

# An Unsteady Flow of Two Immiscible Viscous Fluids in Porous Medium Between two Impermeable Parallel Plates, Impulsively Stopped From Relatively Motion

Research Article

Manju Agarwal<sup>1</sup> and Deepak Kumar<sup>1\*</sup><sup>1</sup> Department of Mathematics and Astronomy, University of Lucknow, Lucknow, India.

**Abstract:** In the present paper an unsteady receding flow of two immiscible viscous incompressible fluids through a porous medium bounded between two impermeable plates is investigated. The flow is generated by the motion of one of the plates with a constant velocity parallel to other while the other plate is kept at rest. After attaining the steady state flow, the moving plate is suddenly stopped and the partial differential equations governing subsequent receding flow are solved by variable separable method to study hydrodynamic properties of the fluids. Separate expressions for fluid velocity and shear stress in both regions are obtained. Variations in fluid velocities and shear stress with time, porosity parameter, viscosity ratio and width of channel are presented graphically.

**MSC:** 76S99, 76T99**Keywords:** Two phase flow, Porous medium, Immiscible fluids, Variable separable method.

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## 1. Introduction

Study of two phase fluid flow in porous media is of principal interest because it is applicable to many natural phenomenas like ground water flows, smog, fog, rain, dust storm, blood flow etc. as well as in many technological problems like refrigeration, power generation, distillation of water etc. Its a major application is in petroleum industry where flow of many hydrocarbon liquids and gases occurs. Also it is applicable in many other industries like steel making, paper manufacturing and food processing etc. Many body fluids like blood and semen are multiphase containing variety of cells.

Both theoretical and experimental work is found in literature on stratified two fluid flow in horizontal pipes. R Hilfer [1] established macroscopic equations of motion for two phase immiscible displacement in Porous media. Packham and Shail [2] analysed a stratified laminar flow of two immiscible liquids in horizontal pipes. The two-phase MHD flow and heat transfer in an inclined channel are studied by Malashetty and Umavathi [3] and Malashetty et al. [4, 5]. Chamkha [6] reported analytical solutions for the flow of two-immiscible fluids in porous and non-porous parallel-plates channels. Two-phase magneto hydrodynamic convective flow of electrically conducting fluid through an inclined channel is studied by Murty and Prakash [7] under the action of a constant transverse magnetic field in a rotating system. Srinivasan and Vafai [8] have reported theoretical study for predicting the movement of the interface for linear encroachment in two immiscible fluids system in porous medium, taking into account the non Darcian boundary and inertia effects.

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\* E-mail: [deepakpatel0412@gmail.com](mailto:deepakpatel0412@gmail.com)

All the above studies are concerned with the steady flow. Rao et.al. [9] analyzed analytically, unsteady two phase viscous ideal fluid flow through a parallel plate channel under a pulsatile pressure gradient with a body acceleration. An unsteady Hartmann flow of two immiscible fluids through a horizontal channel with time-dependent oscillatory wall transpiration velocity is investigated by Umavathi et.al. [10]. An unsteady magneto hydrodynamic (MHD) two-layered fluids flow and heat transfer in a horizontal channel between two parallel plates in the presence of an applied magnetic and electric field are investigated, when the whole system is rotated about an axis perpendicular to the flow by Raju and Nagavalli [11]. An experimental study of two phase flow in bi-dispersed porous media is made by Chen et.al. [12]. Hussain and Ramacharyulu [13] studied unsteady viscous incompressible single fluid flow in a porous medium between two impermeable plates impulsively stopped from relatively motion.

Various methods are proposed in literature to find analytical solution of partial differential equation governing flow in porous media e.g. Laplace transform [13, 14], Fourier transform [15] and separation of variables [16, 17] etc. In spite of Various methods, exact solutions of two phase flow in porous media can only be obtained by making some assumptions yet they are very helpful to verify practical datas.

Here we investigate an unsteady flow of a two immiscible viscous incompressible fluids through a porous medium bounded between two impermeable plates. In this study we follow the momentum equation (Yamamoto and Yoshida [18]) for flow through porous media including fluid inertia and viscous stress in addition to Darcy's law.

$$\rho\left\{\frac{\partial V}{\partial t} + (\nabla \cdot V)\nabla\right\} = -\nabla P + \mu\nabla^2 V - \frac{\mu}{k}V$$

where

$\rho$  is the fluid density

$\mu$  is the coefficient of viscosity

$P$  is the pressure

$V$  is the fluid velocity vector

$K$  is the permeability of the medium

We consider the Couette flow generated by motion of one of the plates with a constant velocity parallel to other. After attaining steady state, the moving plate is suddenly stopped and the subsequent receding flow is investigated analytically using the Variable separable method. Expressions for velocity, stress tensor on both plates are obtained and variations with flow parameters are depicted graphically.

## 2. Mathematical Formulations and Solutions

The geometry under consideration consist of two infinite impermeable parallel plates extending in  $X$  and  $Z$  directions (Fig. 1). The regions  $0 \leq y \leq h_1$  and  $h_1 \leq y \leq h_2$  are denoted as Region-I and Region-II. The fluid in Region-I have density  $\rho_1$ , viscosity  $\mu_1$  and kinematic viscosity  $\nu_1$ , Region-II is filled with a fluid having density  $\rho_2$ , viscosity  $\mu_2$  and kinematic viscosity  $\nu_2$ . The lower plate ( $y = 0$ ) is considered to be fixed while the upper plate ( $y = h_2$ ) is moving parallel to  $x$ -axis.

