International Journal of Mathematics And its Applications
Volume 5, Issue 1–B (2017), 247–255.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

On Complements and Relative Complements in Distributive Soft Rough Lattice

Research Article

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Abstract: Soft sets and rough sets are both mathematical tools for dealing with problems that contain uncertainty. Soft rough set is a connection between these two mathematical approaches to vagueness and it is the generalization of rough set with respect to the soft approximation space. Nagarajan et al. [9] defined the direct product of soft lattices. The complements of soft lattice has been given by Nagarajan et al. [10]. The complements of rough set has been defined by Ameri et al. [1]. In this paper, we first define complements of soft rough set and also define the complements and relative complements of soft rough lattice. We discuss some theorems related to the complements and relative complements of distributive soft rough lattice with an example.

MSC: 06C15, 06D99.

Keywords: Soft set, rough set, soft rough set, soft rough lattice, distributive soft rough lattice, complement of soft rough set, complement of soft rough lattice, relatively complement of soft rough lattice.

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1. Introduction

In 1999, Molodtsov [7] introduced soft set as a mathematical tool for dealing with uncertainty. Maji et al. [6] defined the operations of soft set and a theoretical study on soft set. The soft lattice structure has been found in [5, 8, 11]. Rough set theory introduced by Pawlak [12] is another mathematical approach to vagueness. Every rough sets are associated with two crisp sets, called lower and upper approximations and viewed as the sets of elements which certainly and possibly belong to the set. Rana and Roy [13] introduced rough set approach on lattice. Soft set theory is a possible way to solve the difficulties of rough set. Thereafter a possible fusion of rough sets and soft sets has been proposed by Feng et al. [2]. Roy and Bera [14] introduced some operations on soft rough set and defined distributive and modular soft rough lattice and discussed is properties. Soft rough set is the generalization of rough set with respect to the soft approximation space. The properties of modular soft rough and distributive soft rough lattice was discussed in [3, 4]. The direct product of soft lattices was defined in [9]. The complements can be found for soft sets as well as rough sets. The complement of soft lattice has been found in [10]. The complement of rough set has been defined in [1]. In this paper, we define the complements of soft rough set and also the complements and relative complements of soft rough lattice and discuss some theorems related to the complements of distributive soft rough lattice with an example.

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2. Preliminaries

In this section, we recall some basic properties and definitions related to soft set, soft rough set and soft rough lattices. Let U be an initial universe of objects and E be the set of parameters and $A \subseteq E$. $P(U)$ is the power set of U.

Definition 2.1. *A pair $S = (F, A)$ is called a soft set over U, where $F : A \to P(U)$ is a set valued mapping.*

Definition 2.2. *Let U be the set of universe and ρ be an equivalence relation on U. An equivalence class of $x(\in U)$ is denoted by $[x]_\rho$ and defined as follows: $[x]_\rho = \{y \in U : x\rho y\}$, where $x\rho y$ imply $(x, y) \in \rho$.*
The lower and upper approximations of $X \subseteq U$ are denoted by $A_*(X)$ and $A^*(X)$ respectively and defined as follows:
$A_*(X) = \{x \in U : [x]_\rho \subseteq X\}$ *and* $A^*(X) = \{x \in U : [x]_\rho \cap X \neq \phi\}$.
The pair (U, ρ) is called an approximation space and is denoted by S. Then $A(X) = (A_*(X), A^*(X))$ is called the rough set of X in S.

Definition 2.3. *Let $A(X) = (A_*(X), A^*(X))$ and $A(Y) = (A_*(Y), A^*(Y))$ be two rough sets under the approximation space $S = (U, \rho)$. Then the rough union is defined by $A(X) \cup A(Y) = (A_*(X) \cup A_*(Y), A^*(X) \cup A^*(Y))$ and the rough intersection is defined by $A(X) \cap A(Y) = (A_*(X) \cap A_*(Y), A^*(X) \cap A^*(Y))$.*

Definition 2.4. *A rough set $A(Y)$ is said to be rough subset of a rough set $A(X)$ if $A_*(Y) \subseteq A_*(X)$ and $A^*(Y) \subseteq A^*(X)$ and it is denoted by $A(Y) \subseteq A(X)$.*

Definition 2.5. *Let $S = (F, A)$ be a soft set over U. Then the pair $P = (U, S)$ is called a soft approximation space. Let $X \subseteq U$. We defined the following operations on P*

$$\underline{apr}(X) = \bigcup_{a \in A} \{F(a) : F(a) \subseteq X\},$$
$$\overline{apr}(X) = \bigcup_{a \in A} \{F(a) : F(a) \cap X \neq \phi\},$$

which are called soft lower and upper approximations respectively of X and the pair $(\underline{apr}(X), \overline{apr}(X))$ is called soft rough set of X with respect to P and is denoted by $S_r(X)$. The set of all soft rough sets over U is denoted by $S_R(U)$ with respect to some soft approximation space P.

