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Viscosity Variation and Thermal Effects in Finite Journal Bearings

Research Article

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Abstract: In this paper In Finite Journal bearing considering the effects of additives in lubrication with viscosity variation and thermal effects are analyzed. The generalized Reynolds equation for two layer fluid is derived and is applied for finite journal bearing. The finite journal bearing with modified Reynolds equation is solved numerically by using Finite Difference Method technique. As the thermal effect increases for two layer fluids increases the pressure and load capacity in the lubrication process.

Keywords: Viscosity, Thermal effect, Finite Journal Bearing, eccentricity, Film-thickness.

1. Introduction

In general, most of the lubricated systems can be considered to consist of moving (stationary) surfaces (plane/curve or loaded/unloaded) with a thin film of an external material (lubricant) between them. The presence of such a thin film between these surfaces not only helps to support considerable load but also minimizes friction. The characteristics such as pressure in the film, frictional force at the surface, flow rate of lubricant etc. of the system depend upon the nature of the surfaces, the nature of the lubricant film boundary conditions etc. The equation governing the pressure generated in the lubricant film can be obtained by coupling the equation of motion with the equation of continuity and was first derived by Reynolds [16] and is known as "Reynolds Equation". In deriving this equation, the thermal effects, compressibility, viscosity variation, slip at the surface, inertia and surface roughness effects were ignored. Later this Reynolds equation is modified by including viscosity and density variation along the fluid film. Dowson [7] unified the various attempts in generalizing the Reynolds equation by considering the variation of fluid properties across as well as along the fluid film-thickness by neglecting slip effects at the bearing surfaces.Bharath Kumar [4] analyzed fluid pressure and load capacity by considering thermal effects. It may be noted that the effects of viscosity at the surface and thermal variation is important on the flow behavior of gases and liquids particularly when the film-thickness is very small and the surface is very smooth. In this paper the modified Reynolds equation with the viscosity variation and thermal effects on finite journal bearing are studied by Finite Difference Method technique. Expression for pressure and load capacity is obtained and analyzed numerically. The effect of peripheral layer thickness, eccentricity, and thermal factor on load capacity and pressure is found. The point of maximum pressure increases due to thermal effect, whereas the load capacity decreases.

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2. Mathematical Formulation of the Problem

The Physical configuration of the journal bearing is shown in fig 1. C be the clearance of the bearing, $c = r - R$ and $\varepsilon = \frac{e}{c}$ be the eccentricity ratio as show in fig 1, h is the total film thickness, is given by

$$h = c(1 + \varepsilon \cos\theta)$$
$$\frac{\partial h}{\partial \theta} = -\varepsilon \sin\theta \tag{1}$$

The equation of governing to the fluid flow in the bearing is given by

$$\frac{\partial}{\partial x}\left[F\frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{h^3}{12\mu}F\frac{\partial p}{\partial y}\right] = U\frac{\partial h}{\partial x} \tag{2}$$
$$F = \frac{(1-\frac{a}{h})^3(k-1)+1}{k} \tag{3}$$

And if

$$\mu = \mu_0\left(\frac{h}{h_0}\right)^q \tag{4}$$

then the non-dimensional parameters are

$$x = R\theta,\ dx = Rd\theta,\ \overline{y} = \frac{y}{L} \Rightarrow y = \overline{y}L,\ dy = Ld\overline{y},\ \overline{a} = \frac{a}{c},\ \overline{h} = \frac{h}{c},\ \lambda = \frac{L}{2R},\ \overline{\mu} = \frac{\mu_0}{{h_0}^q} \tag{5}$$

$$\frac{\partial}{R\partial\theta}\left[F\frac{h^3}{12\mu_0(\frac{h}{h_0})^q}\frac{\partial p}{R\partial\theta}\right] + \frac{\partial}{L\partial\overline{y}}\left[\frac{h^3}{12\mu_0(\frac{h}{h_0})^q}F\frac{\partial p}{L\partial\overline{y}}\right] = \frac{U}{R}\frac{\partial h}{\partial\theta} \tag{6}$$