The present problem is solved in two stages. In the first stage the flow is generated by the motion of the upper plate ( $y = h_2$ ) with a constant velocity parallel to lower plate while lower plate ( $y = 0$ ) is kept at rest. The second stage concerns with the subsequent unsteady receding flow when the moving plate is stopped.

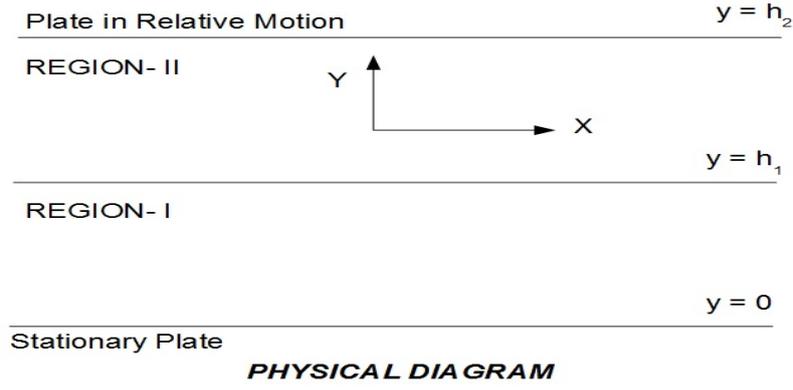


Figure 1:

## 2.1. Stage-1:

This is the steady state of the fluid flow generated by the motion of the plate ( $y = h_2$ ) with a constant velocity  $u_0$ . Let  $\bar{u}_1(y)$  and  $\bar{u}_2(y)$  be the velocities of the fluids flowing in Region-I and Region-II respectively. In this case momentum equations in both regions are

Region-I

$$\frac{d^2 \bar{u}_1}{dy^2} - \frac{\bar{u}_1}{k_1} = 0 \quad (1)$$

Region-II

$$\frac{d^2 \bar{u}_2}{dy^2} - \frac{\bar{u}_2}{k_1} = 0 \quad (2)$$

With the hydrodynamic boundary and interface conditions

$$\begin{aligned} \bar{u}_1(y) &= 0 & \text{at } y = 0 \\ \bar{u}_2(y) &= u_0 & \text{at } y = h_2 \\ \bar{u}_1(y) &= \bar{u}_2(y) & \text{at } y = h_1 \\ \alpha \frac{\partial \bar{u}_1(y)}{\partial y} &= \frac{\partial \bar{u}_2(y)}{\partial y} & \text{at } y = h_1 \end{aligned} \quad (3)$$

where  $k_1$  is the permeability of the porous medium and  $\alpha = \frac{\mu_1}{\mu_2}$  the ratio of viscosities.

Solving equations (1) and (2), we have

$$\bar{u}_1(y) = C_1 e^{m_1 y} + C_2 e^{-m_1 y} \quad (4)$$

$$\bar{u}_2(y) = C_3 e^{m_1 y} + C_4 e^{-m_1 y} \quad (5)$$

where  $m_1^2 = \frac{1}{k_1}$ . and  $C_1, C_2, C_3$  and  $C_4$  are arbitrary constants.

Applying hydrodynamic boundary and interface conditions(3) in equations (4) and (5), we get

$$\begin{aligned} C_1 &= \frac{2u_0}{A}, \quad C_2 = -\frac{2u_0}{A} \\ C_3 &= \left\{ (1 + \alpha) - (1 - \alpha)e^{-2m_1 h_1} \right\} \frac{u_0}{A} \\ C_4 &= \left\{ -(1 + \alpha) + (1 - \alpha)e^{2m_1 h_1} \right\} \frac{u_0}{A} \end{aligned} \quad (6)$$

Substituting constants, obtained in equation (6) in equations(4) and (5), velocities in both regions are

$$\bar{u}_1(y) = (e^{m_1 y} - e^{-m_1 y}) \frac{2u_0}{A} \quad (7)$$

$$\bar{u}_2(y) = \{(1 + \alpha)(e^{m_1 y} - e^{-m_1 y}) - (1 - \alpha)(e^{-2m_1 h_1} e^{m_1 y} - e^{2m_1 h_1} e^{-m_1 y})\} \frac{u_0}{A} \quad (8)$$

where  $A$  is given as

$$A = (1 - \alpha)(e^{2m_1 h_1 - m_1 h_2} - e^{-(2m_1 h_1 - m_1 h_2)}) + (1 + \alpha)(e^{m_1 h_2} - e^{-m_1 h_2}) \quad (9)$$

## 2.2. Stage-2

After attaining the steady state flow considered in stage-1, the moving plate ( $y = h_2$ ) is suddenly stopped. Let  $u_1^*(y, t)$  and  $u_2^*(y, t)$  be the fluid velocities in Region-I and Region-II respectively. In this case momentum equations in both regions are

Region-I

$$\frac{\partial u_1^*}{\partial t} = \nu_1 \frac{\partial^2 u_1^*}{\partial y^2} - \nu_1 \frac{u_1^*}{k_1} \quad (10)$$