Definition 2.6. *Let $S_r(X) = (\underline{apr}(X), \overline{apr}(X))$ and $S_r(Y) = (\underline{apr}(Y), \overline{apr}(Y))$ be two soft rough set. Then soft rough union and soft rough intersection of $S_r(X)$ and $S_r(Y)$ are defined by*

$$S_r(X) \sqcup S_r(Y) = (\underline{apr}(X) \cup \underline{apr}(Y), \overline{apr}(X) \cup \overline{apr}(Y)) \text{ and}$$
$$S_r(X) \sqcap S_r(Y) = (\underline{apr}(X) \cap \underline{apr}(Y), \overline{apr}(X) \cap \overline{apr}(Y)) \text{ respectively,}$$

where the symbols $\sqcup$ and $\sqcap$ stand for soft rough union and intersection respectively.

Definition 2.7. *Let $S_r(X) = (\underline{apr}(X), \overline{apr}(X))$ and $S_r(Y) = (\underline{apr}(Y), \overline{apr}(Y))$ be two soft rough set. Then $S_r(Y)$ is said to be soft rough subset of $S_r(X)$, denoted by $S_r(Y) \sqsubseteq S_r(X)$ if $\underline{apr}(Y) \subseteq \underline{apr}(X)$ and $\overline{apr}(Y) \subseteq \overline{apr}(X)$, where $\sqsubseteq$ stands for soft rough inclusion relation.*

Let $S = (F, A)$ be a soft set over U and $P = (U, S)$ be a soft approximation space and $S_R(U)$ be the set of all soft rough sets with respect to P.

Definition 2.8. *Let $\mathcal{L} \subseteq S_R(U)$, $\vee$ and $\wedge$ be two binary operations on $\mathcal{L}$. The algebraic structure $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be soft rough lattice if*

(i). $\vee$ and $\wedge$ are associative,

(ii). $\vee$ and $\wedge$ are commutative,

(iii). $\vee$ and $\wedge$ satisfied absorption laws.

Definition 2.9. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice. Then $(\mathcal{K}, \vee, \wedge, \preceq)$ is said to be a soft rough sublattice of $(\mathcal{L}, \vee, \wedge, \preceq)$ if and only if $\mathcal{K} \subseteq \mathcal{L}$ and it is closed under both operations $\vee$ and $\wedge$, $S_r(X) \vee S_r(Y) \in \mathcal{K}$ and $S_r(X) \wedge S_r(Y) \in \mathcal{K}$ for all $S_r(X), S_r(Y) \in \mathcal{K}$.*

Definition 2.10. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be distributive soft rough lattice if for every $S_r(X), S_r(Y), S_r(Z) \in \mathcal{L}$, then*

$$S_r(X) \wedge (S_r(Y) \vee S_r(Z)) = (S_r(X) \wedge S_r(Y)) \vee (S_r(X) \wedge S_r(Z)).$$

Definition 2.11. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be modular soft rough lattice if for every $S_r(X), S_r(Y), S_r(Z) \in \mathcal{L}$, with $S_r(X) \succeq S_r(Y)$ the following equality holds*

$$S_r(X) \wedge (S_r(Y) \vee S_r(Z)) = S_r(Y) \vee (S_r(X) \wedge S_r(Z)).$$

Definition 2.12. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ and$(\mathcal{M}, \vee, \wedge, \preceq)$ be two soft rough lattices. Consider the cartesian product $\mathcal{L} \times \mathcal{M}$ $= \{(S_r(X), S_r(Y))/S_r(X) \in \mathcal{L}, S_r(Y) \in \mathcal{M}\}$. Define the binary operations $\vee$ and $\wedge$ on $\mathcal{L} \times \mathcal{M}$ as follows: For all $(S_r(X_1), S_r(Y_1))$ and $(S_r(X_2), S_r(Y_2)) \in \mathcal{L} \times \mathcal{M}$,*

$$\begin{aligned}(S_r(X_1), S_r(Y_1)) \vee (S_r(X_2), S_r(Y_2)) &= (S_r(X_1) \vee_1 S_r(X_2), S_r(Y_1) \vee_2 S_r(Y_2)) \\ &= (\overline{apr}(X_1) \vee_1 \overline{apr}(X_2), \overline{apr}(Y_1) \vee_2 \overline{apr}(Y_2)) \quad \textit{and} \\ (S_r(X_1), S_r(Y_1)) \wedge (S_r(X_2), S_r(Y_2)) &= (S_r(X_1) \wedge_1 S_r(X_2), S_r(Y_1) \wedge_2 S_r(Y_2)) \\ &= (\overline{apr}(X_1) \wedge_1 \overline{apr}(X_2), \overline{apr}(Y_1) \wedge_2 \overline{apr}(Y_2)).\end{aligned}$$