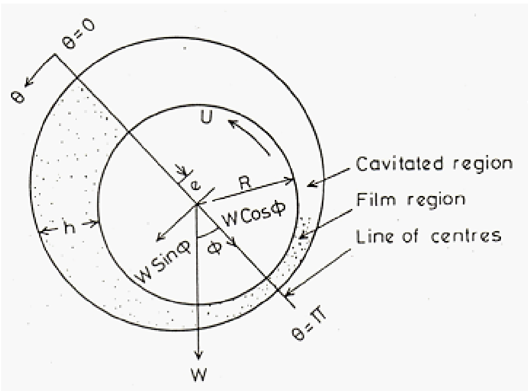


Figure 1: Journal bearing configuration

$$\frac{\partial}{R\partial\theta}\left[F\frac{h_0^q h^{3-q}}{12\mu_0}\frac{\partial p}{R\partial\theta}\right] + \frac{\partial}{L\partial\overline{y}}\left[\frac{h_0^q h^{3-q}}{12\mu_0}F\frac{\partial p}{L\partial\overline{y}}\right] = \frac{U}{R}\frac{\partial h}{\partial\theta} \tag{7}$$

By substituting equation (5) in (7), then the modified Reynolds equation in a non-dimensional form can be written a

$$\frac{\partial}{R\partial\theta}\left[\overline{F}\frac{\overline{h}^{3-q}}{12\overline{\mu}}\frac{\partial p}{R\partial\theta}\right] + \frac{\partial}{L\partial\overline{y}}\left[\frac{\overline{h}^{3-q}}{12\overline{\mu}}\overline{F}\frac{\partial p}{L\partial\overline{y}}\right] = \frac{U}{R}\frac{\partial h}{\partial\theta} \tag{8}$$

Where $\lambda^2 = \frac{L^2}{4R^2}$

$$\overline{F} = \frac{(1-\frac{\overline{a}}{\overline{h}})^3(k-1)+1}{k}\overline{h} \qquad = c(1+\varepsilon\cos\theta) \tag{9}$$

By solving the above equation (7), we get the non-dimensional pressure as

$$\overline{p} = \frac{pc^2}{\mu u R} \tag{10}$$

Now the equation (7) reduced to

$$\frac{\partial}{\partial\theta}\left[\overline{Fh}^{3-q}\frac{\partial\overline{p}}{\partial\theta}\right] + \frac{1}{4\lambda^2}\frac{\partial}{\partial\overline{y}}\left[\overline{Fh}^{3-q}\frac{\partial\overline{p}}{\partial\overline{y}}\right] = -12\varepsilon\sin\theta \tag{11}$$

The boundary conditions for fluid film pressure are

$$\begin{aligned}\overline{p} &= 0 \ at \ \ \theta = 0\\ \overline{p} &= 0 \ at \ \ \theta = \pi\end{aligned} \tag{12}$$

The modified Reynolds equation is solved numerically using Finite difference method. The film domain under consideration is divided by grid points as shown in Fig 2. In finite increment format, the terms of equation (11) can be written as