Region-II

$$\frac{\partial u_2^*}{\partial t} = \nu_2 \frac{\partial^2 u_2^*}{\partial y^2} - \nu_2 \frac{u_2^*}{k_1} \quad (11)$$

with the boundary conditions

$$u_1^*(0, t) = 0, \quad u_2^*(h_2, t) = 0 \quad (12)$$

and interface condition

$$u_1^*(y, t) = u_2^*(y, t) \quad \text{at } y = h_1 \quad (13)$$

and initial value conditions

$$u_1^*(y, 0) = \bar{u}_1(y), \quad u_2^*(y, 0) = \bar{u}_2(y) \quad (14)$$

where  $\nu_1 = \frac{\mu_1}{\rho_1}$  and  $\nu_2 = \frac{\mu_2}{\rho_2}$  are kinematic viscosities of the fluids in Region-I and Region-II.  $\bar{u}_1(y)$  and  $\bar{u}_2(y)$  are the velocities given by equations(7) and (8) respectively. For simplification, taking assumption that the Kinematic viscosities of both fluids are same i. e.  $\nu_1 = \nu_2 = \nu$  and introducing following non dimensional quantities

$$y = h_2 y', \quad t = \frac{h_2^2}{\nu} t', \quad k_1 = h_2^2 k', \quad u_i^*(y, t) = \frac{\nu}{h_2} u_i'(y, t), \quad i = 1, 2$$

in equations (10) and (11) and dropping the dashes, equations expressed in non dimensional form are

$$\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_1}{\partial y^2} - \frac{u_1}{k} \quad (15)$$

$$\frac{\partial u_2}{\partial t} = \frac{\partial^2 u_2}{\partial y^2} - \frac{u_2}{k} \quad (16)$$

Boundary conditions (12) and interface condition (13) in non dimensional form are

$$u_1(0, t) = 0, \quad u_2(1, t) = 0 \quad (17)$$

and

$$u_1(y, t) = u_2(y, t) \quad \text{at } y = h \quad (18)$$

Also using non-dimensional parameters initial conditions (14) becomes

$$u_1(y, 0) = \left(\frac{h_2}{\nu}u_0\right)(e^{my} - e^{-my})\frac{2}{A} \quad (19)$$

$$u_2(y, 0) = \left(\frac{h_2}{\nu}u_0\right)\{(1 + \alpha)(e^{my} - e^{-my}) - (1 - \alpha)(e^{-2mh}e^{my} - e^{2mh}e^{-my})\}\frac{1}{A} \quad (20)$$

where

$$A = (1 - \alpha)(e^{2mh-m} - e^{-(2mh-m)}) + (1 + \alpha)(e^m - e^{-m}) \quad (21)$$

In above equations  $m = m_1h_2 = \frac{1}{\sqrt{K}}$  and  $h = \frac{h_1}{h_2}$ . Solving equations (15) and (16) by Variable separable method for the boundary conditions (17) and (18) and the initial conditions (19) and (20), we get

**Velocities:**

In Flow Region-I

$$u_1(y, t) = \sum_{n=1}^{\infty} 8\pi \left\{ \frac{(-1)^{n+1}n \sinh(m)}{A(m^2 + n^2\pi^2)} \frac{h_2u_0}{\nu} \right\} \sin(n\pi y)e^{-\lambda^2 t} \quad (22)$$

In Flow Region-II

$$u_2(y, t) = \sum_{n=1}^{\infty} 2\pi \left\{ \frac{-n}{A(m^2 + n^2\pi^2)} \{e^{-2hm}(e^{4hm} - 1)(\alpha - 1)\} + (-1)^{n+1}e^{-(2h+1)m} \{-e^{2m}(\alpha - 1) + e^{4hm}(\alpha - 1) + e^{2hm}(\alpha + 1) - e^{2(1+h)m}(\alpha + 1)\} \right\} \frac{h_2u_0}{\nu} \sin(n\pi y)e^{-\lambda^2 t} \quad (23)$$

Here constant  $\lambda$  are given as

$$\lambda = \sqrt{\frac{1}{K} + (n\pi)^2}$$

**Shear Stress:**

In Flow Region-I

$$\tau_1 = \sum_{n=1}^{\infty} 8\mu_1 \left\{ \frac{(-1)^{n+1}(n\pi)^2 \sinh(m)}{A(m^2 + n^2\pi^2)} \frac{h_2u_0}{\nu} \right\} \cos(n\pi y)e^{-\lambda^2 t} \quad (24)$$

In Flow Region-II

$$\tau_2 = \sum_{n=1}^{\infty} 2\mu_2 \left\{ \frac{-(n\pi)^2}{A(m^2 + n^2\pi^2)} \{e^{-2hm}(e^{4hm} - 1)(\alpha - 1)\} + (-1)^{n+1}e^{-(2h+1)m} \{-e^{2m}(\alpha - 1) + e^{4hm}(\alpha - 1) + e^{2hm}(\alpha + 1) - e^{2(1+h)m}(\alpha + 1)\} \right\} \frac{h_2u_0}{\nu} \cos(n\pi y)e^{-\lambda^2 t} \quad (25)$$

Stress at the upper plate ( $y = 1$ ) is

$$\tau_{upper} = \sum_{n=1}^{\infty} 2\mu_2(-1)^n \left\{ \frac{-(n\pi)^2}{A(m^2 + n^2\pi^2)} \{e^{-2hm}(e^{4hm} - 1)(\alpha - 1)\} + (-1)^{n+1}e^{-(2h+1)m} \{-e^{2m}(\alpha - 1) + e^{4hm}(\alpha - 1) + e^{2hm}(\alpha + 1) - e^{2(1+h)m}(\alpha + 1)\} \right\} \frac{h_2u_0}{\nu} e^{-\lambda^2 t} \quad (26)$$