3. Complement of Soft Rough Set

In this section, we introduce the definition of complement on soft rough sets and illustrate it with an example. Let U be the initial universe of objects and E be the set of parameters and $A \subseteq E$. Let $S = (F, A)$ be a soft set over U and $P = (U, S)$ be a soft approximation space and $S_R(U)$ be the set of all soft rough sets with respect to P.

Definition 3.1. *If $S_r(X) = (\underline{apr}(X), \overline{apr}(X)) = (\phi, \phi) = S_r(\phi)$ for all $x \in E$, then $S_r(\phi)$ is called a null soft rough set.*

Definition 3.2. *If $S_r(X) = (\underline{apr}(X), \overline{apr}(X)) = (U, U) = S_r(U)$ for all $x \in A$, then $S_r(U)$ is called a A-universal soft rough set.*

Definition 3.3. *If $A = E$ and $S_r(X) = (\underline{apr}(X), \overline{apr}(X)) = (U, U) = S_r(U)$ for all $x \in E$, then $S_r(U)$ is called a universal soft rough set.*

Definition 3.4. *The soft rough complement of the soft rough set $S_r(X)$ over the soft approximation space $P = (U, S)$ is defined by $S_r^c(X) = (U \setminus \overline{apr}(X), U \setminus \underline{apr}(X)) = (\overline{apr}^c(X), \underline{apr}^c(X))$ and it is denoted by $S_r^c(X)$.*

Example 3.5. *Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ and $A = \{e_1, e_2, e_3, e_4, e_5\}$. Let $S = (F, A)$ be a soft set over U given by $F(e_1) = \{u_1, u_3, u_6\}$, $F(e_2) = \{u_5, u_6\}$, $F(e_3) = \{u_2, u_4\}$, $F(e_4) = \{u_6\}$, $F(e_5) = \{u_2, u_4, u_6\}$. Let $X_1 = \phi$, $X_2 = \{u_2, u_4\}$, $X_3 = \{u_6\}$, $X_4 = \{u_1, u_2, u_3, u_4, u_5, u_6\}$. The soft rough sets on the soft approximation space $P = (U, S)$ are given by $S_r(X_1) = (\phi, \phi)$, $S_r(X_2) = (u_2u_4, u_2u_4u_6)$, $S_r(X_3) = (u_6, u_1u_2u_3u_4u_5u_6)$, $S_r(X_4) = (u_1u_2u_3u_4u_5u_6, u_1u_2u_3u_4u_5u_6)$. The complements of the soft rough sets on the soft approximation space $P = (U, S)$ of $S_r(X_1), S_r(X_2), S_r(X_3), S_r(X_4)$ are given by $S_r^c(X_1) = (u_1u_2u_3u_4u_5u_6, u_1u_2u_3u_4u_5u_6)$, $S_r^c(X_2) = (u_1u_3u_5, u_1u_3u_5u_6)$, $S_r^c(X_3) = (\phi, u_1u_2u_3u_4u_5)$, $S_r^c(X_4) = (\phi, \phi)$ respectively.*

4. Complements and Relative Complements of Distributive Soft Rough Lattice

In this section, we define complements and relative complements on soft rough lattice and discuss some theorems related to the complements of distributive soft rough lattice with an example. Let $\mathcal{L} \subseteq S_R(U)$, the set of all soft rough sets over U with respect to the soft approximation space $P = (U, S)$.

Definition 4.1. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice and $S_r(I), S_r(J) \in (\mathcal{L}, \vee, \wedge, \preceq)$. Suppose $S_r(I) \preceq S_r(J)$, $[S_r(I), S_r(J)] = \{S_r(X) \in \mathcal{L} / S_r(I) \preceq S_r(X) \preceq S_r(J)\}$. Then $[S_r(I), S_r(J)]$ is said to be a soft rough complemented if for every $S_r(X) \in [S_r(I), S_r(J)]$ there exists $S_r(Y) \in [S_r(I), S_r(J)]$ such that $S_r(X) \vee S_r(Y) = S_r(J)$ and $S_r(X) \wedge S_r(Y) = S_r(I)$. Here $S_r(Y)$ is the soft rough relative complement of $S_r(X)$.*

