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Signed Product Cordial and Even Sum Cordial Labeling for the Extended Duplicate Graph of Splitting Graph of Path

Research Article

R.Avudainayaki[1], B.Selvam[2]* and P.P.Ulaganathan[2]

1 Department of Mathematics, Sri Sairam Institute of Technology, Chennai, India.

2 Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai, India.

Abstract: In this paper, we prove that the extended duplicate graph of splitting graph of path admits signed product cordial, total signed product cordial and even sum cordial labeling.

MSC: 05C78.

Keywords: Graph labeling, Duplicate graph, Splitting graph, Signed product cordial, Even sum cordial.

1. Introduction

The origin of graph labeling can be attributed to Rosa. E.Sampthkumar [1,2] introduced the concept of splitting graph and duplicate graph. Gallian provide the literature on survey of different types of graph labeling. The idea of signed product cordial labeling was introduced by J.BaskarBabujee [4] and he proved that many graphs admits signed product cordial labeling. R.Vikrama Prasad, R.Dhavaseelan and S.Abhirami [5] have proved the splitting graphs on even sum cordial labeling of graphs. P.Lawrence Rozario Raj and S.Koilraj [3] have proved the cordial labeling for the splitting graph of some standard graphs. K.Thirusangu, B.Selvam and P.P.Ulaganathan have proved that the extended duplicate graph of twig graphs is cordial and total cordial [6].

2. Preliminaries

In this section, we give some basic definitions which are relevant to this paper. Let $G(V, E)$ be a finite, simple and undirected graph with p vertices and q edges.

Definition 2.1 (Splitting Graph). *For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph Spl(G) thus obtained is called splitting graph of G.*

Illustration 2.2. *The Splitting graph of Path Spl(P_6)*

* *E-mail: kbmselvam@gmail.com*

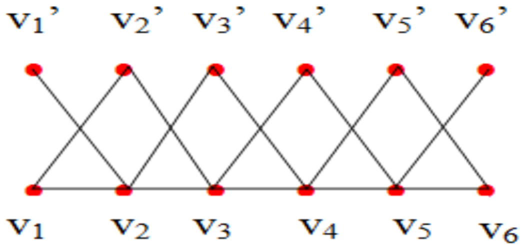

Figure 1. **Spl(P_6)**

Definition 2.3 (Duplicate Graph)**.** *Let $G(V,E)$ be a simple graph and the duplicate graph of G is $DG = (V_1, E_1)$, where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$ for convenience) and the edge set E_1 of DG is defined as the edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.*

Definition 2.4 (Extended duplicate graph of Splitting graph of Path)**.** *Let $DG = (V_1, E_1)$ be a duplicate graph of splitting graph of path $G(V,E)$. Extended duplicate graph of splitting graph of path is obtained by adding the edge v_2v_2' to the duplicate graph. It is denoted by EDG Spl(P_m). Clearly it has $4m$ vertices and $6m-5$ edges, $m \geq 2$.*