Stress at the lower plate ( $y = 0$ ) is

$$\tau_{lower} = \sum_{n=1}^{\infty} 8\mu_1 \left\{ \frac{(-1)^{n+1}(n\pi)^2 \sinh(m)}{A(m^2 + n^2\pi^2)} \frac{h_2u_0}{\nu} \right\} e^{-\lambda^2 t} \quad (27)$$

**For same fluid:** If we take same fluids then  $\alpha = 1$  i.e.  $\mu_1 = \mu_2 = \mu$  and  $\rho_1 = \rho_2 = \rho$ . Then the velocities and shear stresses becomes velocity

$$u_1(y, t) = u_2(y, t) = \sum_{n=1}^{\infty} 2n\pi \left\{ \frac{(-1)^{n+1}}{(m^2 + n^2\pi^2)} \frac{h_2u_0}{\nu} \right\} \sin(n\pi y)e^{-\lambda^2 t} \quad (28)$$

shear stress

$$\tau_1 = \tau_2 = \sum_{n=1}^{\infty} 2\mu \left\{ \frac{(-1)^{n+1} (n\pi)^2}{(m^2 + n^2\pi^2)} \frac{h_2 u_0}{\nu} \right\} \cos(n\pi y) e^{-\lambda^2 t} \quad (29)$$

where

$$\lambda = \sqrt{\frac{1}{K} + (n\pi)^2} \quad (30)$$

Above results match with the results obtained by Hussain and Ramacharyulu[13] for single fluid flow.

### 3. Results and Discussion

The problem of two immiscible fluids flow in a horizontal porous channel with moving plate is investigated analytically. The Problem is solved in two stages, Stage-I is the steady state of fluid flow generated by motion of the upper plate ( $y = h_2$ ) at constant velocity  $u_0$ . In Stage-II, moving plate is suddenly stopped after attaining the steady state flow considered in stage-I and the subsequent receding flow is investigated. Although correctness of the obtained results is not verified, the fact that the solutions satisfy all the boundary, interface and initial conditions (as will be seen in graphical results ) lend some confidence. Results are depicted graphically from figures 2 to 17 to elucidate interesting features of the hydrodynamic state of the flow.

Figures 2 to 6 represents velocity profile verses time at different values of porosity parameter  $m$ . The velocity profiles verses porosity parameter at different time instants are shown in figures 7 to 10. It is observed that effect of porosity on the velocity profile is of flatten type with the velocity attaining maximum near the middle of the plates due to friction. it decreases towards the plates. Figures show that this effect is more predominant as  $m$  increases. Further the point of maximum velocity is shifted towards the upper plate. The effect of viscosity ratio  $\alpha$  on the velocity of the fluids is shown if figures 11 and 12. It is noticed that as  $\alpha$  increases, the velocity of the fluids in both regions decreases. This happens because as  $\alpha$  increases, the viscous effect increases. Figures 13 and 14 depict the effect of channel width on the velocity profile. From figures it is clear that as the channel width decreases, velocities of the fluids in both regions decreases. Variation of shear stress with respect to porosity parameter  $m$  is shown in figures 15 and 16. It is clear from the figures, shear stress on the lower plate decreases exponentially as  $m$  increases while shear stress on the upper plate increases as  $m$  increases.

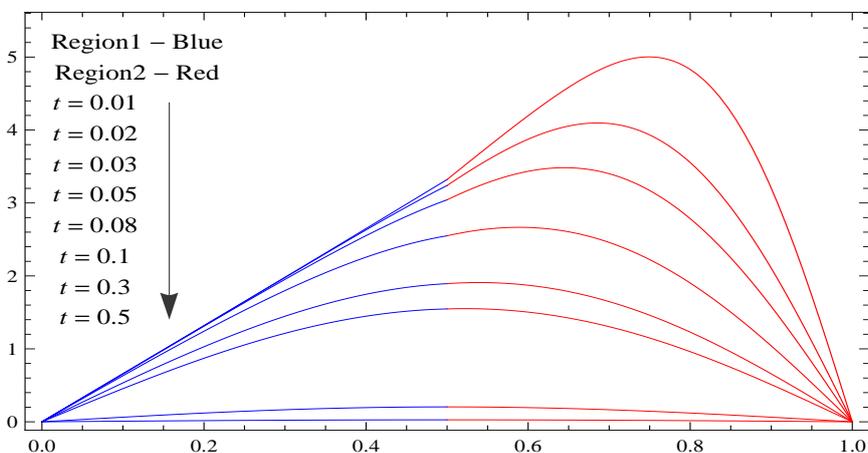


Figure 2: Velocity profile at different time instants when  $m = 0.5$ .