Definition 4.2. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be relatively complemented if every interval in a soft rough lattice is complemented.*

Definition 4.3. *Let $\mathcal{L} \subseteq S_R(U)$. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice if there exists a soft rough set $S_r(X) \in \mathcal{L}$ such that $S_r(X) \preceq S_r(Y)$ for all $S_r(Y) \in \mathcal{L}$, then $S_r(X)$ is called the least element of $(\mathcal{L}, \vee, \wedge, \preceq)$.*

Definition 4.4. *Let $\mathcal{L} \subseteq S_R(U)$. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice if there exists a soft rough set $S_r(X) \in \mathcal{L}$ such that $S_r(X) \succeq S_r(Y)$ for all $S_r(Y) \in \mathcal{L}$, then $S_r(X)$ is called the greatest element of $(\mathcal{L}, \vee, \wedge, \preceq)$.*

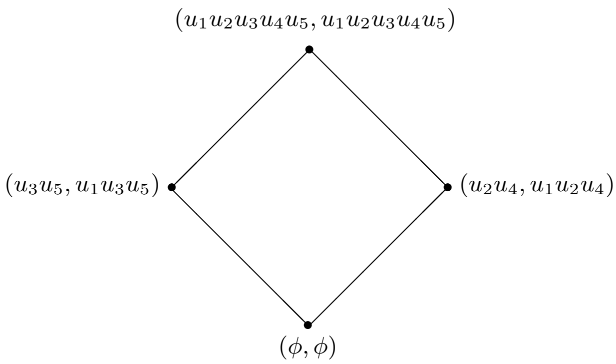
Definition 4.5. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be a bounded soft rough lattice if it has both the greatest element $S_r(U)$ and the least element $S_r(\phi)$.*

Definition 4.6. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a bounded soft rough lattice, for any $S_r(X) \in \mathcal{L}$, there exists $S_r^c(X) \in \mathcal{L}$ such that $S_r(X) \vee S_r^c(X) = S_r(U)$ and $S_r(X) \wedge S_r^c(X) = S_r(\phi)$. Then $S_r^c(X)$ is called the soft rough complement of $S_r(X)$.*

Remark 4.7. *The complement of the crisp set is not valid for the complement of the soft rough set in a soft rough lattice. i.e., $S_r(X) \vee S_r^c(X) = (\underline{apr}(\widehat{X}), \overline{apr}(U)) \sqsubseteq S_r(U)$ and $S_r(X) \wedge S_r^c(X) = (\underline{apr}(\phi), \overline{apr}(\widehat{X})) \sqsupseteq S_r(\phi)$ where $(\underline{apr}(\widehat{X}))^c = \overline{apr}(\widehat{X}))$.*

Note 4.8. *A bounded soft rough lattice in $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be a complemented soft rough lattice if every soft rough set of $(\mathcal{L}, \vee, \wedge, \preceq)$ has atleast one soft rough complement.*

Example 4.9. *Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ and $A = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$. Let $S = (F, A)$ be a soft set over U given by $F(e_1) = \{u_2, u_4\}$, $F(e_2) = \{u_1, u_4\}$, $F(e_3) = \{u_2\}$, $F(e_4) = \{u_1, u_3\}$, $F(e_5) = \{u_3\}$, $F(e_6) = \{u_1, u_5\}$, $F(e_7) = \{u_5\}$. Let $X_1 = \phi$, $X_2 = \{u_3, u_5\}$, $X_3 = \{u_2, u_4\}$, $X_4 = \{u_1, u_2, u_3, u_4, u_5\}$. The soft rough sets on the soft approximation space $P = (U, S)$ are given by $S_r(X_1) = (\phi, \phi)$, $S_r(X_2) = (u_3u_5, u_1u_3u_5)$, $S_r(X_3) = (u_2u_4, u_1u_2u_4)$, $S_r(X_4) = (u_1u_2u_3u_4u_5, u_1u_2u_3u_4u_5)$. Then the set $\mathcal{L} = \{S_r(X_1), S_r(X_2), S_r(X_3), S_r(X_4)\}$ is a relatively complemented distributive soft rough lattice with the operations $\sqcup, \sqcap$ and $\sqsubseteq$. Hence $(\mathcal{L}, \sqcup, \sqcap, \sqsubseteq)$ is a complemented distributive soft rough lattice.*



Complemented distributive soft rough lattice

Theorem 4.10. *In a distributive soft rough lattice, soft rough complements are unique.*

