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Attractivity Result For Second Order Random Differential Equation

Research Article

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Abstract: In this paper, the existence and attractivity results are proved for nonlinear second order ordinary random differential equations by random version of Schauder's fixed point theorem.

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1. Introduction

Let R denote the real line and R_+, the set of nonnegative real numbers. Let $C(R_+, R)$ denote the class of real-valued functions defined and continuous on R_+. Given a measurable space (Ω, A) and a measurable function $x : \Omega \to C(R_+, R)$, we consider the initial value problem of nonlinear second order ordinary random differential equations (RDE)

$$x''(t,\omega) + k(t,\omega)x(t,\omega) = f(t, x(t,\omega), \omega) \quad a.\ e.\ t \in R_+$$
$$x(0,\omega) = q(\omega)\,, x'(0,\omega) = 0. \tag{1}$$

for all $\omega \in \Omega$, where $k : R_+ \times \Omega \to R_+$, $q : \Omega \to R$ and $f : R_+ \times R \times \Omega \to R$. By a random solution of the RDE (1). We mean a measurable function $x : \Omega \to AC(R_+, R)$ that satisfies the equations in (1), where $AC(R_+, R)$ is the space of absolutely continuous real-valued functions defined on R_+. The initial value problem of random differential equations have been discussed in the literature for existence theorems on bounded intervals, however, the study of such random equations has not been studied on unbounded intervals of the real line for any aspects of the random solutions. Some results appear in Itoh [10], Bharucha-Reid [1] and Dhage [6]. Therefore, nonlinear random differential equations on unbounded intervals need to pay attention to the existence as well as the different characterizations of the random solutions. The present paper discuss the existence and attractivity results for random differential equations (1) on the right half R_+ of the real line R. The classical fixed point theory, in particular, random version of Schauder's fixed point theorem will be employed to prove the main result of this paper. This results generalize the stability results of Burton and Furumochi [2] in some sense.

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2. Auxiliary Results

The following theorem is used in the study of nonlinear discontinuous random differential equations.

Theorem 2.1 (Itoh [10])**.** . *Let X be a non-empty, closed convex bounded subset of the separable Banach space E and let $Q : \Omega \times X \to X$ be a compact and continuous random operator. Then the random equation $Q(\omega)x = x$ has a random solution, that is there is a measurable function $\xi : \Omega \to X$ such that $Q(\omega)\xi(\omega) = \xi(\omega)$ for all $\omega \in \Omega$.*

Theorem 2.2 (Carathodory)**.** *Let $Q : \Omega \times E \to E$ be a mapping such that $Q(\cdot, x)$ is measurable for all $x \in E$ and $Q(\omega, \cdot)$ is continuous for all $\omega \in \Omega$. Then the map $(\omega, x) \to Q(\omega, x)$ is jointly measurable.*

3. Characterizations of Random Solutions

The random solutions of RDE (1) in the Banach space $BC(R_+, R)$ of real-valued functions defined,continuous and bounded on R_+ with supremum norm $\|\cdot\|$ defined by $\|x\| = \sup_{x \in R_+} |x(t)|$. By $L^1(R_+, R)$ we denote the space of Lebesgue Measurable real-valued functions defined on R_+. By $\|\cdot\|_{L^1}$.e denote the usual norm in $L^1(R_+, R)$ defined by $\|x\|_{L^1} = \int_0^\infty |x(t)|dt$. In order to introduce the further concepts used in this paper, let us denote $E = BC(R_+, R)$ and S be a non-empty subset of E. Let $Q : \Omega \times E \to E$ be a mapping and consider the following random equation

$$Q(\omega)x(t) = x(t, \omega) \tag{2}$$

for $t \in R_+$ and $\omega \in \Omega$. A measurable function $x : \Omega \to E$ is called a random solution of random Equation (2) if it satisfies (2) on R_+. Herewith, we gives different characterizations of the random solutions for random Equation (2) onR_+.