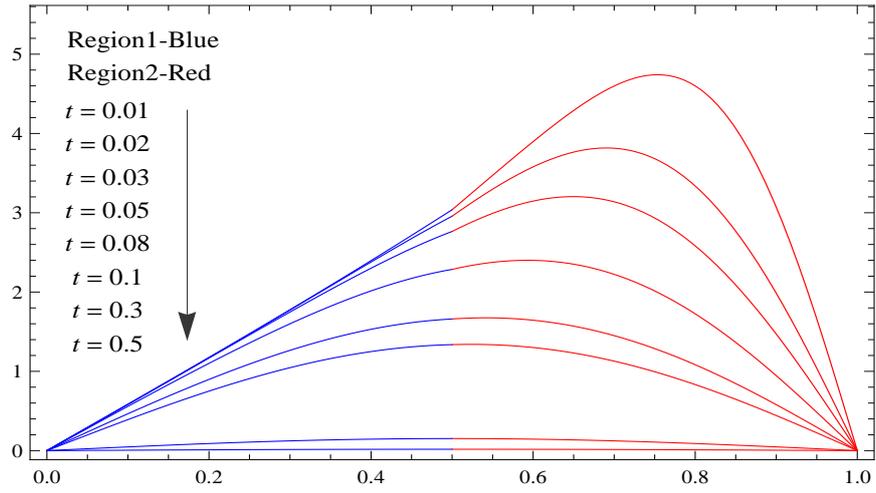


Figure 3: Velocity profile at different time instants when  $m = 1$ .

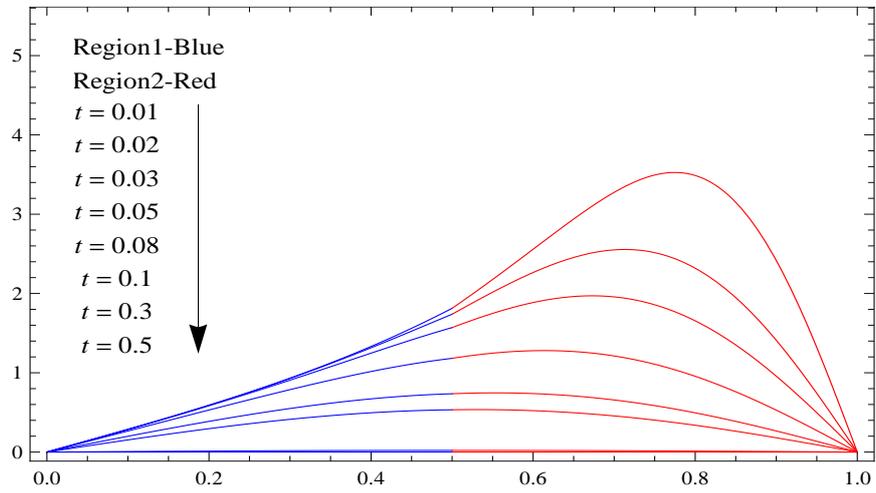


Figure 4: Velocity profile at different time instants when  $m = 2.5$ .

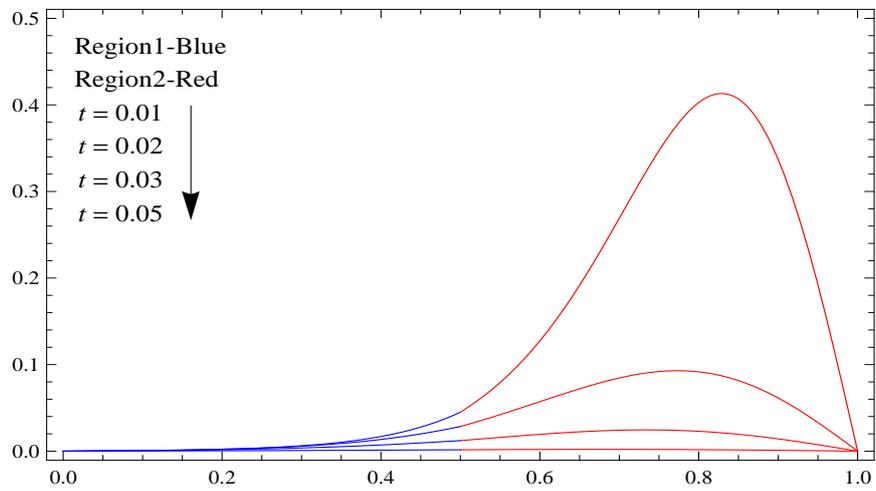


Figure 5: Velocity profile at different time instants when  $m = 10$ .

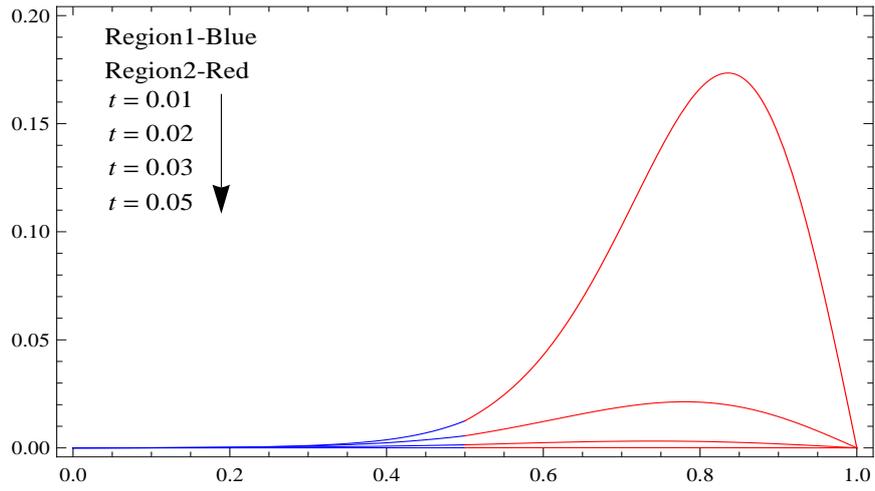


Figure 6: Velocity profile at different time instants when  $m = 12$ .