Proof. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a distributive soft rough lattice. Let $S_r(X), S_r(Y), S_r(Z) \in (\mathcal{L}, \vee, \wedge, \preceq)$. Let $S_r(X)$ and $S_r(Y)$ be two soft rough complements of $S_r(Z)$ in $(\mathcal{L}, \vee, \wedge, \preceq)$. Then $S_r(X) \wedge S_r(Z) = S_r(\phi) = S_r(Y) \wedge S_r(Z)$ and $S_r(X) \vee S_r(Z) = S_r(U) = S_r(Y) \vee S_r(Z)$. Now to prove $S_r(X) = S_r(Y)$ for all $S_r(X), S_r(Y), S_r(Z) \in (\mathcal{L}, \vee, \wedge, \preceq)$.

$$\begin{aligned}
S_r(X) &= S_r(X) \vee (S_r(X) \wedge S_r(Z)) \\
&= S_r(X) \vee (S_r(Y) \wedge S_r(Z)) \\
&= (S_r(X) \vee S_r(Y)) \wedge (S_r(X) \vee S_r(Z)) \\
&= (S_r(X) \vee S_r(Y)) \wedge (S_r(Y) \vee S_r(Z)) \\
&= S_r(Y) \vee (S_r(X) \wedge S_r(Z)) \\
&= S_r(Y) \vee (S_r(Y) \wedge S_r(Z)) \\
&= S_r(Y) \\
\therefore\ S_r(X) &= S_r(Y)
\end{aligned}$$

□

Theorem 4.11. *A complemented distributive soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is a relatively complemented distributive soft rough lattice.*

Proof. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a complemented distributive soft rough lattice. Let $S_r(X), S_r(Y) \in (\mathcal{L}, \vee, \wedge, \preceq)$. Let $S_r(Y)$ be a soft rough complements of $S_r(X)$ in $(\mathcal{L}, \vee, \wedge, \preceq)$. Then $S_r(X) \wedge S_r(Y) = S_r(\phi)$ and $S_r(X) \vee S_r(Y) = S_r(U)$. Let $[S_r(I), S_r(J)]$ be any interval in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $S_r(X) \in [S_r(I), S_r(J)], S_r(I) \preceq S_r(X) \preceq S_r(J)$. Take $S_r(Z) = S_r(I) \vee (S_r(J) \wedge S_r(Y))$ in $(\mathcal{L}, \vee, \wedge, \preceq)$ where $S_r(Z) \in [S_r(I), S_r(J)]$. Now to prove $S_r(Z)$ is a soft rough relative complement of $S_r(X)$ in $[S_r(I), S_r(J)]$.

$$\begin{aligned}
S_r(X) \wedge S_r(Z) &= S_r(X) \wedge (S_r(I) \vee (S_r(J) \wedge S_r(Y))) \\
&= (S_r(X) \wedge S_r(I)) \vee (S_r(X) \wedge (S_r(J) \wedge S_r(Y))) \\
&= S_r(I) \vee (S_r(X) \wedge (S_r(J) \wedge S_r(Y))) \\
&= S_r(I) \vee (S_r(J) \wedge (S_r(X) \wedge S_r(Y))) \\
&= S_r(I) \vee (S_r(J) \wedge S_r(\phi)) \\
&= S_r(I) \vee S_r(\phi) \\
\therefore\ S_r(X) \wedge S_r(Z) &= S_r(I)
\end{aligned}$$

$$\begin{aligned}S_r(X) \vee S_r(Z) &= S_r(X) \vee (S_r(I) \vee (S_r(J) \wedge S_r(Y)))\\ &= (S_r(X) \vee S_r(I)) \vee (S_r(J) \wedge S_r(Y))\\ &= S_r(X) \vee (S_r(J) \wedge S_r(Y))\\ &= (S_r(X) \vee S_r(J)) \wedge (S_r(X) \vee S_r(Y))\\ &= (S_r(X) \vee S_r(J)) \wedge S_r(U)\\ &= S_r(X) \vee S_r(J)\\ \therefore\ S_r(X) \vee S_r(Z) &= S_r(J)\end{aligned}$$

$\therefore$ $S_r(Z)$ is a soft rough relative complement of $S_r(X)$ in $[S_r(I), S_r(J)]$. Hence, a complemented distributive soft rough lattice is relatively complemented distributive soft rough lattice. □

Note 4.12. *The converse of the above theorem is obviously true only when the distributive soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is bounded.*

Theorem 4.13. *In a distributive soft rough lattice, soft rough relative complements are unique.*