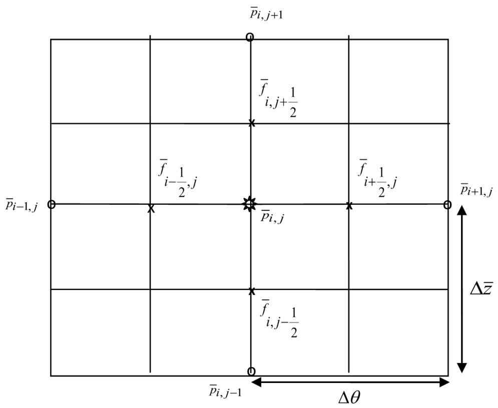


Figure 2: Grid point notation for film domain

$$\begin{aligned}&\frac{\overline{h}^{3-q}}{\Delta\theta}\left[\overline{F}_{i+{}^1\!/_2,\,j}\left(\frac{\overline{p}_{i,j}-\overline{p}_{i-1,j}}{\Delta\theta}\right)\right] - \left[\overline{F}_{i-{}^1\!/_2,\,j}\left(\frac{\overline{p}_{i+1,j}-\overline{p}_{i,j}}{\Delta\theta}\right)\right] +\\ &\frac{\overline{h}^{3-q}}{4\lambda^2}\frac{1}{\Delta\overline{y}}\left[\overline{F}_{i,\,j+{}^1\!/_2}\left(\frac{\overline{p}_{i,j}-\overline{p}_{i,j-1}}{\Delta\overline{y}}\right) - \left[\overline{F}_{i,\,j-{}^1\!/_2}\left(\frac{\overline{p}_{i,j+1}-\overline{p}_{i,j}}{\Delta\overline{y}}\right)\right]\right] = -12\varepsilon\sin\theta\end{aligned} \tag{13}$$

By solving we get as

$$\begin{aligned}&\frac{\overline{h}^{3-q}}{(\Delta\theta)^2}\left[\overline{F}_{i+1/2,j}\,\overline{p}_{i,j} - \overline{F}_{i+1/2,j}\,\overline{p}_{i+1,j} - \overline{F}_{i-1/2,j}\,\overline{p}_{i+1,j} + \overline{F}_{i-1/2,j}\,\overline{p}_{i,j}\right] +\\ &\frac{\overline{h}^{3-q}}{4\lambda^2}\frac{1}{\Delta\overline{y}^2}\left[\overline{F}_{i,j+1/2}\,\overline{p}_{i,j} - \overline{F}_{i,j+1/2}\,\overline{p}_{i,j-1} - \overline{F}_{i,j-1/2,}\,\overline{p}_{i+1,j} + \overline{F}_{i,j-1/2}\,\overline{p}_{i,j}\right] = -12\varepsilon\sin\theta\\ &4\lambda^2 r^2\overline{h}^{3-q}\left[\overline{F}_{i+1/2,j}\,\overline{p}_{i,j} - \overline{F}_{i+1/2,j}\,\overline{p}_{i-1,j} - \overline{F}_{i-1/2,j}\,\overline{p}_{i+1,j} + \overline{F}_{i-1/2,j}\,\overline{p}_{i,j}\right] +\\ &\overline{h}^{3-q}\left[\overline{F}_{i,j+1/2}\,\overline{p}_{i,j} - \overline{F}_{i,j+1/2}\,\overline{p}_{i,j-1} - \overline{F}_{i,j-1/2,}\,\overline{p}_{i,j+1} + \overline{F}_{i,j-1/2}\,\overline{p}_{i,j}\right] = -12\varepsilon 4\lambda^2\sin\theta\,\Delta\overline{y}^2\end{aligned} \tag{14}$$

By substituting this equation in (10), we get

$$\begin{aligned}\overline{p}_{i,j}\overline{h}^{3-q}\left[4\lambda^2 r^2\left(\overline{F}_{i+1/2,j} + \overline{F}_{i-1/2,j}\right) + \left(\overline{F}_{i,j+1/2} + \overline{F}_{i,j-1/2}\right)\right] = -12\varepsilon 4\lambda^2\Delta\overline{y}^2\sin\theta + \overline{h}^{3-q}[4\lambda^2 r^2\overline{F}_{i+1/2,j}\overline{p}_{i-1,j}\\ + 4\lambda^2 r^2\overline{F}_{i-1/2,j}\overline{p}_{i+1,j} + \overline{F}_{i,j+1/2}\overline{p}_{i,j-1} + \overline{F}_{i,j-1/2}\overline{p}_{i,j+1}]\end{aligned}$$

$$c_0\overline{p}_{i,j}\overline{h}^{3-q} = \overline{h}^{3-q}[4\lambda^2 r^2 \overline{F}_{i+1/2,j}\overline{p}_{i-1,j} + 4\lambda^2 r^2 \overline{F}_{i-1/2,j}\overline{p}_{i+1,j} + \overline{F}_{i,j+1/2}\overline{p}_{i,j-1} + \overline{F}_{i,j-1/2}\overline{p}_{i,j+1}] - 12\varepsilon 4\lambda^2 \Delta\overline{y}^2 \sin\theta$$

$$\overline{p}_{i,j} = c_1\overline{p}_{i-1,j} + c_2\overline{p}_{i+1,j} + c_3\overline{p}_{i,j-1} + c_4 c_{i,j+1} + c_5 \tag{15}$$