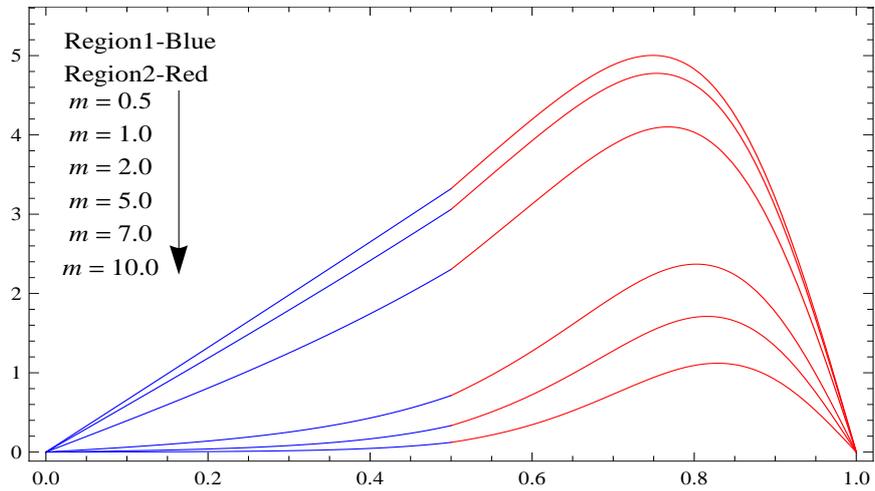


Figure 7: Velocity profile versus  $m$  at  $t = 0.01$ .

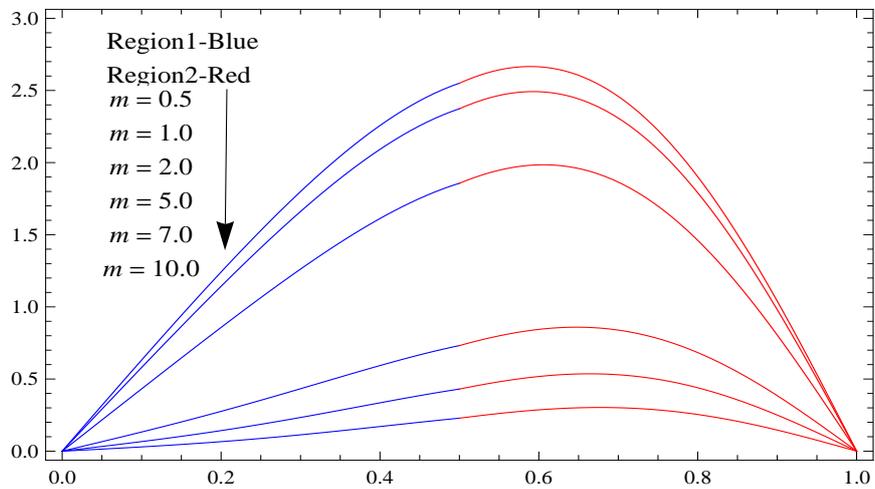


Figure 8: Velocity profile versus  $m$  at  $t = 0.05$ .

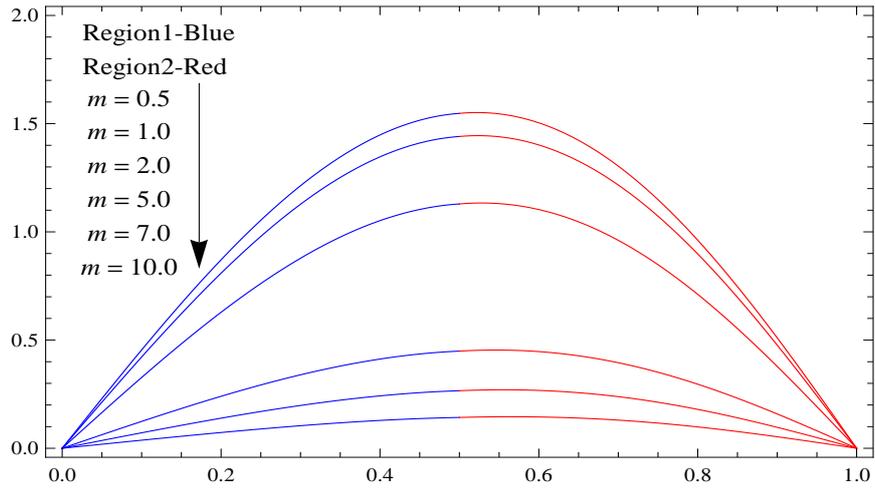


Figure 9: Velocity profile verses  $m$  at  $t = 0.1$ .

