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Complemented and Relatively Complemented Distributive Fuzzy Soft Lattice

Research Article

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Abstract: Soft set theory was introduced by Molodtsov [11] in 1999 as a mathematical tool for dealing with problems that contain uncertainty. Frauk Karasslan [5] defined the concept of fuzzy soft lattices. H.K.Baruah [6] extended the definition of fuzzy set and redefined the complement of a fuzzy set. Tridiv Jyoti Neog et.al [16, 17] reintroduced the notation of complement of a fuzzy soft set. In this paper, we introduce the definition of complements and relative complements of fuzzy soft lattices and discuss some theorems related to distributive fuzzy soft lattice with an example.

MSC: 06D72.

Keywords: soft set, fuzzy set, fuzzy soft set, complement of fuzzy soft set, complement of fuzzy soft lattice, relative complement of fuzzy soft lattice.

1. Introduction

Soft Set theory was firstly introduced by Molodtsov [11] in 1999 as a mathematical tool for dealing with uncertainty. The operations of soft sets are defined in [2]. The lattice structures of soft sets have been studied by some authors [9, 10, 12, 13, 15]. The fuzzy set theory was initiated by Zadeh in 1965. By embedding the ideas of fuzzy sets, many interesting applications of soft set theory have been expanded [1, 3, 4, 7]. Karaaslan [5] introduced the concept of fuzzy soft lattices and discussed its basic properties.Some theorems based on distributive fuzzy soft lattice and modular fuzzy soft lattice was discussed in [8]. E.K.R.Nagarajan et al. [14] introduced the concept of direct product of soft lattices. It has been accepted that for a fuzzy set A and its complement A^c neither $A \cap A^c$ is the null set, nor $A \cup A^c$ is the universal set. Baruah [6] defined two functions, namely fuzzy membership function and fuzzy reference function are necessary to represent a fuzzy set and reintroduced the notation of complement of a fuzzy set in a different way. Also,he put forward an extended definition of fuzzy sets,union and intersection of two fuzzy sets. An extended definition of fuzzy set which enables us to define complement of fuzzy set in a way that give us $A \cap A^c$= the null set and $A \cup A^c$= the universal set. Tridiv Jyoti Neog et.al [17] has given a new definition of complement of a fuzzy soft set on basis of Baruah in a way that gives $(F, A) \cap (F, A)^c = \tilde{\phi}$, the null fuzzy soft set and $(F, A) \cup (F, A)^c = \tilde{A}$, the absolute fuzzy soft set. In this paper, we define the complements and relative complements of fuzzy soft lattice and discuss some theorems related to the complements and relative complements of distributive fuzzy soft lattice with an example.

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2. Preliminaries

In this section, we recall the revised extended definition of fuzzy sets, union and intersection of two fuzzy sets, the extended definition of complement of a fuzzy set and fuzzy subset by H.K.Baruah [6]. Null fuzzy soft set and absolute fuzzy soft set given by Tridiv Jyoti Neog [16, 17]. We also recall definition and operations on fuzzy soft sets by Karaaslan [5].

Definition 2.1. *Let* $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ *and* $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ *be two fuzzy sets defined over the same universe U. To avoid degenerate cases we assume that* $min(\mu_1(x),\mu_3(x))\geqslant max(\mu_2(x),\mu_4(x))\forall\, x\in U$*. Then the operations intersection and union are defined as*

$$A(\mu_1,\mu_2)\cap B(\mu_3,\mu_4)=\{x,min(\mu_1(x),\mu_3(x)),max(\mu_2(x),\mu_4(x));x\in U\}\quad \text{and}$$

$$A(\mu_1,\mu_2)\cup B(\mu_3,\mu_4)=\{x,max(\mu_1(x),\mu_3(x)),min(\mu_2(x),\mu_4(x));x\in U\}$$

Definition 2.2. *For usual fuzzy sets* $A(\mu,0)=\{x,\mu(x),0;x\in U\}$ *and* $B(1,\mu)=\{x,1,\mu(x);x\in U\}$ *defined over the same universe U, we have* $A(\mu,0)\cap B(1,\mu)=\{x,min(\mu(x),1),max(0,\mu(x));x\in U\}=\{x,\mu(x),\mu(x);x\in U\}$*, which is nothing but null set* ϕ*.* $A(\mu,0)\cup B(1,\mu)=\{x,max(\mu(x),1),min(0,\mu(x));x\in U\}=\{x,1,0;x\in U\}$*, which is nothing but the universal set U. This means if we define a fuzzy set* $(A(\mu,0)^c)=\{x,1,0;x\in U\}$*, it is nothing but the complement of* $A(\mu,0)=\{x,\mu(x),0;x\in U\}$*.*

