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An Extension of Hadamard-Rybezynski Solution

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Abstract: This paper is concerned with the motion of a liquid drop surrounded by a shell of immiscible liquid, having the same density but different viscosities, and both being embedded in a liquid medium differing from them in both density and viscosity. This is an extension of Hadamard and Rybezynski Levich [7].

Keywords: Slow viscous flow, drops and bubbles.

1. Introduction

The study of liquid drop having density ρ and viscosity μ surrounded by a concentric liquid shell having the same density and viscosity μ, both being immersed in a uniform stream of viscosity U is considered here.

2. Formulation of the Problem

A viscous drop of radius b and a concentric liquid shell in a uniform stream of velocity U are considered. The Renolds number of the flow in the three regions $[I, II, III]$, as shown in figure 1, is assumed to be small. Consequently the equation of motions in three regions are taken to be given by tokes flow for slow motion, namely

$$-\nabla\rho + \mu\nabla^2\underline{v} = 0 \tag{1}$$

And the equation of continuity

$$\nabla\cdot\underline{v} = 0 \tag{2}$$

Spherical polar coordinates (r, θ, φ) are used, and since the flow is axisymmetric

$$v = (v_r, v_\theta, 0) \tag{3}$$

At large distance from origin the flow is uniform which implies

$$\left.\begin{aligned} v_r &\approx U\cos\theta \\ v_\theta &\approx -U\sin\theta \end{aligned}\right\} \tag{4}$$

The liquid drop, the liquid shell and the external uniform stream will be called III, II and I which implies respectively. The boundary conditions at the surfaces of the liquid drop and sell are:

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(i). Continuity of pressure.

(ii). The normal components both inner and outer velocities must be zero.

(iii). Tue component of the rat of strain tensor e_{r_θ} must be continuous, where

$$e_{r_\theta} = r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta} \tag{5}$$

(iv). The velocity must be finite in region (III).

(v). The total pressure in the outer liquid (region I) is $(p - \pi_1)$. The boundary conditions for the normal and tangential stress on $r = a$ are

$$\left.\begin{aligned} -(p-\pi_1) + 2\mu\left(\tfrac{\partial v_r}{\partial r}\right) &= -p' + 2\mu'\left(\tfrac{\partial v_r'}{\partial r}\right) && \text{at } r = a \\ \mu\left(\tfrac{1}{r}\tfrac{\partial v_r}{\partial \theta} + \tfrac{\partial v_r}{\partial r} - \tfrac{v_\theta}{r}\right) &= \mu'\left(\tfrac{1}{r}\tfrac{\partial v_r'}{\partial \theta} + \tfrac{\partial v_r'}{\partial r} - \tfrac{v_\theta'}{r}\right) && \text{at } r = a \end{aligned}\right\} \tag{6}$$

where $\pi = (p - p')\, gx$ and at $r = b$ are:

$$\left.\begin{aligned} -(p-\pi_1) + 2\mu\left(\tfrac{\partial v_r'}{\partial r}\right) &= -p'' + 2\mu'\left(\tfrac{\partial v_r''}{\partial r}\right) && \text{at } r = a \\ \mu'\left(\tfrac{1}{r}\tfrac{\partial v_r'}{\partial \theta} + \tfrac{\partial v_r'}{\partial r} - \tfrac{v_\theta'}{r}\right) &= \mu''\left(\tfrac{1}{r}\tfrac{\partial v_r''}{\partial \theta} + \tfrac{\partial v_r''}{\partial r} - \tfrac{v_\theta''}{r}\right) && \text{at } r = a \end{aligned}\right\} \tag{7}$$

where $\pi_2 = (p' - p'')\, gx$. Notice that $p' = p''$ at $r = b$.

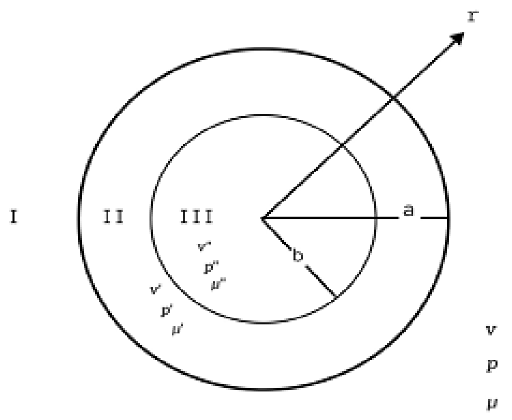

Figure 1. **Drop and sell immersed in fluid**

3. Method of Solution

The solution of equation (1) and (2) which satisfy the above boundary conditions can be found by standard techniques so that

In Region (I)

$$\left.\begin{aligned} v_r &= \left[\tfrac{A}{r^3} + \tfrac{B}{r} - C + Dr^2\right]\cos\theta \\ v_\theta &= \left[\tfrac{A}{r^3} + \tfrac{B}{2r} - C - 2Dr^2\right]\sin\theta \\ p &= \tfrac{\mu B}{r^2}\cos\theta \end{aligned}\right\} \tag{8}$$

In Region (II)

$$\left.\begin{aligned} v'_r &= \left[\tfrac{A'}{r^3} + \tfrac{B'}{r} - C' + D'r^2\right]\cos\theta \\ v'_\theta &= \left[\tfrac{A'}{r^3} + \tfrac{B'}{2r} - C' - 2D'r^2\right]\sin\theta \\ p' &= \mu'\left(\tfrac{B'}{r^2} + 10D'r\right)\cos\theta \end{aligned}\right\} \tag{9}$$

In Region (III)

$$\left.\begin{aligned} v''_r &= \left[\tfrac{A''}{r^3} + \tfrac{B''}{r} - C'' + D''r^2\right]\cos\theta \\ v''_\theta &= \left[\tfrac{A''}{r^3} + \tfrac{B''}{2r} - C'' - 2D''r^2\right]\sin\theta \\ p'' &= \mu''\left[10D''r\right]\cos\theta \end{aligned}\right\} \tag{10}$$

Now solving equations (6) - (10) for the arbitrary constants and with the appropriate substitutions we obtain, after lengthy calculations, the velocities and pressure components. Applying the boundary conditions (i)-(v), one finally obtains

$$U = \frac{2\left(p - p'\right) g a^2}{3\mu}\left[\frac{\left(3\mu'' + 2\mu'\right)\left(\mu + \mu'\right) a^5 - \left(\mu' - \mu''\right)\left(2\mu' - 3\mu\right) b^5}{\left(3\mu'' + 2\mu'\right)\left(3\mu' + 2\mu\right) a^5 - 6\left(\mu' - \mu''\right)\left(\mu' - \mu\right) b^5}\right] \tag{11}$$

Which is an extension to Hadamard-Rybczynski Solution.

4. Conclusion

On putting $\mu = \mu'$ or $b = 0$, in equation (11), one recovers the solution

$$U = \frac{2\left(p - p'\right) g a^2}{3\mu}\,\frac{\left(\mu' + \mu\right)}{\left(3\mu' + 2\mu\right)} \tag{12}$$

This conjecture may have far reaching effects in multi- phase flows, or other related ones, such as in the velocity of rise or fall, and on the drag force on the drop.

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