The coefficient c_0, c_1, c_2, c_3, c_4, c_5, defined as

$$c_0 = \left[4\lambda^2 r^2 \left(\overline{F}_{i+1/2,j} + \overline{F}_{i-1/2,j}\right) + \left(\overline{F}_{i,j+1/2} + \overline{F}_{i,j-1/2}\right)\right], \quad c_1 = 4\lambda^2 r^2 \overline{F}_{i+1/2,j}/c_0, \ c_2 = 4\lambda^2 r^2 \overline{F}_{i-1/2,j}/c_0$$

$$c_3 = \overline{F}_{i,j+1/2}/c_0, \quad c_4 = \overline{F}_{i,j-1/2}/c_0, \quad c_5 = -48\lambda^2 \Delta\overline{y}^2 \varepsilon \sin\theta / c_0 \overline{h}^{3-q} \tag{16}$$

$\overline{r} = \frac{\Delta\overline{z}}{\Delta\overline{\theta}}$. The pressure p is calculated numerically with grid spacing of $\Delta\overline{\theta} = 0.05$ and $\Delta\overline{z} = 9^\circ$. The load carrying capacity of the bearing W, generated by the film pressure is obtained by

$$W = \int_{\theta=0}^{\theta=\pi} \int_{z=0}^{z=1/2} p\cos\theta \, d\theta . dz \tag{17}$$

By using (11) in (16), we get non dimensional load as

$$\overline{W} = \frac{WC^2}{\mu UR} = \int_{\theta=0}^{\theta=\pi} \int_{z=0}^{z=1/2} p\cos\theta \, d\overline{\theta} . d\overline{z} \tag{18}$$

$$\approx \overline{w} = \sum_{i=0}^{M} \sum_{j=0}^{N} \overline{p}_{ij} \, \Delta\overline{\theta} \, \Delta\overline{z} \tag{19}$$

Where $M + 1$ and $N + 1$ are the grid point numbers in the x and z direction respectively.

3. Result and Discussion

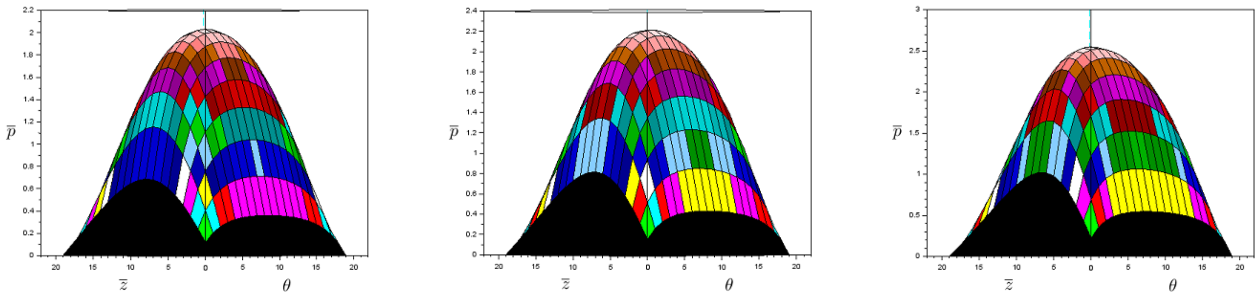
The pressure in equation (15) the mesh of the film domain has 20 equal intervals along the bearing length and circumference. The coefficient matrix of the system of algebraic equation is of pentadiagonal form. These equations have been solved by using sci-lab tools.