Illustration 2.5. *Extended duplicate graph of Splitting graph of Path*

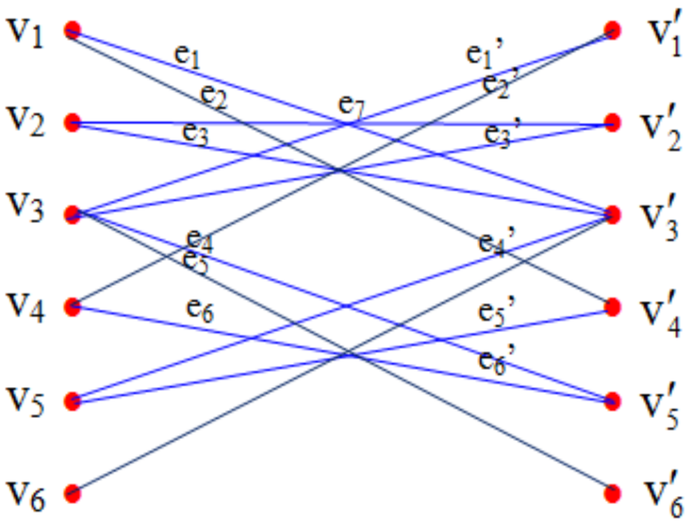

Figure 2. **EDG(Spl(P_3))**

Definition 2.6 (Signed product cordial labeling)**.** *A vertex labeling of graph G, $f : V(G) \to \{-1, 1\}$ with induced edge labeling $f^* : E(G) \to \{-1, 1\}$ defined by $f^*(uv) = f(u) \times f(v)$ is called a signed product cordial labeling if $|v_f(-1) - v_f(1)| = 1$ and $|e_f(-1) - e_f(1)| = 1$, where $v_f(-1)$ is the number of vertices labeled with '-1', $v_f(1)$ is the number of vertices labeled with 1, $e_f(-1)$ is the number of edges labeled with '-1' and $e_f(1)$ is the number of edges labeled with 1.*

Definition 2.7 (Total signed product cordial labeling)**.** *A function $f : V \to \{-1, 1\}$ such that each edge uv receives the label $f(u) \times f(v)$ is said to be total signed product cordial labeling if the number of vertices and edges labeled '-1' and the number of vertices and edges labeled '1' differ by at most one.*

Definition 2.8 (Even sum cordial labeling)**.** *Let $G = (V,E)$ be a simple graph and $f : V \to \{1, 2, 3 \ldots |V|\}$ be a bijection. For each edge uv, assign the label '1' if $f(u) + f(v)$ is even and the label '0' otherwise. f is called an even sum cordial labeling* *if $|e_f(0) - e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and number of edges labeled with 0 respectively.*

3. Main Results

3.1. Signed Product Cordial Labeling

In this section, we present an algorithm and prove the existence of signed product cordial labeling for the extended duplicate graph of splitting graph of path P_m, $m \geq 2$.

Algorithm 3.1. *Procedure [Signed Product cordial labeling for EDG Spl(P_m), $m \geq 2$]*

$V \leftarrow \{v_1, v_2, \ldots, v_{2m}, v'_1, v'_2, \ldots, v'_{2m}\}$

$E \leftarrow \{e_1, e_2, \ldots, e_{3m-3}, e_{3m-2}, e'_1, e'_2, \ldots, e'_{3m-3}\}$

$v_1 \leftarrow -1, v_2 \leftarrow 1, v'_1 \leftarrow 1, v'_2 \leftarrow -1, v'_3 \leftarrow 1$

for $i = 0$ *to* $(m-2)$ *do*

$v_{3+2i} \leftarrow -1$

$v_{4+2i} \leftarrow 1$

end for

for $i = 0$ *to* $[(m-2)/2]$ *do*

$v'_{4+4i} \leftarrow -1$

end for

for $i = 0$ *to* $[(m-3)/2]$ *do*

$v'_{5+4i} \leftarrow -1$

$v'_{6+4i} \leftarrow 1$

end for

for $i = 0$ *to* $[(m-4)/2]$ *do*

$v'_{7+4i} \leftarrow 1$

end for

end procedure.

Theorem 3.2. *The extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ admits signed product cordial labeling.*

Proof. Let Spl(P_m), $m \geq 2$ be a splitting graph of path. Let EDG Spl(P_m), $m \geq 2$ be the extended duplicate graph of splitting graph of path. To label the vertices, define a function $f : V \to \{-1, 1\}$ as given in Algorithm 3.1. The vertices v_1, v_2, v'_1, v'_2 and v'_3 receive label '-1', '1', '1' ,'-1' and '1' respectively; for $0 \leq i \leq (m-2)$, the vertices v_{3+2i} receive label '-1' and the vertices v_{4+2i} receive label '1'; for $0 \leq i \leq [(m-2)/2]$, the vertices v'_{4+4i} receive label '-1'; for $0 \leq i \leq [(m-3)/2]$, the vertices v'_{6+4i} receive label '1' and the vertices v'_{5+4i} receive label '-1'; for $0 \leq i \leq [(m-4)/2]$, the vertices v'_{7+4i} receive label '1'. Hence the entire $4m$ vertices are labeled such that the number of vertices labeled '-1' is $2m$ which is same as the number of vertices labeled '0' and satisfies the required condition. To obtain the label for edges, we define the induced function $f^* : E \to \{-1, 1\}$ such that