Definition 3.1. *We say that random solutions of the random Equation (2)are locally attractive on R_+ if there exit a closed ball $\bar{B}_r(x_0)$ in the space $BC(R_+, R)$ for some $x_0 \in BC(R_+, R)$ and for some real number $r > 0$, such that for arbitrary random solutions $x = x(t, \omega)$ and $y = y(t, \omega)$ of random Equation (2) belonging to $\bar{B}_r(x_0) \cap S$ we have that*

$$\lim_{t \to \infty} (x(t, \omega) - y(t, \omega)) = 0 \tag{3}$$

for all $\omega \in \Omega$. In this case when the limit (3) is uniform with respect to the set $\bar{B}_r(x_0) \cap S$, that is, when for each $\epsilon > 0$ there exist a $T > 0$ such that for all $t \geq T$,

$$|x(t, \omega) - y(t, \omega)| \leq \varepsilon \tag{4}$$

for all $\omega \in \Omega$ and for all $x, y \in \bar{B}_r(x_0) \cap S$ being the random solutions of (2), I will say that the random solutions are uniformly locally attractive on R_+.

Definition 3.2. *I say that random solutions of the random equation (2) are globally attractive on R_+, if for arbitrary random solutions $x = x(t, \omega)$ and $y = y(t, \omega)$ of the random equation (2) belonging to S the condition (3) is satisfied. In the case when (3) is satisfied uniformly with respect to the set S in E, that is, for $\varepsilon > 0$ there exists a $T > 0$ such that $t \geq T$, the inequality (4) holds for all $x, y \in S$ being the random solutions for the random equation (2), I will say that the random solutions of the random equation (2) are uniformly globally attractive on R_+.*

Definition 3.3. *Let $c \in R$ be fixed. A line $y(t, \omega) = c$ for all $c \in R_+$ and $\omega \in \Omega$, is called an attractor **f** or the random solution $x : \Omega \to E$ to the random equation (2) if $\lim_{t \to \infty} [(x(t, \omega) - c)] = 0$ for all $\omega \in \Omega$. In this case the random solution x of the random equation (2) is said to be be asymptotic to the line $y = c$ and the line is an asymptote for the random solution x on R_+.*

Definition 3.4. *The random solutions for the random equation (2) are said to be locally asymptotically attractive if there exists a closed ball $\bar{B}_r(x_0)$ in E for some $x_0 \in E$ and for some real number $r > 0$, such that for any two random solutions $x = x(t,\omega)$ and $y = y(t,\omega)$ to the random equation (2) belonging to $\bar{B}_r(x_0) \cap S$ there is a line which is a common attractor to them onR_+. When x and y are uniformly locally attractive and there is a line as a common attractor, I will say that the random solutions of the random equation (2) are uniformly locally attractive on R_+.*

Definition 3.5. *The random solutions for the random equation (2) are said to be globally asymptotically attractive if for any two globally attractive solutions x and y of (2) there is a line which is a common attractor to them on R_+. Furthermore, if the random solutions for the random equation (2) are uniformly globally attractive then they are called uniformly globally asymptotically attractive on R_+.*

4. Attractivity Results

We need the following definition in the sequel

Definition 4.1. *A function $f : R_+ \times R \times \Omega \to R$ is called random Carathodory if (i) the map $\omega \to f(t,x,\omega)$ is measurable for all $t \in R_+$ and $x \in R$ and (ii) the map $(t,x) \to f(t,x,\omega)$ is jointly continuous for all $\omega \in \Omega$. Furthermore, a random Carathodory function $f : R_+ \times R \times \Omega \to R$ is called random L^1-Carathodory, if there exists a function $h \in L^1(R_+, R)$ such that $|f(t,x,\omega)| \leqslant h(t)$ a.e. $t \in R_+$, for all $\omega \in \Omega$ and $x \in R$. The function h is the growth function of f on $R_+ \times R \times \Omega$. Consider the following set of hypotheses:*

(A_0) The function $k : R_+ \times R \to R$ is measurable and bounded.

(A_1) The function $q : \Omega \to R$is measurable and bounded. Moreover,

$$ess \sup_{\omega\in\Omega} |q(\omega)| = c_1$$

for some real number $c_1 > 0$.

(A_2) The function f is randomL^1-Carathodory with growth function h on R_+. Moreover,

$$\lim_{t\to\infty} \int_0^t (t-s)\, h\, ds$$

where $h = [f(s, x(s,\omega), \omega) - k(s,\omega)\, x(s,\omega)]$ for all$\omega \in \Omega$.

