n)$ starts from u_2 , then we need to consider two subcases

Subcase(ii)(a): If n is even, then

Define a function, $f : V(A(T_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by $f(u_1) = 1$,

$$f(u_i) = 3i - 3, \forall 2 \leq i \leq n,$$

$$f(v_i) = 6i - 4, \forall 1 \leq i \leq \frac{(n-2)}{2},$$

$$f(w_i) = 6i - 1, \forall 1 \leq i \leq \frac{(n-2)}{2}.$$

Edges are labeled by,

$$f(u_i u_{i+1}) = 3i - 2, \forall i = 1, 3, \dots, n - 1$$

$$f(u_i u_{i+1}) = 3i - 1, \forall i = 2, 4, \dots, n$$

$$f(u_{2i+1} v_i) = 6i - 3, \forall 1 \leq i \leq \frac{(n-2)}{2}$$

$$f(u_{2i} v_i) = 6i - 4, \forall 1 \leq i \leq \frac{(n-2)}{2}$$

$$f(u_{2i+1} w_i) = 6i, \forall 1 \leq i \leq \frac{(n-2)}{2}$$

$$f(u_{2i} w_i) = 6i - 2, \forall 1 \leq i \leq \frac{(n-2)}{2}$$

In this case f is a Heronian mean labeling. The following figure shows the labeling pattern.



Figure 4.

Subcase(ii)(b): If n is odd, then

Define a function, $f : V(A(T_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by $f(u_1) = 1$,

$$f(u_i) = 3i - 3, \forall 2 \leq i \leq n,$$

$$f(v_i) = 6i - 4, \forall 1 \leq i \leq \frac{(n-1)}{2},$$

$$f(w_i) = 6i - 1, \forall 1 \leq i \leq \frac{(n-1)}{2}.$$

Edges are labeled by,

$$\begin{aligned}
 f(u_i u_{i+1}) &= 3i - 2, \forall i = 1, 3, \dots, n \\
 f(u_i u_{i+1}) &= 3i - 1, \forall i = 2, 4, \dots, n - 1. \\
 f(u_{2i+1} v_i) &= 6i - 3, \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(u_{2i} v_i) &= 6i - 4, \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(u_{2i+1} w_i) &= 6i, \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(u_{2i} w_i) &= 6i - 2, \forall 1 \leq i \leq \frac{(n-1)}{2}
 \end{aligned}$$

In this case f is a Heronian mean labeling. The labeling pattern is,

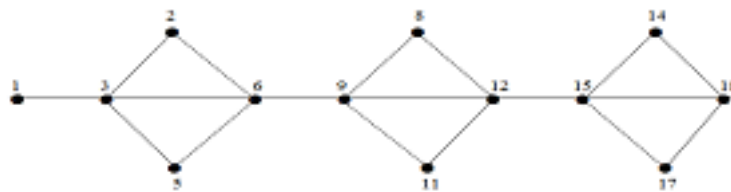


Figure 5.

From Case (i) and Case (ii), We conclude that Alternate Double Triangular snakes are Heronian mean graphs. □

Theorem 2.4. Any Double Quadrilateral snake $D(Q_n)$ is a Heronian mean graph.

Proof. Let $D(Q_n)$ be a Double Quadrilateral snake. Consider a path u_1, u_2, \dots, u_n . Join u_i, u_{i+1} with new vertices v_i, w_i and x_i, y_i $1 \leq i \leq n - 1$. Define a function, $f : V(A(T_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned}
 f(u_i) &= 7i - 6, \forall 1 \leq i \leq n, \\
 f(v_i) &= 7i - 5, \forall 1 \leq i \leq n - 1 \\
 f(w_i) &= 7i - 4, \forall 1 \leq i \leq n - 1 \\
 f(x_i) &= 7i - 1, \forall 1 \leq i \leq n - 1 \\
 f(y_i) &= 7i, \forall 1 \leq i \leq n - 1
 \end{aligned}$$

Edges are labeled by,

$$\begin{aligned}
 f(u_i u_{i+1}) &= 7i - 3, \forall 1 \leq i \leq n - 1 \\
 f(u_i v_i) &= 7i - 6, \forall 1 \leq i \leq n - 1 \\
 f(u_{i+1} w_i) &= 7i - 2, \forall 1 \leq i \leq n - 1 \\
 f(v_i w_i) &= 7i - 5, \forall 1 \leq i \leq n - 1 \\
 f(u_i x_i) &= 7i - 4, \forall 1 \leq i \leq n - 1 \\
 f(u_{i+1} y_i) &= 7i, \forall 1 \leq i \leq n - 1 \\
 f(x_i y_i) &= 7i - 1, \forall 1 \leq i \leq n - 1
 \end{aligned}$$

Obviously f is a Heronian mean labeling and $D(Q_n)$ is a Heronian mean graph. □

Example 2.5. A Heronian mean labeling of $D(Q_3)$ is given below.