$$f^*(v_i v_j) = f(v_i) \times f(v_j); \quad v_i, v_j \in V$$

The induced function yields the label '-1' for the edges e_1, e'_1 and e'_{3m-2}; the label '1' for the edges e_2, e_3, e'_2 and e'_3; for $0 \leq i \leq [(m-2)/2]$, the edges e_{1+6i} receive label '-1' and for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the edges e_{2+6i+j} receive label '1'; for $0 \leq i \leq [(m-2)/2]$, the edges e_{4+6i} receive label '1' and the edges e'_{4+6i} receive label '-1'; for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the edges e_{5+6i+j} receive label '-1' and the edges e'_{5+6i+j} receive label '1'; for $0 \leq i \leq [(m-3)/2]$, the edges e_{7+6i} receive label '-1' and the edges e'_{7+6i} receive label '1'; for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the edges e'_{8+6i+j} receive label '1' and the edges e'_{8+6i+j} receive label '-1'. Hence all the $6m-5$ edges are labeled such that $3m-2$ edges receive label '1' and $3m-3$ edges receive label '-1' which differ by at most one and satisfies the required condition. Hence the extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ is signed product cordial. $\square$

Theorem 3.3. *The extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ admits total signed product cordial labeling.*

Proof. In Theorem 3.2, $2m$ vertices were assigned the label '-1' and $2m$ vertices were assigned the label '1' and it has been proved that the number of edges labeled '-1' is $(3m-3)$ and the number of edges labeled '1' is $(3m-2)$. From this, we conclude that the number of vertices and edges labeled '-1' is $2m+(3m-3) = 5m-3$ and the number of vertices and edges labeled '1' is $2m+(3m-2) = 5m-2$, which differ by one. Hence the extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ is total signed product cordial. $\square$

Illustration 3.4. *Signed product cordial labeling for the extended duplicate graph of Splitting graph P_6.*