Remark 4.2. *If the hypothesis (H_2) holds, then the function $w : R_+ \times \Omega \to R_+$ defined by*

$$w(t,\omega) = \int_0^t (t-s)\,[f(s, x(s,\omega), \omega) - k(s,\omega)\, x(s,\omega)]\, ds$$

is continuous and the number

$$w = \sup_{t\geqslant 0} w(t,\omega) = \sup_{t\geqslant 0} \int_0^t (t-s)\,[f(s, x(s,\omega), \omega) - k(s,\omega)\, x(s,\omega)]\, ds$$

exists for all $\omega \in \Omega$. See for example, Dhage [5], Burton and Furumochi [2].

5. Main Result

Theorem 5.1. *Assume that the hypotheses* (A_0) *to* (A_2) *hold. Then the RDE(1) admits a random solution. Moreover, random solutions are uniformly globally asymptotically attractive to the zero random solution on* R_+.

Proof. Now RDE(1) is equivalent to the random equation

$$x(t,\omega) = q(\omega) + \int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds \tag{5}$$

for all $t \in R_+$ and $\omega \in \Omega$. Set $E = BC(R_+, R)$. For a given function $x : \Omega \to E$, define a mapping Q on $\Omega \times E$ by

$$Q(\omega)x(t,\omega) = q(\omega) + \int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds \tag{6}$$

for all $t \in R_+$ and $\omega \in \Omega$. For the sake of convenience, We write $Q(\omega)x(t,\omega) = Q(\omega)x(t)$ omitting the double appearance of ω merge it into $Q(\omega)$. Clearly, Q defines a mapping $Q : \Omega \times E \to E$. To see this, let $x \in E$ be arbitrary. Then for each $\omega \in \Omega$, the continuity of map $t \to Q(\omega)x(t)$ follows from the fact that the indefinite integral

$$\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds$$

are continuous functions of t on R_+. Next, We show that the function $Q(\omega)x : R_+ \to R$ is bounded for each $\omega \in \Omega$. Now by hypotheses (A_1) and (A_2),

$$|Q(\omega)x(t)| \leq |q(\omega)| + \left| \int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds \right| \leq c_1 + W \tag{7}$$

for all $\omega \in \Omega$. As a result, $Q : \Omega \times E \to E$. Define a closed ball $\bar{B}_r(0)$ in the Banach space E centered at origin of radius $r = c_1 + W$ from (7), $|Q(\omega)x| \leq c_1 + W$ for all $\omega \in \Omega$ and $x \in E$. Hence $Q : \Omega \times E \to \bar{B}_r(0)$, and in particular, Q defines a map $Q : \Omega \times \bar{B}_r(0) \to \bar{B}_r(0)$. Now we show that Q satisfies all the conditions of Theorem 2.1 with $X = \bar{B}_r(0)$.

Firstly, We show that Q is a random operator on $\Omega \times \bar{B}_r(0)$ in to $\bar{B}_r(0)$. By hypothesis (H_2), the map $\omega \to f(t,x,\omega)$ is measurable by the Caratheodory theorem. Since a continuous function is measurable, so the $t \to [f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]$ is measurable in ω for all $t \in R_+$ and $x \in R$. Since the integral is a limit of the finite sum of measurable functions, We have that the function $\omega \to \int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds$ is measurable. Similarly, the map

$$\omega \to q(\omega) + \int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds$$

is measurable for all $t \in R_+$.Consequently, the map $\omega \to Q(\omega)x$ is measurable for all $x \in E$ and that Q is a random operator on $\Omega \times \bar{B}_r(0)$.

Secondly, We show that the random operator $Q(\omega)$ is continuous on $\bar{B}_r(0)$. By hypothesis (A_2),

$$\lim_{t\to\infty} w(t,\omega) = \lim_{t\to\infty} \int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds = 0,$$

there is a real number $T > 0$ such that $w(t) < \frac{\varepsilon}{4}$ for all $t \geq T$. We show that the continuity of the random operator $Q(\omega)$ in the following two cases:

Case I: Let $t \in [0,T]$ and let $\{x_n\}$ be a sequence of points in $\bar{B}_r(0)$ such that $x_n \to x$ as $n \to \infty$. Then, by the dominated convergence theorem,

$$\lim_{n\to\infty} Q(\omega)x_n(t) = \lim_{n\to\infty}\left(q(\omega) + \int_0^t (t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right)$$
$$\lim_{n\to\infty} Q(\omega)x_n(t) = q(\omega) + \lim_{n\to\infty}\left(\int_0^t (t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right)$$
$$\lim_{n\to\infty} Q(\omega)x_n(t) = q(\omega) + \left(\int_0^t (t-s)\lim_{n\to\infty}[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right)$$
$$= q(\omega) + \left(\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right) = Q(\omega)x(t)$$

for all $t \in [0,T]$ and $\omega \in \Omega$.

