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Cordial Labeling for Swastik Graph

Research Article

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Abstract: I studied a new graph which is called swastik graph. I proved that the swastik graph is cordial. I have investigated some swastik graph related families of connected cordial graphs. I proved that path union of swastik graph, cycle of swastik graph and star of swastik graph are cordial.

Keywords: Cordial labeling, swastik graph, path union of graphs, cycle of graphs, star of a graph.

1. Introduction

The concept of cordial labeling was introduced by Cahit [2] in 1987 as a weaker version of graceful and harmonious labelings. It is found from Gallian survey [5] that Many researchers have studied cordiality of graphs. Large numbers of papers are found with variety of applications in coding theory. A depth details about applications of graph labeling is found in Bloom and Golomb [1]. I accepts all notations and terminology from Harary [5]. First of all let us recall some definitions, which are used in this paper.

1.1. Definitions

Definition 1.1. *A function $f : V(G) \rightarrow \{0,1\}$ is called binary vertex labeling of a graph G and f(v) is called label of the vertex v of G under f. For an edge $e = (u,v)$, the induced function $f^* : E(G) \rightarrow \{0,1\}$ defined as $f^*(e) = |f(u) - f(v)|$. Let vf(0), vf(1) be number of vertices of G having labels 0 and 1 respectively under f and let ef(0), ef(1) be number of edges of G having labels 0 and 1 respectively under f^*. A binary vertex labeling f of a graph tt is called cordial labeling if $|vf(0) - vf(1)| \leq 1$ and $|ef(0) - ef(1)| \leq 1$. A graph which admits cordial labeling is called cordial graph.*

Definition 1.2. *Let G be a graph and $G_1, G_2, \ldots, G_n$, $n \geq 2$ be n copies of graph G. Then the graph obtained by adding an edge from G_i to G_{i+1} $(1 \leq i \leq n-1)$ is called path union of G.*

Definition 1.3. *For a cycle C_n, each vertex of C_n is replaced by connected graphs $G_1, G_2, \ldots, G_n$ and is known as cycle of graphs. We shall denote it by $C(G_1, G_2, \ldots, G_n)$. If we replace each vertex by a graph G, i.e. $G_1 = G$, $G_2 = G, \ldots$, $G_n = G$, such cycle of a graph G is denoted by $C(n \cdot G)$.*

Above definition is introduced by Kaneria et al [4].

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Definition 1.4. *Let G be a graph on n vertices. The graph obtained by replacing each vertex of the star $K_{1,n}$ by a copy of G is called a star of G and is denoted by G^*.*

Definition 1.5. *Swastik graph is an union of four copies on C_{4n}. If $V_{i,j}$ $(\forall\ i = 1,2,3,4,\ \ \forall\ j = 1,2,\ldots,4n)$ be vertices of ith copy of $C_{4n}(i)$ then we shell combine $V_{1,4t}$ & $V_{2,1}$, $V_{2,4t}$ & $V_{3,1}$, $V_{3,4t}$ & $V_{4,1}$ and $V_{4,4t}$ & $V_{1,1}$ by a single vertex. So graph seems like a plus sign. If we bend branches of graph toward clockwise at the middle then the graph looks a swastik. It is denoted as S_{wn} of n size, where $n \in N - \{1\}$. Obviously $|V(S_{wn})| = 16(n) - 4$ and $|E(S_{wn})| = 16(n)$.*

In this paper I introduced cordialness of swastik graph, path union of swastik graph, cycle of swastik graph and star of swastik graph. For detail survey of graph labeling I refer Gallian [6].

2. Main Results

Theorem 2.1. *A swastik graph S_{wn} is a cordial graph, where $n \in N - \{1\}$.*