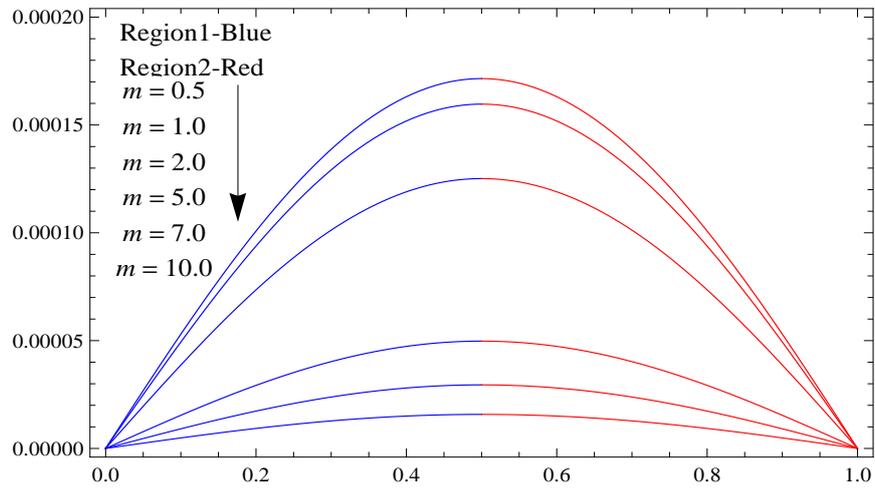


Figure 10: Velocity profile verses  $m$  at  $t = 1$ .

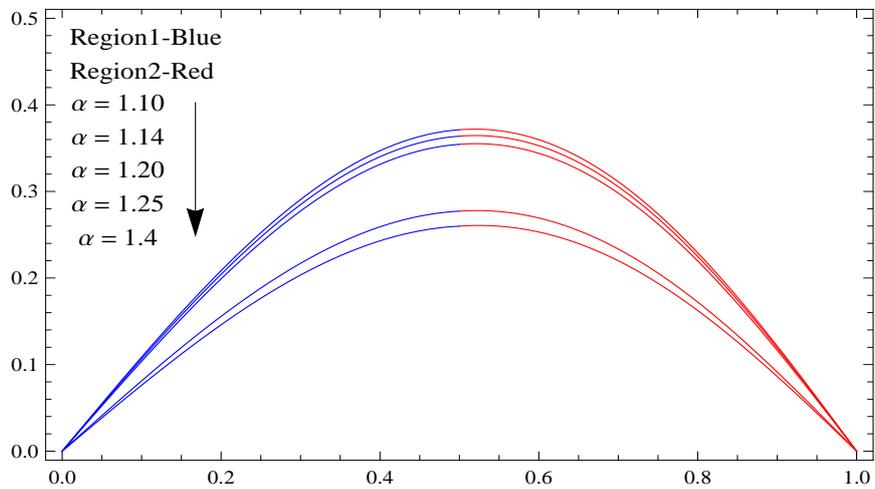


Figure 11: Velocity profile verses viscosity ratio  $\alpha$  at  $t = 0.1, m = 1$ .

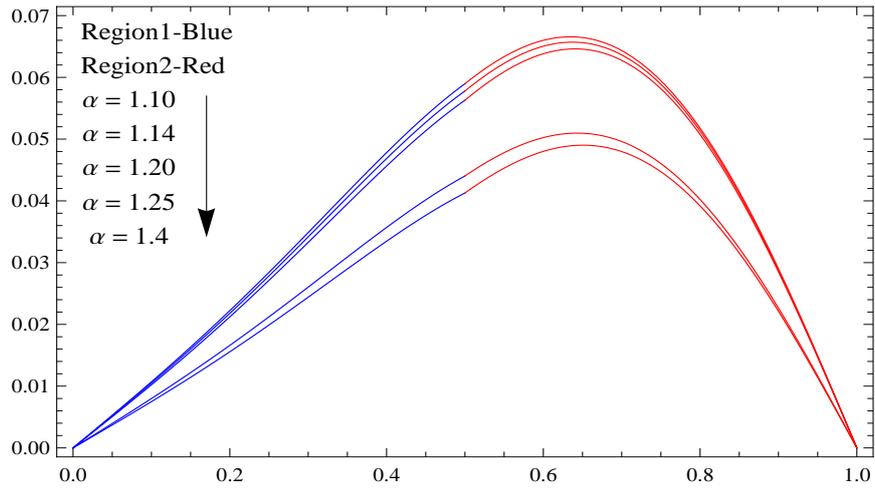


Figure 12: Velocity profile verses viscosity ratio  $\alpha$  at  $t = 0.05, m = 5$ .

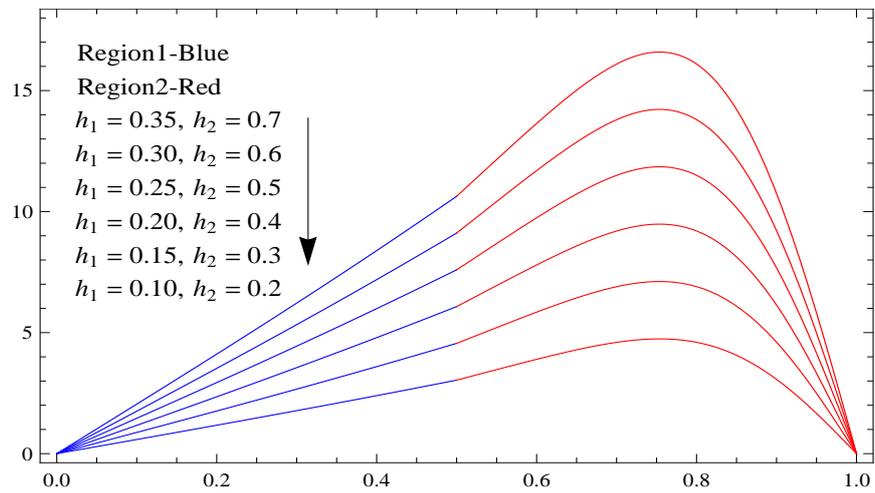


Figure 13: Velocity profile verses channel width at  $t = 0.01, m = 1$ .