Case II: Suppose that $t \geq T$. Then we have

$$\begin{aligned}
&|Q(\omega)x_n(t) - Q(\omega)x(t)|\\
&= \left|\left(\int_0^t (t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right) - \left(\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right|\\
&\leq \left|\left(\int_0^t (t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right)\right| + \left|\left(\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right|\\
&\leq 2\omega(t) < \varepsilon
\end{aligned}$$

for all $t \geq T$ and $\omega \in \Omega$. Since ε is arbitrary, one has $\lim\limits_{n\to\infty} Q(\omega)x_n(t) = Q(\omega)x(t)$ all $t \geq T$ and $\omega \in \Omega$. Now combining the Case I with Case II, We conclude that $Q(\omega)$ is a point-wise continuous random operator on $\bar{B}_r(0)$ into itself. Further, it is shown below that the family of functions $\{Q(\omega)x_n\}$ is an equi-continuous set in E for a fixed $\omega \in \Omega$. Hence, the above convergence is uniform on R_+ and consequently, $Q(\omega)$ is a continuous random operator on $\bar{B}_r(0)$ into itself. Next, We show that $Q(\omega)$ is a compact random operator on $\bar{B}_r(0)$. Let $\omega \in \Omega$ be fixed and consider a sequence $\{Q(\omega)x_n\}$ of points in $\bar{B}_r(0)$. To finish, it is enough to show that the sequence $\{Q(\omega)x_n\}$ has a Cauchy subsequence for each $\omega \in \Omega$. Clearly $\{Q(\omega)x_n\}$ is a uniformly bounded subset of $\bar{B}_r(0)$. We show that it is an equi-continuous sequence of functions on R_+.

Let $\varepsilon > 0$ be given. Since $\lim\limits_{t\to\infty} w(t,\omega) = 0$, there exists a real number $T > 0$ such for $t < \frac{\varepsilon}{4}$ for $t \geq T$. We consider the following three cases:

Case I: Let $t_1, t_2 \in [0,T]$. Then, we have

$$\begin{aligned}
&|Q(\omega)x_n(t_1) - Q(\omega)x(t_2)|\\
&= \left|\left(\int_0^{t_1} (t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right) - \left(\int_0^{t_2} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right|\\
&= \left|\left(\int_0^{t_1} (t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right) + \left(\int_0^{t_1} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right.\\
&\left. - \left(\int_0^{t_2} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right) - \left(\int_0^{t_1} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right|\\
&\leq \left|\left(\int_0^{t_1} (t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right) - \int_0^{t_1} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right|\\
&+ \left|\left(\int_0^{t_2} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right) - \int_0^{t_1} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right|\\
&\leq \left|\left(\int_0^{T} (t-s)h(s)ds\right)\right|\\
&+ \left|\left(\int_0^{t_2} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right) - \int_0^{t_1} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right|\\
&\leq \left|\left(\int_0^{T} (t-s)h(s)ds\right)\right| + \left|\left(\int_{t_2}^{t_1} (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right|\\
&\leq |p(t_1) - p(t_2)|
\end{aligned}$$

$p(t) = \int_0^t (t-s)h(s)ds$ Since the function $p(t)$ is continuous on $[0,T]$, they are uniformly continuous there. Hence, $|Q(\omega)x_n(t_1) - Q(\omega)x(t_2)| \to 0$ as $t_1 \to t_2$ uniformly for all $t_1, t_2 \in [0,T]$ and for all $n \in N$.