Pressure: The variation of non-dimensional pressure $\overline{p}$ for different values of q with $\overline{a} = 0.1$ and $r = 1.5$ is shown in Fig 3. It is observed that $\overline{p}$ increases for increasing value of q. Fig 4 shows The variation of film pressure $\overline{p}$ and different values of peripheral layer thickness $\overline{a}$ with $k = 0.5$ and $r = 1.5$. It is observed that $\overline{p}$ decreases for increasing values of $\overline{a}$. Fig 5 shows the variation of film pressure $\overline{p}$ and different values of k with $q = 0.1$. It is observed that $\overline{p}$ increasing for increasing values of k.

Load carrying capacity: Fig 6 shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with ε for different values of q at $k = 0.5$. It is observed that the increasing values of q decrease the $\overline{W}$ and the corresponding values of load at different q are shown in Table 1. Fig 7 shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with q for different values of ε at $k = 0.5$. It is observed that the increasing values of ε increase the $\overline{W}$ and the corresponding values of load at different ε are shown in Table 2.

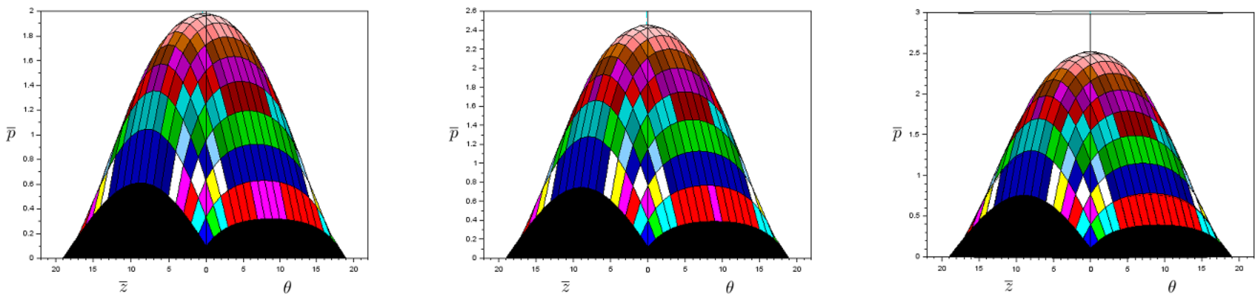
Fig 8 shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with q for different values of k at $\overline{a} = 0.1$. It is observed that the increasing values of k increase the $\overline{W}$ and the corresponding values of load $\overline{W}$ at different k are shown in Table 3. Fig 9 shows that the variation of non-dimensional load carrying capacity $\overline{W}$ with q for different values of $\overline{a}$. It is observed that the increasing values of $\overline{a}$ decrease the $\overline{W}$ and the corresponding values of load $\overline{W}$ at different $\overline{a}$ are shown in Table 4.

4. Graphs



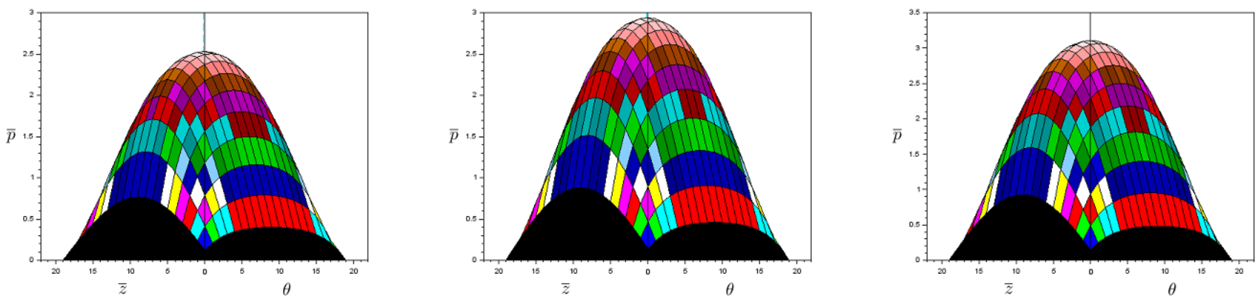
(a) q=1(2.03230127990072) (b) q=2(2.214355030045597) (c) q=3(2.545465945596239)