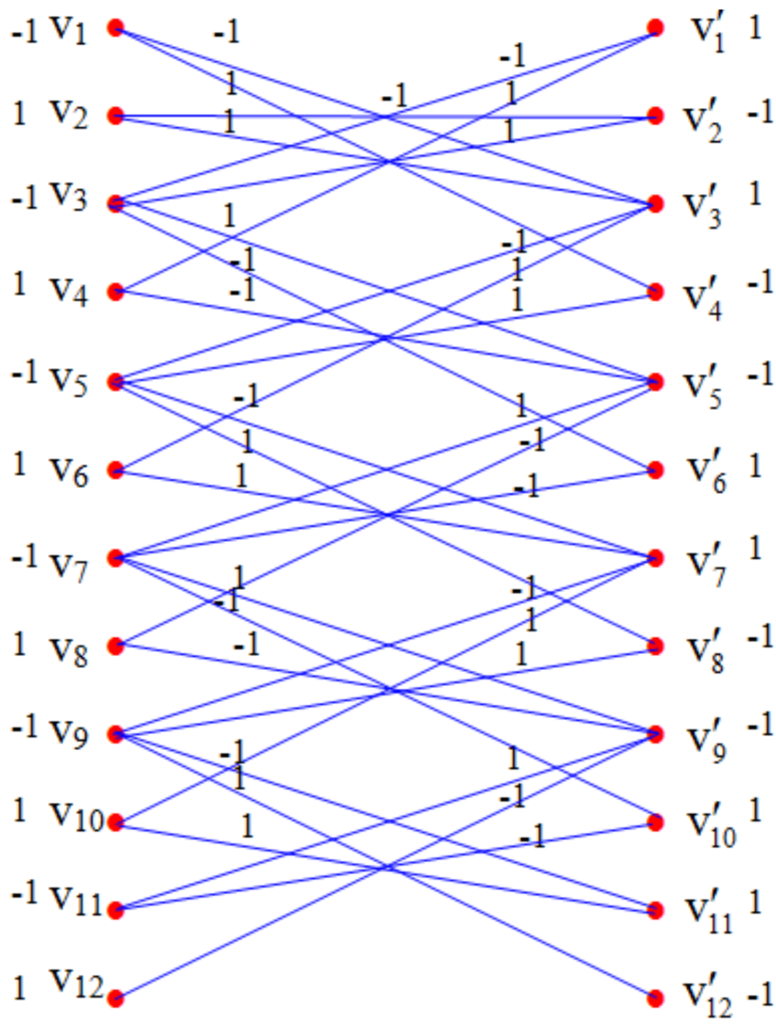

Figure 3. EDG(Spl(P_6))

3.2. Even Sum Cordial Labeling

In this section, we present an algorithm and prove the existence of even sum cordial labeling for the extended duplicate graph of splitting graph of path P_m, $m \geq 2$.

Algorithm 3.5. *Procedure [Even sum cordial labeling for EDG Spl(P_m), $m \geq 2$]*

$V \leftarrow \{v_1, v_2, \dots, v_{2m}, v'_1, v'_2, \dots, v'_{2m}\}$

$E \leftarrow \{e_1, e_2, \dots, e_{3m-3}, e_{3m-2}, e'_1, e'_2, \dots, e'_{3m-3}\}$

$v_1 \leftarrow 1, v_2 \leftarrow 3, v'_1 \leftarrow 2, v'_2 \leftarrow 4$

for $i = 0$ *to* $[(m-2)/2]$ *do*

$v_{3+4i} \leftarrow 4m - 3$

$v_{4+4i} \leftarrow 4m$

$v'_{3+4i} \leftarrow 4m - 1$

$v'_{4+4i} \leftarrow 4m - 2$

end for

for $i = 0$ *to* $[(m-3)/2]$ *do*

$v_{5+4i} \leftarrow 4m - 3$

$v_{6+4i} \leftarrow 4m - 1$

$v'_{5+4i} \leftarrow 4m - 2$

$v'_{6+4i} \leftarrow 4m$

end for

end procedure.

Theorem 3.6. *The extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ admits even sum cordial labeling.*

Proof. Let Spl(P_m), $m \geq 2$ be a splitting graph of path. Let EDG Spl(P_m), $m \geq 2$ be the extended duplicate graph of splitting graph of path. To label the vertices , define a function $f : V \rightarrow \{1, 2, 3, \dots, 4m\}$ as given in Algorithm 3.5. The vertices v_1, v_2, v'_1 and v'_2 receive label '1', '3', '2' and ' 4' respectively; for $0 \leq i \leq [(m-2)/2]$, the vertices v_{3+4i} receive label '$4m-3'$, the vertices v_{4+4i} receive label '$4m'$, the vertices v'_{3+4i} receive label '$4m-1'$ and the vertices v'_{4+4i} receive label '$4m-2'$; for $0 \leq i \leq [(m-3)/2]$, the vertices v_{5+4i} receive label '$4m-3'$, the vertices v_{6+4i} receive label '$4m-1'$, the vertices v'_{5+4i} receive label '$4m-2'$ and the vertices v'_{6+4i} receive label '$4m'$. Hence the entire $4m$ vertices are labeled. To obtain the label for edges, we define the induced function $f^* : E \rightarrow \{0, 1\}$ such that

$$f^*(v_iv_j) = 1 \text{ if } f(v_i) + f(v_j) \text{ is even and}$$

$$f^*(v_iv_j) = 0 \text{ if } f(v_i) + f(v_j) \text{ is odd where } v_i, v_j \in V.$$