Definition 2.3. *Let* $A(\mu_1,\mu_2)=\{x,\mu_1(x),\mu_2(x);x\in U\}$ *and* $B(\mu_3,\mu_4)=\{x,\mu_3(x),\mu_4(x);x\in U\}$ *be two fuzzy sets defined over the same universe U.The fuzzy set* $A(\mu_1,\mu_2)$ *is a subset of the fuzzy set* $B(\mu_3,\mu_4)$ *if* $\forall\ x\in U$*,* $\mu_1(x)\leqslant\mu_3(x)$ *and* $\mu_4(x)\leqslant\mu_2(x)$*.*

Definition 2.4. *Let E be a crisp set. Then a fuzzy set* μ *over E is a function from E into [0,1]. (ie)* $\mu:E\longrightarrow[0,1]$*.*

Definition 2.5. *Let U be a universe, P(U) be the power set of U, E be a set of all parameters. Then, a soft set* f_A *over U is a function from E into P(U) such that* $f_A(x)=\phi,x\notin A$*. (ie)* $f_A:E\longrightarrow P(U)$ *such that* $f_A(x)=\phi,x\notin A$*, where* f_A *is called approximate function of the soft set* f_A *and the value* $f_A(x)$ *is a set called x-element of the soft set for all* $x\in E$*.*

Definition 2.6. *Let U be an initial universe, F(U) be the set of all fuzzy sets over U, E be a set of parameters and A*$\subset$ *E. Then, a fuzzy soft set (f,A) over U as a function from E into F(U). (ie)* $f_A:E\longrightarrow F(U)$*.*

Definition 2.7. *Let* f_A *and* f_B *be fuzzy soft sets.Then,*f_A *is a fuzzy soft subset of* f_B*,denoted by* $f_A\subseteq f_B$*, if* $\mu_A\subseteq\mu_B$ *and* $f_A(x)\subseteq f_B(x)$ *for all* $x\in E$*.*

Definition 2.8. *Let* f_A*and* f_B *be two fuzzy soft sets. Then, union of* f_A *and* f_B*, denoted by* $f_A\cup\ f_B$ *if* $\mu_A\cup\mu_B=max\{\mu_A(x),\mu_B(x)\}$ *and* $f_{A\cup B}(x)=f_A(x)\cup f_B(x)$ *for all* $x\in E$*.*

Definition 2.9. *Let* f_A*and* f_B *be two fuzzy soft sets.Then,intersection of* f_A *and* f_B*, denoted by* $f_A\cup f_B if \mu_A\cap\mu_B=min\{\mu_A(x),\mu_B(x)\}$ *and* $f_{A\cap B}(x)=f_A(x)\cap f_B(x)$ *for all* $x\in E$*.*

Definition 2.10. *Let U be a universe and E be a parameters.Then the Pair* (U,E) *denotes the collection of all fuzzy soft sets on U with parameters from E and is called a fuzzy soft class.*

Definition 2.11. *A Soft set* f_A *over U is said to be null fuzzy soft set denoted by* $\tilde{\phi}$ *if* $\forall\,\varepsilon\in A$*,* $f_A(\varepsilon)$ *is the null fuzzy set* ϕ*. (ie)* $\forall\,\varepsilon\in A,f_A(\varepsilon)=\{x,\mu_{f_A(\varepsilon)},\mu_{f_A(\varepsilon)};x\in U\}$*.*

Definition 2.12. *A Soft set* f_A *over U is said to be absolute fuzzy soft set denoted by* $\tilde{A}$ *if* $\forall\,\varepsilon\in A$*,* $f_A(\varepsilon)$ *is the absolute fuzzy set* $\tilde{A}$*. (ie)* $\forall\,\varepsilon\in A,f_A(\varepsilon)=\{x,1,0;x\in U\}$*.*