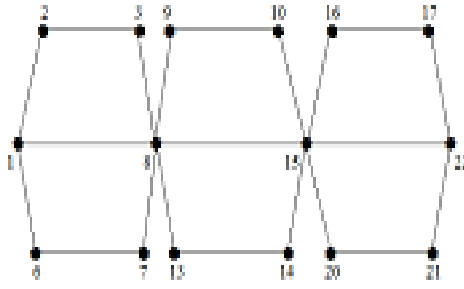


Figure 6.

Theorem 2.6. Alternate Double Quadrilateral Snakes $A(D(Q_n))$ are Heronian mean graphs.

Proof. Let G be the graph $A(D(Q_n))$. Consider a path u_1, u_2, \dots, u_n . To construct G , Join u_i, u_{i+1} (alternatively) with four new vertices v_i, w_i and $x_i, y_i, 1 \leq i \leq n - 1$. Here we consider two different cases.

Case (i): If the Double Quadrilateral Snake $A(D(Q_n))$ starts from u_1 , then we need to consider two subcases

Subcase(i)(a): If n is even, then

Define a function $f : V(A(Q_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned} f(u_i) &= 4i - 1, \forall 1 \leq i \leq n \\ f(v_i) &= 8i - 7, \forall 1 \leq i \leq \frac{n}{2}, \\ f(w_i) &= 8i - 3, \forall 1 \leq i \leq \frac{n}{2}, \\ f(x_i) &= 8i - 6, \forall 1 \leq i \leq \frac{n}{2}, \\ f(y_i) &= 8i - 2, \forall 1 \leq i \leq \frac{n}{2}, \end{aligned}$$

Edges are labeled by,

$$\begin{aligned} f(u_i u_{i+1}) &= 4i + 1, \forall i = 1, 3, \dots, n - 1 \\ f(u_i u_{i+1}) &= 4i, \forall i = 2, 4, \dots, n \\ f(u_i v_i) &= 8i - 7, \forall 1 \leq i \leq \frac{n}{2} \\ f(u_{i+1} w_i) &= 8i - 2, \forall 1 \leq i \leq \frac{n}{2} \\ f(v_i w_i) &= 8i - 5, \forall 1 \leq i \leq \frac{n}{2} \\ f(u_i x_i) &= 8i - 6, \forall 1 \leq i \leq \frac{n}{2} \\ f(u_{i+1} y_i) &= 8i - 1, \forall 1 \leq i \leq \frac{n}{2} \\ f(x_i y_i) &= 8i - 4, \forall 1 \leq i \leq \frac{n}{2} \end{aligned}$$

In this case f is a Heronian mean labeling. The labeling pattern is given below.

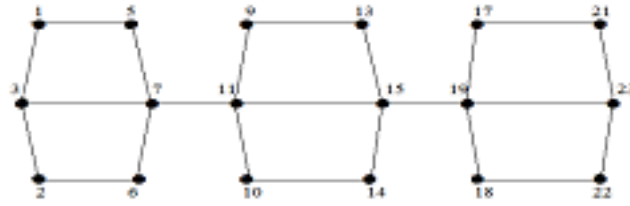


Figure 7.

Subcase(i)(b): If n is odd, then

Define a function $f : V(A(Q_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned}
 f(u_i) &= 4i - 1, \quad \forall 1 \leq i \leq n \\
 f(v_i) &= 8i - 7, \quad \forall 1 \leq i \leq \frac{(n-1)}{2}, \\
 f(w_i) &= 8i - 3, \quad \forall 1 \leq i \leq \frac{(n-1)}{2}, \\
 f(x_i) &= 8i - 6, \quad \forall 1 \leq i \leq \frac{(n-1)}{2}, \\
 f(y_i) &= 8i - 2, \quad \forall 1 \leq i \leq \frac{(n-1)}{2},
 \end{aligned}$$

Edges are labeled by,

$$\begin{aligned}
 f(u_i u_{i+1}) &= 4i + 1, \quad \forall i = 1, 3, \dots, n \\
 f(u_i u_{i+1}) &= 4i, \quad \forall i = 2, 4, \dots, n - 1 \\
 f(u_i v_i) &= 8i - 7, \quad \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(u_{i+1} w_i) &= 8i - 2, \quad \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(v_i w_i) &= 8i - 5, \quad \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(u_i x_i) &= 8i - 6, \quad \forall 1 \leq i \leq \frac{(n-1)}{2}, \\
 f(u_{i+1} y_i) &= 8i - 1, \quad \forall 1 \leq i \leq \frac{(n-1)}{2}, \\
 f(x_i y_i) &= 8i - 4, \quad \forall 1 \leq i \leq \frac{(n-1)}{2}
 \end{aligned}$$

In this case f is a Heronian mean labeling. The labeling pattern is shown below.

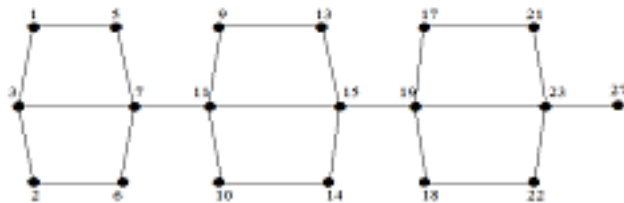


Figure 8.