The induced function yields the label '0' for the edge e'_{3m-2}; for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the edges $e_{1+6i+2j}$ receive label '1'; for $0 \leq i \leq [(m-2)/2]$, the edges e_{2+6i} receive label '0'; for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the edges e_{4+6i+j} receive label '0'; for $0 \leq i \leq [(m-3)/2]$, the edges e_{6+6i} receive label '1'; for $0 \leq i \leq [(m-2)/2]$ and $0 \leq j \leq 1$, the edges $e'_{1+6i+2j}$ receive label '0'; for $0 \leq i \leq [(m-2)/2]$, the edges e'_{2+6i} receive label '1'; for $0 \leq i \leq [(m-3)/2]$ and $0 \leq j \leq 1$, the edges e'_{4+6i+j} receive label '1'; for $0 \leq i \leq [(m-3)/2]$, the edges e'_{6+6i} receive label '0'. Hence all the $6m-5$ edges are labeled such that $3m-2$ edges receive label '1' and $3m-3$ edges receive label '0' which differ by at most one and satisfies the required condition. Hence the extended duplicate graph of splitting graph of path Spl(P_m), $m \geq 2$ is even sum cordial. □

Illustration 3.7. *Even sum cordial labeling for the extended duplicate spilitting graph of path P_6.*

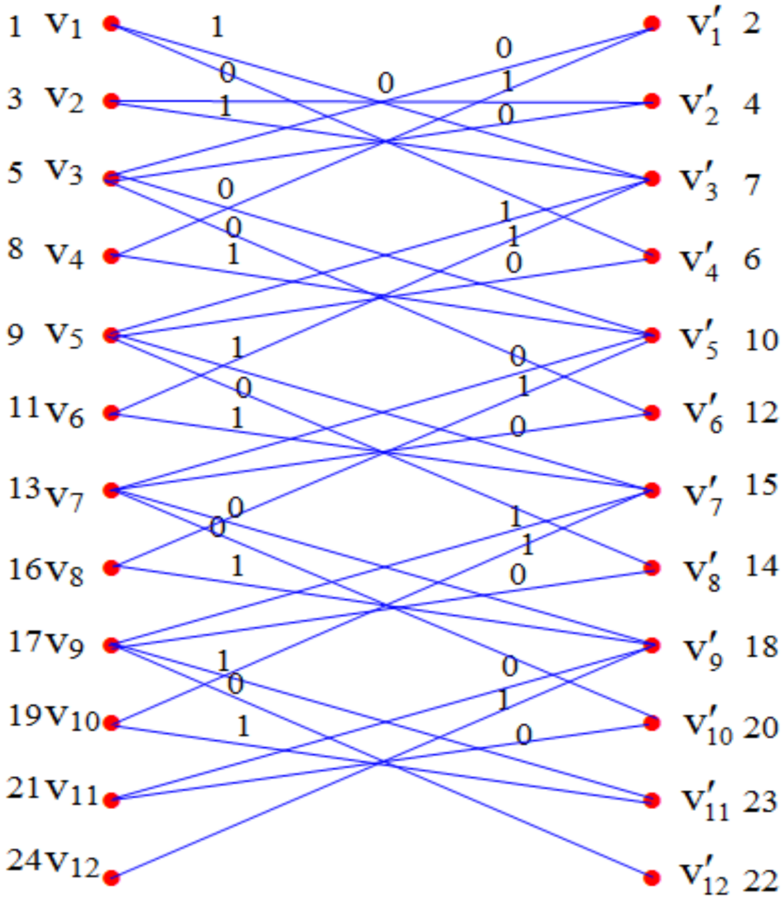

Figure 4. $\mathbf{EDG(Spl}(P_6))$

4. Conclusion

In this paper, we presented algorithms and prove that the extended duplicate graph of splitting graph of path P_m, $m \geq 2$ admits signed product cordial, total signed product cordial and even sum cordial labeling.

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