Case II. If $t_1,\ t_2 \in\ [T,\infty]$, then

$$\begin{aligned}
&|Q(\omega)x_n(t_1) - Q(\omega)x(t_2)| \\
&\qquad = \left|\left(\int_0^{t_1}(t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right) - \left(\int_0^{t_2}(t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right| \\
&\qquad \le \left|\left(\int_0^{t_1}(t-s)[f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right)\right| + \left|\left(\int_0^{t_2}(t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds\right)\right| \\
&\qquad \le w(t_1) + w(t_2) \\
&\qquad \le \frac{\varepsilon}{4} + \frac{\varepsilon}{4} < \varepsilon.
\end{aligned}$$

Since ε is arbitrary, one has $|Q(\omega)x_n(t_1) - Q(\omega)x(t_2)| \to 0$ as $t_1 \to t_2$ uniformly for all $n \in N$.

Case III. If $t_1 < T < t_2$, then

$$|Q(\omega)x_n(t_1) - Q(\omega)x_n(t_2)| \le |Q(\omega)x_n(t_1) - Q(\omega)x_n(T)| + |Q(\omega)x_n(T) - Q(\omega)x_n(t_2)|$$

As $t_1 \to t_2$, $t_1 \to T$ and $t_2 \to T$, and so

$$|Q(\omega)x_n(t_1) - Q(\omega)x_n(T)| \to 0$$

and

$$|Q(\omega)x_n(T) - Q(\omega)x_n(T)| \to 0$$

as $t_1 \to t_2$ uniformly for all $n \in N$. Hence

$$|Q(\omega)x_n(t_1) - Q(\omega)x(t_2)| \to 0 \text{ as } t_1 \to t_2$$

uniformly for all $t_1 < T$ and $t_2 > T$ and for all $n \in N$. Thus, in all three cases

$$|Q(\omega)x_n(t_1) - Q(\omega)x_n(t_2)| \to 0 \text{ as } t_1 \to t_2$$

uniformly for all $t_1, t_2 \in R_+$ and for all $n \in N$. This shows that $\{Q(\omega)x_n\}$ is a equicontinuous sequence in X. Now an application of Arzelá-Ascoli theorem yields that $\{Q(\omega)x_n\}$ has a uniformly convergent subsequence on the compact subset $[0,T]$ of R. Without loss of generality, call the subsequence to be the sequence itself. We show that $\{Q(\omega)x_n\}$ is Cauchy in X. Now shows that $|Q(\omega)x_n(t) - Q(\omega)x(t)| \to 0$ as $n \to \infty$ for all $t \in [0,T]$. Then for given $\varepsilon > 0$ there exist $n_0 \in N$ such that

$$\sup_{0\le p\le T}\left|\left(\int_0^p (t-s)[f(s,x_m(s,\omega),\omega) - k(s,\omega)x_m(s,\omega)] - [f(s,x_n(s,\omega),\omega) - k(s,\omega)x_n(s,\omega)]ds\right)\right| < \frac{\varepsilon}{2}$$

for all $m, n \ge n_0$. Therefore, if $m, n \ge n_0$, then we have

$$\begin{aligned}
&|Q(\omega)x_m - Q(\omega)x_n| \\
&\quad = \sup_{0\le p\le T}\left(\begin{array}{c}\left|\left(\int_0^p (t-s)\left[f(s,x_m(s,\omega),\omega) - k(s,\omega)\,x_m(s,\omega)\right] - \left[f(s,x_n(s,\omega),\omega) - k(s,\omega)\,x_n(s,\omega)\right]ds\right)\right| + \\ \\ \left|\left(\int_0^p (t-s)\left[f(s,x_m(s,\omega),\omega) - k(s,\omega)\,x_m(s,\omega)\right] + \left[f(s,x_n(s,\omega),\omega) - k(s,\omega)\,x_n(s,\omega)\right]ds\right)\right|\end{array}\right) \\
&\quad \le \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon
\end{aligned}$$

This shows that $\{Q(\omega)x_n\} \subset Q(\omega)(\bar{B}_r(0)) \subset (\bar{B}_r(0))$ is Cauchy. Since X is complete, $\{Q(\omega)x_n\}$ converges to a point in X. As $Q(\omega)(\bar{B}_r(0))$ is closed, the sequence $\{Q(\omega)x_n\}$ converges to a point in $Q(\omega)(\bar{B}_r(0))$. Hence $Q(\omega)(\bar{B}_r(0))$ is relatively compact for each $\omega \in \Omega$ and consequently Q is a continuous and compact random operator on $\Omega \times (\bar{B}_r(0))$. Now an application of Theorem 2.1 to the operator $Q(\omega)$ on $(\bar{B}_r(0))$ yields that Q has a fixed point in $(\bar{B}_r(0))$ which further implies that the RDE (1) random solution on R_+.