Figure 3: Non-Dimensionless pressure $\overline{p}$ for different values of q with $a = 0.1$, $r = 1.5$, $\varepsilon = 0.4$, $\lambda = 0.75$



(a) a=0.1(1.983636565406811) (b) a=0.01(2.455285750254697) (c) a=0.001(2.522529964714077)

Figure 4: Non-Dimensionless pressure $\overline{p}$ for different values of $\overline{a}$ with $k = 0.5$, $q = 0.1$, $\varepsilon = 0.4$, $\lambda = 0.75$



(a) K=1(2.530321510631443) (b) K=2(2.939949783363757) (c) K=3(3.109080349522665)

Figure 5: Non-Dimensionless pressure $\overline{p}$ for different values of k with $\overline{a} = 0.1$, $q = 0.1$, $\varepsilon = 0.4$, $\lambda = 0.75$

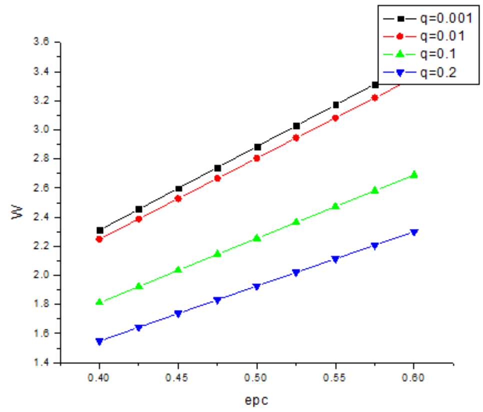


Figure 6: Dimensionless load $\overline{W}$ Vs ε for different q

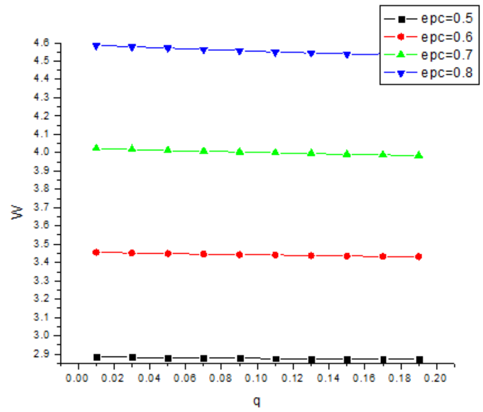


Figure 7: Dimensionless load $\overline{W}$ Vs q for different values of ε

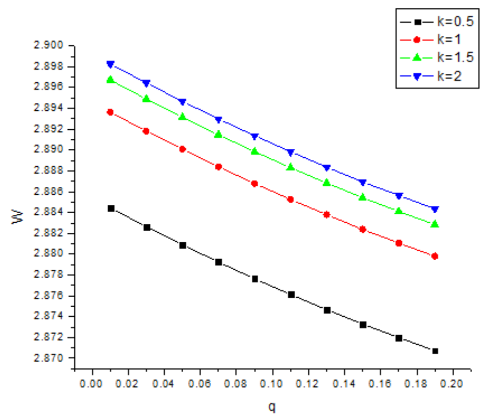


Figure 8: Dimensionless load $\overline{W}$ Vs q for different values of k

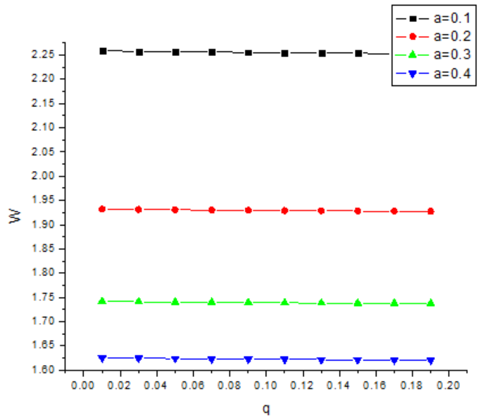


Figure 9: Dimensionless load $\overline{W}$ Vs q for different values of a

5. Tables

ε	$q = 0.001$	$q = 0.01$	$q = 0.1$	$q = 0.2$
0.4	2.3112	2.24867	1.81281	1.548982
0.425	2.45493	2.38813	1.924	1.644106
0.45	2.59852	2.52738	2.03478	1.738926
0.475	2.74197	2.66642	2.14515	1.833426
0.5	2.88528	2.80521	2.25506	1.927591
0.525	3.02842	2.94376	2.36449	2.021404
0.55	3.1714	3.08203	2.47341	2.11485
0.575	3.3142	3.22001	2.58179	2.207917
0.6	3.45683	3.35769	2.68959	2.30059

Table 1: Dimensionless load $\overline{W}$ Vs Eccentricity Ratio ε for different q

q	epc=0.5	epc=0.6	epc=0.7	epc=0.8
0.01	2.88444	3.45537	4.02293	4.58642
0.03	2.88264	3.45222	4.01785	4.57862
0.05	2.8809	3.4492	4.01297	4.57115
0.07	2.87924	3.4463	4.0083	4.56401
0.09	2.87765	3.44353	4.00384	4.55718
0.11	2.87613	3.44088	3.99958	4.55067
0.13	2.87468	3.43836	3.99553	4.54448
0.15	2.8733	3.43597	3.99168	4.53861
0.17	2.87198	3.43369	3.98803	4.53305
0.19	2.87074	3.43154	3.98458	4.52779