Proof. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a distributive soft rough lattice. Let $S_r(X), S_r(Y), S_r(Z) \in (\mathcal{L}, \vee, \wedge, \preceq)$. Let $S_r(X)$ and $S_r(Y)$ be two soft rough relative complements of $S_r(Z)$ in $[S_r(I), S_r(J)]$. Then $S_r(X) \wedge S_r(Z) = S_r(I) = S_r(Y) \wedge S_r(Z)$ and $S_r(X) \vee S_r(Z) = S_r(J) = S_r(Y) \vee S_r(Z)$. Now to prove $S_r(X) = S_r(Y)$ for all $S_r(X), S_r(Y), S_r(Z) \in \mathcal{L}$.

$$\begin{aligned}S_r(X) &= S_r(X) \vee (S_r(X) \wedge S_r(Z))\\ &= S_r(X) \vee (S_r(Y) \wedge S_r(Z))\\ &= (S_r(X) \vee S_r(Y)) \wedge (S_r(X) \vee S_r(Z))\\ &= (S_r(X) \vee S_r(Y)) \wedge (S_r(Y) \vee S_r(Z))\\ &= S_r(Y) \vee (S_r(X) \wedge S_r(Z))\\ &= S_r(Y) \vee (S_r(Y) \wedge S_r(Z))\\ \therefore\ S_r(X) &= S_r(Y)\end{aligned}$$

□

Theorem 4.14. *If $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are two distributive soft rough relatively complemented lattice then $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is a relatively complemented distributive soft rough lattice.*

Proof. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ be relatively complemented distributive soft rough lattices. Also $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are bounded distributive soft rough lattices. Let $[S_r(X_1), S_r(X_2)]$ and $[S_r(Y_1), S_r(Y_2)]$ be intervals in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ respectively. Let $S_r(X) \in [S_r(X_1), S_r(X_2)]$ and $S_r(Y) \in [S_r(Y_1), S_r(Y_2)]$. Then

$$S_r(X_1) \preceq S_r(X) \preceq S_r(X_2) \text{ and } S_r(Y_1) \preceq S_r(Y) \preceq S_r(Y_2) \tag{1}$$

Since $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are relatively complemented distributive soft rough lattices. By note [4.12], we have $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are complemented distributive soft rough lattices. Let $S_r(X)$ and $S_r(Y)$ has complements $S_r^c(X)$ and $S_r^c(Y)$ in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ respectively. Then $S_r(X) \wedge S_r^c(X) = S_r(\phi), S_r(X) \vee S_r^c(X) =$

$S_r(U)$. Also $S_r(Y) \wedge S_r^c(Y) = S_r(\phi), S_r(Y) \vee S_r^c(Y) = S_r(U)$. Let $(S_r(X), S_r(Y)), (S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2)) \in (\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$. Let $[(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))]$ be an interval in $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ and $(S_r(X), S_r(Y)) \in [(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))]$ be any element. From (1), we get $(S_r(X_1), S_r(Y_1)) \preceq (S_r(X), S_r(Y)) \preceq (S_r(X_2), S_r(Y_2))$. Now

$$\begin{aligned}(S_r(X), S_r(Y)) \wedge (S_r^c(X), S_r^c(Y)) &= (S_r(X) \wedge S_r^c(X), S_r(Y) \wedge S_r^c(Y))\\ &= (S_r(\phi), S_r(\phi))\\ (S_r(X), S_r(Y)) \vee (S_r^c(X), S_r^c(Y)) &= (S_r(X) \vee S_r^c(X), S_r(Y) \vee S_r^c(Y))\\ &= (S_r(U), S_r(U))\end{aligned}$$

$\Rightarrow (\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is complemented distributive soft rough lattice. Let $(S_r(X'), S_r(Y'))$ be an element in $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ where $(S_r(X'), S_r(Y')) \in [(S_r(X_1), S_r(Y_1), (S_r(X_2), S_r(Y_2)]$. Take $(S_r(X'), S_r(Y')) = (S_r(X_1), S_r(Y_1) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y)))$