Proof. Let $G = S_{wn}$ be any swastik graph of size n, where $n \in N - \{1\}$. I mention each vertices of S_{wn} like $V_{i,j}$ $(i = 1,2,3,4,\ j = 1,2,\ldots,4n)$. I see the numbers of vertices in G is $|V(S_{wn})| = p = 16(n) - 4$ and $|E(S_{wn})| = q = 16(n)$. I define labeling function $f : V(G) \to \{0,1\}$ as follows

$$\begin{aligned}
f(v_{1,j}) &= 0;\ \ j = 1,2,5,6,9,10,\ldots,4n-3,4n-2 \\
&= 1;\ \ j = 3,4,7,8,\ldots,4n-1,4n \\
f(v_{2,j}) &= 0;\ \ j = 2,3,6,7,\ldots,4n-2,4n-1 \\
&= 1;\ \ j = 1,4,5,\ldots,4n-3,4n \\
f(v_{3,j}) &= 0;\ \ j = 3,4,7,8,\ldots,4n-1,4n \\
&= 1;\ \ j = 1,2,5,6,\ldots,4n-3,4n-2 \\
f(v_{4,j}) &= 0;\ \ j = 4,5,8,9,\ldots,4n-3,4n \\
&= 1;\ \ j = 2,3,6,7,\ldots,4n-2,4n-1
\end{aligned}$$

□

Illustration 2.2. *S_{w4} and its cordial labeling shown in Figure 1.*

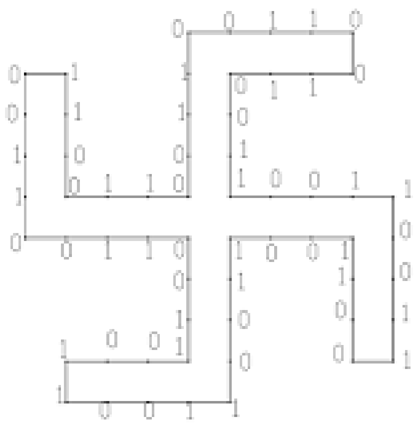

Figure 1. S_{w4}**, swastik graph with** $n = 4$ **and its cordial labeling**

Theorem 2.3. *Path union of finite copies of the swastik graph S_{wn} is a cordial graph, where $n \in N - \{1\}$.*

Proof. Let $G = P(r \cdot S_{wn})$ be a path union of r copies for the swastik graph S_{wn}, where $n \in N - \{1\}$. Let f be the cordial labeling of S_{wn} as I mentioned in Theorem 2.1. In graph G, I see that the vertices $|V(G)| = P = r(16(n) - 4)$ and the edges $|E(G)| = Q = (r - 1) + r \cdot 16(n)$. Let $u_{k,i,j}$ $(\forall \ i = 1, 2, 3, 4, \ \forall \ j = 1, 2, \ldots, 4n.)$ be the vertices of k^{th} copy of S_{wn}, $\forall \ k = 1, 2, \ldots, r$. Where the vertices of k^{th} copy of S_{wn} is $p = 16(n) - 4$ and edges of k^{th} copy of S_{wn} is $q = 16n$ Join vertices $u_{k,1,4n-1}$ with $u_{k+1,i,4n-1}$ for $k = 1, 2 \ldots, r - 1$ by an edge to form the path union of r copies of swastik graph. To define labeling function $g : V(G) \to \{0, 1\}$ as follows

$$\begin{aligned}
g(u_{1,i,j}) &= f(u_{i,j}) \\
g(u_{2,i,j}) &= g(u_{1,i,j}) + 1; \quad j = 1, 2, 5, 6, \ldots, 4n - 3, 4n - 2 \\
&= g(u_{1,i,j}) - 1; \quad j = 3, 4, 7, 8 \ldots, 4n - 1, 4n \\
g(u_{3,i,j}) &= g(u_{2,i,j}) \\
g(u_{k+1,i,j}) &= g(u_{k,i,j}) + 1; \quad k = 1, 2, 5, 6 \ldots, 4n - 3, 4n - 2 \\
&= g(u_{k,i,j}) - 1; \quad k = 3, 4, 7, 8 \ldots, 4n - 1, 4n \\
g(u_{4k-2,i,j}) &= g(u_{4k-1,i,j}); \quad k = 1, 2, 3 \ldots
\end{aligned}$$

Above labeling patten give rise a cordial labeling to given graph G. So path union of finite copies of the swastik graph is cordiall graph. □

Illustration 2.4. *Path union of 3 copies of S_{w3} and its cordial labeling shown in Figure 2.*