Definition 2.13. *The Complement of a fuzzy soft set f_A is denoted by ${f_A}^c$ and is defined by ${f_A}^c = f_A^c$ where ${f_A}^c$: $A \longrightarrow \tilde{P}(U)$ is a mapping given by ${f_A}^c(x) = (f_A(x))^c\ \forall\ x \in A$. (ie) $\forall\ \varepsilon \in A$ if $f_A(\varepsilon) = \{x, \mu_{f_A(\varepsilon)}, 0; x \in U\}$,then ${f_A}^c(\varepsilon) = \{x, 1, \mu_{f_A(\varepsilon)}; x \in U\}$.*

Definition 2.14. *Let f_L be a fuzzy soft set over U, and $\curlyvee$ and $\curlywedge$ be two binary operation on f_L. If, elements of f_L are equipped with two commutative and associative binary operations $\curlyvee$ and $\curlywedge$ which are connected by the absorbtion law, then algebraic structure $(f_L, \curlyvee, \curlywedge)$ is called a fuzzy soft lattice.*

Definition 2.15. *Let $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft lattice. Then, f_L is called distributive fuzzy soft lattice. If it satisfies the following axiom:*

$$f_L(x) \curlywedge (f_L(y) \curlyvee f_L(z)) = (f_L(x) \curlywedge f_L(y)) \curlyvee (f_L(x) \curlywedge f_L(z)).$$
$$f_L(x) \curlyvee (f_L(y) \curlywedge f_L(z)) = (f_L(x) \curlyvee f_L(y)) \curlywedge (f_L(x) \curlyvee f_L(z)).$$

Definition 2.16. *If $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft lattice. Then, f_L is called modular fuzzy soft lattice. If it satisfies the following axiom:*

$$f_L(z) \preccurlyeq f_L(x) \Longrightarrow f_L(x) \curlywedge (f_L(y) \curlyvee f_L(z)) = (f_L(x) \curlywedge f_L(y)) \curlyvee f_L(z).$$

3. Complements and Relative Complements of Distributive Fuzzy Soft Lattice

In this section, we introduce the definition of complements and relative complements of fuzzy soft lattice and discuss some theorems related to complements and relative complements of distributive fuzzy soft lattice with an example.

Definition 3.1. *Let $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft lattice. If there exist an element $f_L(x) \in f_L$ such that $f_L(x) \preccurlyeq f_L(y)$ for all $f_L(y) \in f_L$, then $f_L(x)$ is called the least element in f_L.*

Definition 3.2. *Let $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft lattice. If there exist an element $f_L(x) \in f_L$ such that $f_L(y) \preccurlyeq f_L(x)$ for all $f_L(y) \in f_L$, then $f_L(x)$ is called the greatest element in f_L.*

Definition 3.3. *A fuzzy soft lattice $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ which has both a null fuzzy soft set $\tilde{\phi}$ and an absolute fuzzy soft set $\tilde{A}$ is called a bounded fuzzy soft lattice.*

Definition 3.4. *In a bounded fuzzy soft lattice$(f_L, \curlyvee, \curlywedge, \preccurlyeq)$, any element $f_L(y) \in f_L$ is said to be fuzzy soft complement of an element $f_L(x) \in f_L$ if $f_L(x) \curlyvee f_L(y) = \tilde{A}$ and $f_L(x) \curlywedge f_L(y) = \tilde{\phi}$.*

Definition 3.5. *A bounded fuzzy soft lattice $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ is said to be a complemented fuzzy soft lattice if every element in f_L has atleast one fuzzy soft complement.*

Definition 3.6. *Let $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft lattice and $f_L(x), f_L(y) \in f_L$ with $f_L(x) \preccurlyeq f_L(y)$. Then the interval is defined as $[f_L(x), f_L(y)] = \{f_L(z) \in f_L; f_L(x) \preccurlyeq f_L(z) \preccurlyeq f_L(y)\}$*