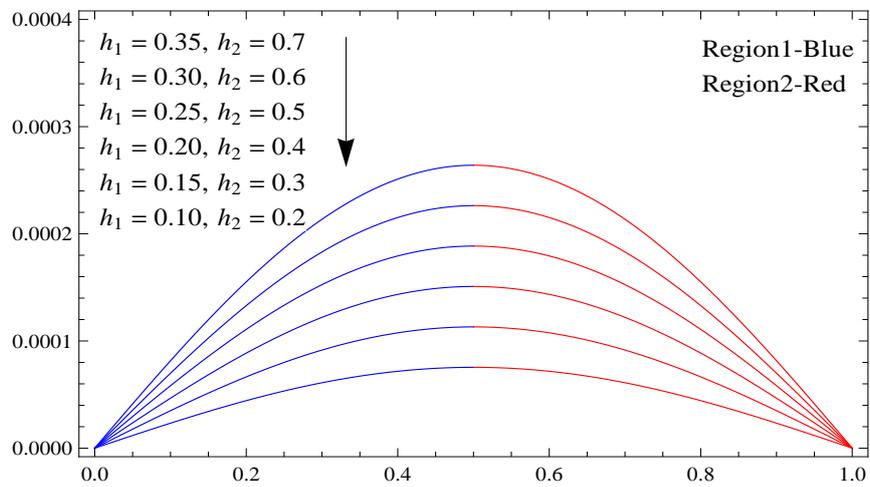


Figure 14: Velocity profile verses channel width at  $t = 0.1, m = 1$ .

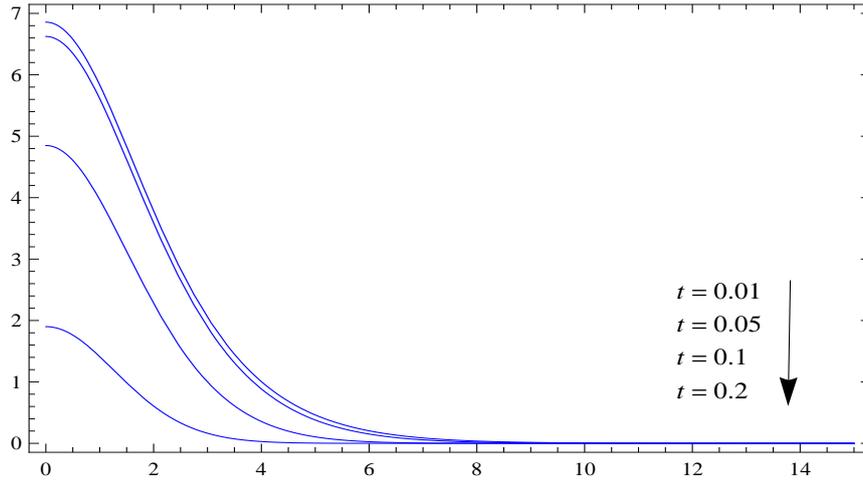


Figure 15: Shear stress on the lower plate verses m at different t.

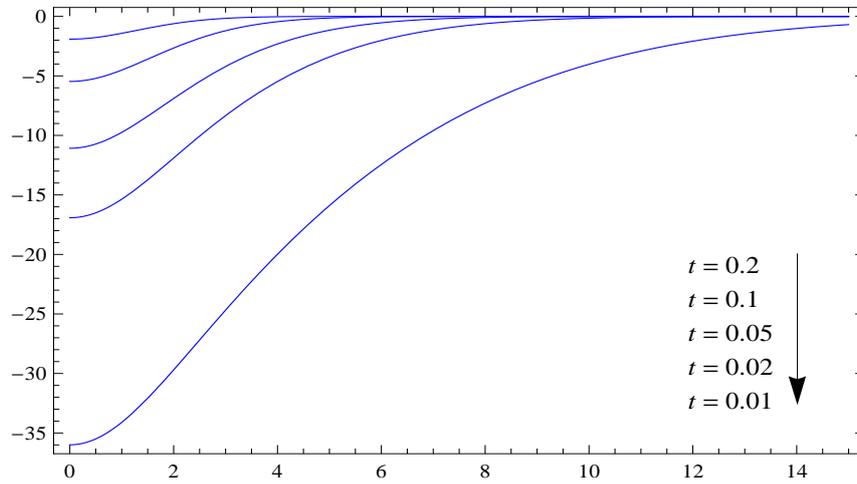


Figure 16: Shear stress on the upper plate verses m at different t.

## 4. Conclusion

The problem of unsteady receding flow of two immiscible viscous incompressible fluids through porous medium bounded between two impermeable parallel plates was investigated analytically. The flow was generated by motion of one of the plates, keeping other at rest. when the steady state had reached, moving plate was suddenly stopped and the subsequent receding flow was investigated. The analytical results were evaluated numerically and depicted graphically for various values of time, porosity parameter, viscosity ratio and channel width. It had been noticed that fluid velocity attained maximum near the middle of the plates and decreased towards the plates. Further as porosity parameter increased, point of maximum velocity was shifted towards the upper plates. Also increment in porosity parameter resulted in an exponential increment in shear stress at the upper plate while an exponential decrement in shear stress at the lower plate. It is found that flow can be effectively controlled by the properties of the two fluids as well as porosity parameter.

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