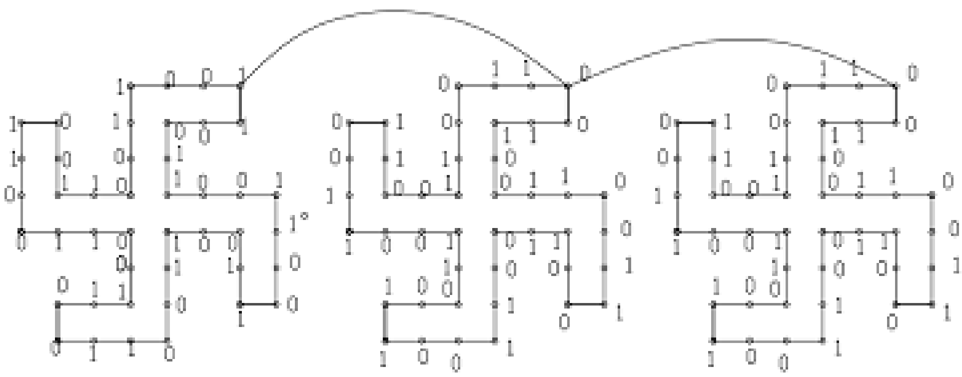

Figure 2. **A Path union of 3 copies of S_{w3} and its cordia labeling**

Theorem 2.5. *Cycle of r copies of swastik graph $C(r \cdot S_{wn})$ is a cordial graph, where $n \in N - \{1\}$ and $r \equiv 0, 3 \ (mod\ 4)$.*

Proof. Let $G = C(r \cdot S_{wn})$ be a cycle of swastik graph S_{wn}, where $n \in N - \{1\}$. Let f be the cordial labeling for S_{wn} as I mentioned in Theorem 2.1. In graph G, I see that the vertices $|V(G)| = P = r(16(n) - 4)$ and the edges $|E(G)| = Q = r(16(n)+1)$. Let $u_{k,i,j}$ $(\forall \ i = 1, 2, 3, 4, \ \forall \ j = 1, 2, \ldots, 4n)$ be the vertices of k copy of S_{wn}, $\forall \ k = 1, 2, \ldots, r$. Where the vertices of k^{th} copy of S_{wn} is $p = 16(n) - 4$ and edges of k^{th} copy of S_{wn} is $q = 16n$. Join vertices $u_{k,1,4n-3}$ with $u_{k+1,1,4n-3}$ for $k = 1, 2 \ldots, r - 1$ and $u_{r,1,4n-3}$ with $u_{1,1,4n-3}$ by an edge to from $C(r \cdot S_{wn})$. We define labeling function

$g : V(x) \to \{0, 1\}$ as follows

$$g(u_{1,i,j}) = f(u, i, j)$$
$$g(u_{2,i,j}) = g(u_{1,i,j}) + 1; \quad j = 1, 2, 5, 6\ldots, 4n-3, 4n-2$$
$$= g(u_{1,i,j}) - 1; \quad j = 3, 4, 7, 8\ldots, 4n-1, 4n$$
$$g(u_{3,i,j}) = g(u_{2,i,j})$$
$$g(u_{4,i,j}) = g(u_{1,i,j}), \quad \forall\ i = 1, 2, 3, 4, \quad \forall\ j = 1, 2, \ldots, 4n$$
$$g(u_{4k-3,i,j}) = g(u_{4k,i,j}), \quad k = 1, 2, 3\ldots$$
$$g(u_{4k-2,i,j}) = g(u_{4k-1,i,j}), \quad k = 1, 2, 3\ldots$$
$$g(u_{k+1,i,j}) = g(u_{k,i,j}) + 1, \quad k = 1, 2, 5, 6\ldots, 4n-3, 4n-2$$
$$= g(u_{k,i,j}) - 1; \quad k = 3, 4, 7, 8, \ldots, 4n-1, 4n$$

Above labeling patten give rise a cordial labeling to cycle of r copies for swastik graph □

Illustration 2.6. *$C(4 \cdot S_{w2})$ and its cordial labeling shown in Figure 3.*