Case (ii): If a Double Quadrilateral Snake $A(D(Q_n))$ starts from u_2 , then we need to consider two subcases

Subcase(ii)(a): If n is even, then

Define a function , $f : V(A(Q_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned} f(u_i) &= 4i - 1, \forall 1 \leq i \leq n \\ f(v_i) &= 8i - 7, \forall 1 \leq i \leq \frac{(n-2)}{2}, \\ f(w_i) &= 8i - 3, \forall 1 \leq i \leq \frac{(n-2)}{2}, \\ f(x_i) &= 8i - 6, \forall 1 \leq i \leq \frac{(n-2)}{2}, \\ f(y_i) &= 8i - 2, \forall 1 \leq i \leq \frac{(n-2)}{2}, \end{aligned}$$

Edges are labeled by,

$$\begin{aligned} f(u_i u_{i+1}) &= 4i + 1, \forall i = 1, 3, \dots, n - 1 \\ f(u_i u_{i+1}) &= 4i, \forall i = 2, 4, \dots, n \\ f(u_i v_i) &= 8i - 7, \forall 1 \leq i \leq \frac{(n-2)}{2} \\ f(u_{i+1} w_i) &= 8i - 2, \forall 1 \leq i \leq \frac{(n-2)}{2} \\ f(v_i w_i) &= 8i - 5, \forall 1 \leq i \leq \frac{(n-2)}{2} \\ f(u_i x_i) &= 8i - 6, \forall 1 \leq i \leq \frac{(n-2)}{2}, \\ f(u_{i+1} y_i) &= 8i - 1, \forall 1 \leq i \leq \frac{(n-2)}{2}, \\ f(x_i y_i) &= 8i - 4, \forall 1 \leq i \leq \frac{(n-2)}{2} \end{aligned}$$

In this case f is a Heronian mean labeling. The labeling pattern is,

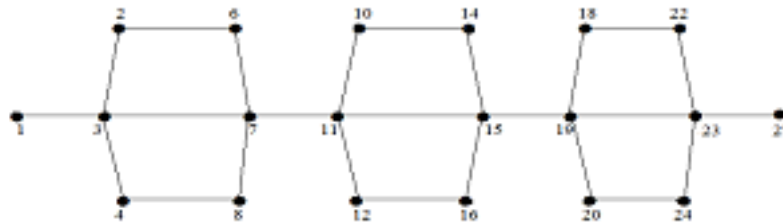


Figure 9.

Subcase(ii)(b): If n is odd, then

Define a function , $f : V(A(Q_n)) \rightarrow \{1, 2, 3, \dots, q + 1\}$ by

$$\begin{aligned} f(u_i) &= 4i - 1, \forall 1 \leq i \leq n \\ f(v_i) &= 8i - 7, \forall 1 \leq i \leq \frac{(n-1)}{2}, \\ f(w_i) &= 8i - 3, \forall 1 \leq i \leq \frac{(n-1)}{2}, \\ f(x_i) &= 8i - 6, \forall 1 \leq i \leq \frac{(n-1)}{2}, \\ f(y_i) &= 8i - 2, \forall 1 \leq i \leq \frac{(n-1)}{2}, \end{aligned}$$

Edges are labeled by,

$$\begin{aligned}
 f(u_i u_{i+1}) &= 4i + 1, \forall i = 1, 3, \dots, n \\
 f(u_i u_{i+1}) &= 4i, \forall i = 2, 4, \dots, n - 1 \\
 f(u_i v_i) &= 8i - 7, \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(u_{i+1} w_i) &= 8i - 2, \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(v_i w_i) &= 8i - 5, \forall 1 \leq i \leq \frac{(n-1)}{2} \\
 f(u_i x_i) &= 8i - 6, \forall 1 \leq i \leq \frac{(n-1)}{2}, \\
 f(u_{i+1} y_i) &= 8i - 1, \forall 1 \leq i \leq \frac{(n-1)}{2}, \\
 f(x_i y_i) &= 8i - 4, \forall 1 \leq i \leq \frac{(n-1)}{2}
 \end{aligned}$$

In this case f is a Heronian mean labeling. The labeling pattern is,

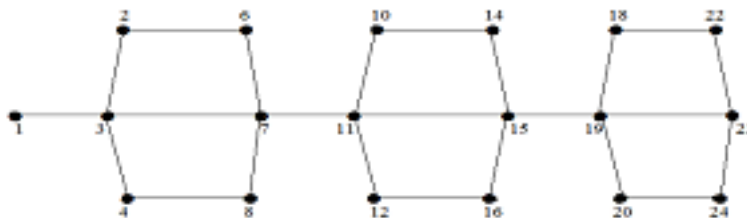


Figure 10.

From case(i) and case(ii), We conclude that Alternate Double Quadrilateral snakes are Heronian mean graphs. □

3. Conclusion

In this paper, we studied the Heronian mean labeling of double triangular and double quadrilateral snakes. The authors are of the opinion that the study of Heronian mean labeling behaviour of graphs obtained from standard graphs shall be quite interesting and also will lead to newer results.

Acknowledgement

The authors are thankful to the referee for their valuable comments and suggestions.

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