Table 2: Dimensionless load $\overline{W}$ Vs Thermal effect q for different ε

q	$K = 0.5$	$K = 1$	$K = 1.5$	$K = 2$
0.01	2.88444	2.89365	2.89673	2.89827
0.03	2.88264	2.89182	2.8949	2.89644
0.05	2.8809	2.89008	2.89315	2.89469
0.07	2.87924	2.8884	2.89146	2.893
0.09	2.87765	2.88679	2.88985	2.89138
0.11	2.87613	2.88525	2.88831	2.88984
0.13	2.87468	2.88379	2.88684	2.88836
0.15	2.8733	2.88239	2.88544	2.88696
0.17	2.87198	2.88106	2.8841	2.88563
0.19	2.87074	2.87981	2.88284	2.88436

Table 3: Dimensionless load $\overline{W}$ Vs Thermal effect for different k

q	a=0.1	a=0.2	a=0.3	a=0.4
0.01	2.25877	1.93204	1.7417	1.62517
0.03	2.25785	1.93137	1.74109	1.62453
0.05	2.25699	1.93075	1.74052	1.62392
0.07	2.25617	1.93018	1.73999	1.62336
0.09	2.25542	1.92966	1.73951	1.62284
0.11	2.25472	1.92918	1.73906	1.62236
0.13	2.25407	1.92875	1.73866	1.62191
0.15	2.25347	1.92836	1.7383	1.62151
0.17	2.25293	1.92802	1.73798	1.62114
0.19	2.25244	1.92772	1.73771	1.62081

Table 4: Dimensionless load $\overline{W}$ Vs Thermal effect for different $\overline{a}$

6. Summary

In this paper Finite Journal bearings considering the effects of additives in lubrication with viscosity variation and thermal effects are analyzed. The generalized Reynolds equation for two layer fluid isderived and is applied for finite journal bearing. The finite journal bearing with modified Reynolds equation is solved numerically by using FDM technique with a grid space of θ=$9^0$and $\Delta\overline{z}$=0.05. As the thermal effect increases for two layer fluids it increases the pressure and decreases viscosity and load capacity .

Nomenclature

a	peripheral layer thickness
p	pressures
h	Film thickness
μ	Viscosity of the fluid
x,y,z	Cartesian coordinates
θ	Circumference angle
c	Clearance
e	Eccentricity
R	Radius of the shaft
K	Ratio of viscosity near the surface to the purely hydrodynamic

u, v,w	Velocity component of the film in x,y,z direction
U	Velocity
ε	Eccentricity ratio
q	Thermal factor

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