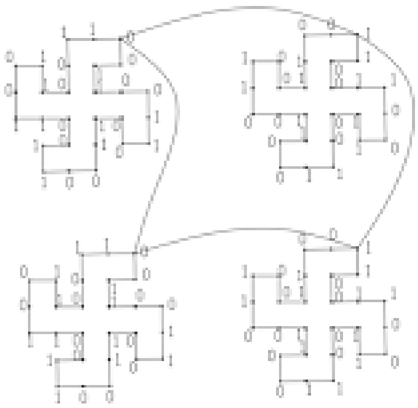

Figure 3. **A cycle of four copies for S_{w2} and its cordial labeling**

Theorem 2.7. *Star of swastik graph $(S_{wn})^*$ is cordial, where $n \in N - \{1\}$.*

Proof. Let $G = (S_{wn})^*$ be a star of swastik graph S_{wn}, where $n \in N - \{1\}$. Let f be the cordial labeling for Swn as we mention in Theorem 2.1. In graph G, we see that the vertices $|V(G)| = P = p(p+1)$ and the edges $|E(G)| = Q = (p+1)q+p$, where $p = 16(n) - 4$ and $q = 16(n)$. Let $u_{k,i,j}$ $(\forall\ i = 1, 2, 3, 4, \ \forall\ j = 1, 2, \ldots, 4n)$ be the vertices of k$^{\text{th}}$ copy of S_{wn}, $\forall\ k = 1, 2, \ldots, p$. Where the vertices of k$^{\text{th}}$ copy of S_{wn} is $p = 16(n) - 4$ and edges of k$^{\text{th}}$ copy of S_{wn} is $q = 16(n)$. I mention that central copy of $(S_{wn})^*$ is $(S_{wn})(0)$ and other copies of $(S_{wn})^*$ is $(S_{wn})(k)$, $\forall\ k = 1, 2, \ldots, p$. We define labeling function $g : V(G) \to \{0, 1\}$ as follows

$$g(u_{0,i,j}) = f(u, i, j)$$
$$g(u_{1,i,j}) = g(u_{0,i,j}) = f(u, i, j)$$
$$g(u_{3,i,j}) = g(u_{1,i,j}) = g(u0, i, j)$$
$$g(u_{5,i,j}) = g(u_{3,i,j})$$
$$\ldots\ldots$$

$$
\begin{aligned}
g(u_{2n-1,i,j}) &= g(u_{0,i,j}), \quad n = 1, 2. \ldots \text{ and} \\
g(u_{2,i,j}) &= g(u_{0,i,j}) + 1, \quad j = 1, 2, 5, 6, \ldots, 4n-3, 4n-2 \\
&= g(u_{0,i,j}) - 1, \quad j = 3, 4, 7, 8, \ldots, 4n-1, 4n \\
g(u_{4,i,j}) &= g(u_{2,i,j}) \\
g(u_{6,i,j}) &= g(u_{4,i,j}) \\
&\ldots\ldots \\
g(u_{2n,i,j}) &= g(u_{2,i,j}), \quad n = 1, 2 \ldots
\end{aligned}
$$

We see that difference of vertices for the central copy $(S_{wn})(0)$ at G and its other copies $(S_{wn})(k)$ $(1 \leq K \leq P)$ is precisely following sequence.

$$
\begin{aligned}
u_{1,i,j} &= u_{0,i,j} \\
u_{2,i,j} &= (u_{0,i,j}) + 1; \quad j = 1, 2, 5, \ldots, 4n-3, 4n-2 \\
&= (u_{0,i,j}) - 1; \quad j = 3, 4, 7, \ldots, 4n-1, 4n \\
u_{3,i,j} &= u_{0,i,j} \\
u_{4,i,j} &= u_{2,i,j} \\
u_{5,i,j} &= u_{0,i,j} \\
&\ldots\ldots \\
u_{2n-1,i,j} &= u_{0,i,j} \\
u_{2n,i,j} &= u_{2,i,j}
\end{aligned}
$$

Using this sequence I can produce required edge label by joing corresponding vertices of $(S_{wn})(0)$ with its other copy $(S_{wn})(k)$ $(1 \leq K \leq P)$ in G. thus G admits cordial labelling. □

3. Concluding Remarks

Here I identified a new graph is called swastik graph. Present work contributes some new results. I discussed cordialness of swastik graphs, path union of swastik graph, cycle of swastik graph and star of swastik graph. The labeling patten is demonstrated by means of illustrations which provide better understanding to derived results.

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