$$\begin{aligned}(S_r(X), S_r(Y)) \wedge (S_r(X'), S_r(Y')) &= (S_r(X), S_r(Y)) \wedge ((S_r(X_1), S_r(Y_1)) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y))))\\ &= ((S_r(X), S_r(Y)) \wedge (S_r(X_1), S_r(Y_1))) \vee ((S_r(X), S_r(Y)) \wedge ((S_r(X_2), S_r(Y_2))\\ &\quad \wedge (S_r^c(X), S_r^c(Y))))\\ &= (S_r(X_1), S_r(Y_1)) \vee ((S_r(X), S_r(Y)) \wedge ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y))))\\ &= (S_r(X_1), S_r(Y_1)) \vee (((S_r(X), S_r(Y)) \wedge (S_r(X_2), S_r(Y_2))) \wedge (S_r^c(X), S_r^c(Y)))\\ &= (S_r(X_1), S_r(Y_1)) \vee ((S_r(X), S_r(Y)) \wedge (S_r^c(X), S_r^c(Y)))\\ &= (S_r(X_1), S_r(Y_1)) \vee (S_r(\phi), S_r(\phi))\\ \therefore\ (S_r(X), S_r(Y)) \wedge (S_r(X'), S_r(Y')) &= (S_r(X_1), S_r(Y_1))\\ (S_r(X), S_r(Y)) \vee (S_r(X'), S_r(Y')) &= (S_r(X), S_r(Y)) \vee ((S_r(X_1), S_r(Y_1)) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y))))\\ &= ((S_r(X), S_r(Y)) \vee (S_r(X_1), S_r(Y_1))) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y)))\\ &= (S_r(X), S_r(Y)) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y)))\\ &= ((S_r(X), S_r(Y)) \vee (S_r(X_2), S_r(Y_2))) \wedge ((S_r(X), S_r(Y)) \vee (S_r^c(X), S_r^c(Y)))\\ &= (S_r(X_2), S_r(Y_2)) \wedge ((S_r(X), S_r(Y)) \vee (S_r^c(X), S_r^c(Y)))\\ &= (S_r(X_2), S_r(Y_2)) \wedge (S_r(U), S_r(U))\\ \therefore\ (S_r(X), S_r(Y)) \vee (S_r(X'), S_r(Y')) &= (S_r(X_2), S_r(Y_2))\end{aligned}$$

Here $(S_r(X'), S_r(Y'))$ is a soft rough relative complement of $(S_r(X), S_r(Y))$ in $[(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))]$. Hence the $[(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))]$ is soft rough complemented. Similarly, every interval in $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is soft rough complemented. Thus, $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is relatively complemented distributive soft rough lattice. □

Theorem 4.15. *If $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is a relatively complemented distributive soft rough lattice then $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are relatively complemented distributive soft rough lattices.*

Proof. Let $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ be a relatively complemented distributive soft rough lattice. Also $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is bounded distributive soft rough lattice. By note [4.12], we have $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is complemented distributive soft rough lattice.

For an element $(S_r(X), S_r(Y)) \in [(S_r(\phi), S_r(\phi)), (S_r(U), S_r(U))]$ there exists a complement $(S_r^c(X), S_r^c(Y))$ relative to $[(S_r(\phi), S_r(\phi)), (S_r(U), S_r(U))]$. So,

$$(S_r(X), S_r(Y)) \wedge (S_r^c(X), S_r^c(Y)) = (S_r(\phi), S_r(\phi)) \text{ and } (S_r(X), S_r(Y)) \vee (S_r^c(X), S_r^c(Y)) = (S_r(U), S_r(U)) \tag{2}$$

Let $[S_r(X_1), S_r(X_2)]$ and $[S_r(Y_1), S_r(Y_2)]$ be an intervals in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ respectively. Let $S_r(X) \in [S_r(X_1), S_r(X_2)]$ and $S_r(Y) \in [S_r(Y_1), S_r(Y_2)]$ be any elements in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ respectively. Then $S_r(X_1) \preceq S_r(X) \preceq S_r(X_2)$ and $S_r(Y_1) \preceq S_r(Y) \preceq S_r(Y_2)$. From (2), we have $(S_r(X) \wedge S_r^c(X), S_r(Y) \wedge S_r^c(Y)) = (S_r(\phi), S_r(\phi))$ and $(S_r(X) \vee S_r^c(X), S_r(Y) \vee S_r^c(Y)) = (S_r(U), S_r(U)) \Rightarrow S_r(X) \wedge S_r^c(X) = S_r(\phi), S_r(Y) \wedge S_r^c(Y) = S_r(\phi)$ and $S_r(X) \vee S_r^c(X) = S_r(U), S_r(Y) \vee S_r^c(Y) = S_r(U)$. Let $S_r(X') \in (\mathcal{L}, \vee, \wedge, \preceq)$, where $S_r(X) \in [S_r(X_1), S_r(X_2)]$. Take $S_r(X') = S_r(X_1) \vee (S_r(X_2) \wedge S_r^c(X))$. Now