Next, I show that the solutions are uniformly attractive on R_+. Let $x, y : \Omega \to \bar{B}_r(0)$ be any two random solutions to the RDE (1) on R_+ then for each $\omega \in \Omega$,

$$\begin{aligned}
&|x(t,\omega) - y(t,\omega)| \\
&\leq \left| \left(\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds \right) - \left(\int_0^t (t-s)[f(s,y(s,\omega),\omega) - k(s,\omega)y(s,\omega)]ds \right) \right| \\
&\leq \left| \left(\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds \right) \right| + \left| \int_0^t (t-s)[f(s,y(s,\omega),\omega) - k(s,\omega)y(s,\omega)]ds \right| \\
&\leq 2w(t)
\end{aligned}$$

for all $t \in R_+$. Since $\lim_{t\to\infty} w(t) = 0$, there is a real number $T > 0$ such that $w(t) < \frac{\varepsilon}{2}$ for all $t \geq T$. Therefore, $|x(t,\omega) - y(t,\omega)| \leq \varepsilon$ for all $t \geq T$ and for all $\omega \in \Omega$. Hence all random solutions of the RDE (1) are uniformly globally attractive on R_+.

Finally, We prove that random solutions are asymptotically attractive to the line y=0 on $\Omega \times R_+$. Let $x : \Omega \to C(R_+, R)$ be a random solution of the RDE (1) on R_+. Then, for each $\omega \in \Omega$,

$$\begin{aligned}
x(t,\omega) &\leq \left| q(\omega) + \left(\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds \right) \right| \\
x(t,\omega) &\leq |q(\omega)| + \left| \left(\int_0^t (t-s)[f(s,x(s,\omega),\omega) - k(s,\omega)x(s,\omega)]ds \right) \right| \\
&\leq c_1 + w(t)
\end{aligned}$$

for all $\omega \in \Omega$. Taking the limit superior in the above inequality as t tends to ∞ yields

$$\lim_{t\to\infty} \sup |x(t,\omega)| \leq c_1 + \lim_{t\to\infty} \sup w(t,\omega) = 0$$

and so, $\lim_{t\to\infty} |x(t,\omega)| = 0$ for all $\omega \in \Omega$. Therefore, for each $\varepsilon > 0$ there exit a real number $T > 0$ such that $|x(t,\omega)| < \varepsilon$ for all $t \geq T$ and $\omega \in \Omega$). Hence, all random solutions of the RDE (1) are uniformly globally asymptotically attractive to the zero random solution on R_+. □

6. Example

Let $\Omega = (-\infty, 0)$ with the usual σ-algebra consisting of Lebesgue measurable subsets of $(-\infty, 0)$. Given a function $x : R_+ \times \Omega \to C(R_+, R)$, consider the following random differential equation as

$$x'(t,\omega) + k(t,\omega)x(t,\omega) = \frac{e^t \cos \omega t x(t,\omega)}{1 + x(t,\omega)}$$

$x(0,\omega) = 1$, $x'(0,\omega) = 0$ for all $t \in R_+$ and $\omega \in \Omega$. Here, $q(\omega) = 1$, $k(t,\omega) = -1$ for all $\omega \in \Omega$. and $f(t,x,\omega) = \frac{e^t \cos \omega t\, x}{1+|x|}$ for $t \in R_+$, $x \in R$ and $\omega \in (-\infty, 0)$. obviously, the function f is random L^1-Caratheodory with growth function

$$h(t) = e^t \geq \left| \frac{e^t \cos \omega t\, x}{1 + |x|} \right| = |f(t,x,\omega)|.$$

Thus, both the hypotheses (A_0) and (A_2) of Theorem 4.1 are satisfied and hence the above RDE has a random solution and all random solutions are uniformly globally asymptotically attractive to the zero random solution on R_+.

7. Conclusion

In this paper, we have been able to extend and generalize some known existence, attractivity or stability results of deterministic nonlinear differential equations obtained in Burton and Furumochi [2] to indeterministic case of random differential equations on unbounded intervals of real line.

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