$$\begin{aligned}
S_r(X) \wedge S_r(X') &= S_r(X) \wedge (S_r(X_1) \vee (S_r(X_2) \wedge S_r^c(X))) \\
&= (S_r(X) \wedge S_r(X_1)) \vee (S_r(X) \wedge (S_r(X_2) \wedge S_r^c(X))) \\
&= S_r(X_1) \vee (S_r(X) \wedge (S_r(X_2) \wedge S_r^c(X))) \\
&= S_r(X_1) \vee ((S_r(X) \wedge S_r^c(X)) \wedge S_r(X_2))) \\
&= S_r(X_1) \vee (S_r(\phi) \wedge S_r(X_2)) \\
&= S_r(X_1) \vee S_r(\phi) \\
\therefore\ S_r(X) \wedge S_r(X') &= S_r(X_1) \\
S_r(X) \vee S_r(X') &= S_r(X) \vee (S_r(X_1) \vee (S_r(X_2) \wedge S_r^c(X))) \\
&= (S_r(X) \vee S_r(X_1)) \vee (S_r(X_2) \wedge S_r^c(X)) \\
&= S_r(X) \vee (S_r(X_2) \wedge S_r^c(X)) \\
&= (S_r(X) \vee S_r(X_2)) \wedge (S_r(X) \vee S_r^c(X)) \\
&= S_r(X_2) \wedge (S_r(X) \vee S_r^c(X)) \\
&= S_r(X_2) \wedge S_r(U) \\
\therefore\ S_r(X) \vee S_r(X') &= S_r(X_2)
\end{aligned}$$

Here $S_r(X')$ is a soft rough relative complement of $S_r(X)$ in $[S_r(X_1), S_r(X_2)]$. Hence $[S_r(X_1), S_r(X_2)]$ is soft rough complemented. Similarly, every interval of $(\mathcal{L}, \vee, \wedge, \preceq)$ is soft rough complemented. $\therefore$ $(\mathcal{L}, \vee, \wedge, \preceq)$ is relatively complemented distributive soft rough lattice. Let $S_r(Y') \in (\mathcal{M}, \vee, \wedge, \preceq)$, where $S_r(Y') \in [S_r(Y_1), S_r(Y_2)]$. Take $S_r(Y') = S_r(Y_1) \vee (S_r(Y_2) \wedge S_r^c(Y))$. Now

$$\begin{aligned}
S_r(Y) \wedge S_r(Y') &= S_r(Y) \wedge (S_r(Y_1) \vee (S_r(Y_2) \wedge S_r^c(Y))) \\
&= (S_r(Y) \wedge S_r(Y_1)) \vee (S_r(Y) \wedge (S_r(Y_2) \wedge S_r^c(Y))) \\
&= S_r(Y_1) \vee (S_r(Y) \wedge (S_r(Y_2) \wedge S_r^c(Y))) \\
&= S_r(Y_1) \vee ((S_r(Y) \wedge S_r^c(Y)) \wedge S_r(Y_2))) \\
&= S_r(Y_1) \vee (S_r(\phi) \wedge S_r(Y_2)) \\
&= S_r(Y_1) \vee S_r(\phi) \\
\therefore\ S_r(Y) \wedge S_r(Y') &= S_r(Y_1)
\end{aligned}$$

$$\begin{aligned}
S_r(Y) \vee S_r(Y') &= S_r(Y) \vee (S_r(Y_1) \vee (S_r(Y_2) \wedge S_r^c(Y))) \\
&= (S_r(Y) \vee S_r(Y_1)) \vee (S_r(Y_2) \wedge S_r^c(Y)) \\
&= S_r(Y) \vee (S_r(Y_2) \wedge S_r^c(Y)) \\
&= (S_r(Y) \vee S_r(Y_2)) \wedge (S_r(Y) \vee S_r^c(Y)) \\
&= S_r(Y_2) \wedge (S_r(Y) \vee S_r^c(Y)) \\
&= S_r(Y_2) \wedge S_r(U) \\
\therefore\ S_r(Y) \vee S_r(Y') &= S_r(Y_2)
\end{aligned}$$

Here $S_r(Y')$ is a soft rough relative complement of $S_r(Y)$ in $[S_r(Y_1), S_r(Y_2)]$. Hence the $[S_r(Y_1), S_r(Y_2)]$ is soft rough complemented. Similarly, every interval of $(\mathcal{M}, \vee, \wedge, \preceq)$ is soft rough complemented. $\therefore$ $(\mathcal{M}, \vee, \wedge, \preceq)$ is relatively complemented distributive soft rough lattice. □

5. Conclusion

Soft rough set is generalization of rough set based on soft set. The complements can be found for soft sets as well as rough sets. In this paper, we have first defined the complements of soft rough set and also the complements and relative complements of soft rough lattice. We also discussed some theorems on complemented and relatively complemented distributive soft rough lattice and illustrate them with an example. We are studying about these distributive soft rough lattices and are expected to give some more results in our future study.

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