Definition 3.7. *Let $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft lattice and $f_L(x), f_L(y) \in f_L$. Suppose $f_L(x) \preccurlyeq f_L(y)$, $[f_L(x), f_L(y)] = \{f_L(z) \in f_L; f_L(x) \preccurlyeq f_L(z) \preccurlyeq f_L(y)\}$.Then $[f_L(x), f_L(y)]$ is said to be fuzzy soft complemented if for every $f_L(z) \in [f_L(x), f_L(y)]$,there exist $f_L(t) \in [f_L(x), f_L(y)]$ such that $f_L(z) \curlyvee f_L(t) = f_L(y)$ and $f_L(z) \curlywedge f_L(t) = f_L(x)$. Here $f_L(t)$ is fuzzy soft complement relative to $f_L(z)$.*

Definition 3.8. *A fuzzy soft lattice $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ is said to be relatively complemented if every interval of the form $[f_L(x), f_L(y)]$ is fuzzy soft complemented .*

Theorem 3.9. *A distributive fuzzy soft lattice $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ has a unique fuzzy soft complement.*

Proof. Let $f_L(x)$ and $f_L(y)$ be two fuzzy soft complements of $f_L(z)$ in a distributive fuzzy soft lattice $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$, where $f_L(x), f_L(y), f_L(z) \in (f_L, \curlyvee, \curlywedge, \preccurlyeq)$. Then,

$$f_L(x) \curlywedge f_L(z) = \tilde{\phi} = f_L(y) \curlywedge f_L(z)$$
$$f_L(x) \curlyvee f_L(z) = \tilde{A} = f_L(y) \curlyvee f_L(z)$$

Take

$$\begin{aligned}
f_L(x) &= f_L(x) \curlywedge (f_L(x) \curlyvee f_L(z)) \\
&= f_L(x) \curlywedge (f_L(y) \curlyvee f_L(z)) \\
&= (f_L(x) \curlywedge f_L(y)) \curlyvee (f_L(x) \curlywedge f_L(z)) \\
&= (f_L(x) \curlywedge f_L(y)) \curlyvee (f_L(y) \curlywedge f_L(z)) \\
&= f_L(y) \curlywedge (f_L(x) \curlyvee f_L(z)) \\
&= f_L(y) \curlywedge (f_L(y) \curlyvee f_L(z)) \\
\therefore\ f_L(x) &= f_L(y)
\end{aligned}$$

Hence the fuzzy soft complements are unique in a distributive fuzzy soft lattice. □

Theorem 3.10. *A complemented distributive fuzzy soft lattice $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ is a relatively complemented distributive fuzzy soft lattice.*

Proof. Let $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be a complemented distributive fuzzy soft lattice. Let $f_L(y)$ be a complement of $f_L(x)$ in $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$. Then $f_L(x) \curlyvee f_L(y) = \tilde{A}$ and $f_L(x) \curlywedge f_L(y) = \tilde{\phi}$. Suppose $f_L(x)$ lies in any interval $[f_L(a), f_L(b)]$. Then $f_L(a) \preccurlyeq f_L(x) \preccurlyeq f_L(b)$. Take $f_L(z) = (f_L(a) \curlyvee f_L(y)) \curlywedge f_L(b)$ where $f_L(z) \in [f_L(a), f_L(b)]$. Now,

$$\begin{aligned}
f_L(x) \curlywedge f_L(z) &= f_L(x) \curlywedge ((f_L(a) \curlyvee f_L(y)) \curlywedge f_L(b)) \\
&= (f_L(x) \curlywedge (f_L(a) \curlyvee f_L(y))) \curlywedge f_L(b) \\
&= ((f_L(x) \curlywedge f_L(a)) \curlyvee (f_L(x) \curlywedge f_L(y)) \curlywedge f_L(b) \\
&= (f_L(a) \curlyvee \tilde{\phi}) \curlywedge f_L(b) \\
&= f_L(a) \curlywedge f_L(b) \\
\therefore\ f_L(x) \curlywedge f_L(z) &= f_L(a) \\
f_L(x) \curlyvee f_L(z) &= f_L(x) \curlyvee ((f_L(a) \curlyvee f_L(y)) \curlywedge f_L(b)) \\
&= (f_L(x) \curlyvee (f_L(a) \curlyvee f_L(y))) \curlywedge (f_L(x) \curlyvee f_L(b)) \\
&= (f_L(x) \curlyvee (f_L(y) \curlyvee f_L(a)) \curlywedge f_L(b) \\
&= ((f_L(x) \curlyvee f_L(y)) \curlyvee f_L(a)) \curlywedge f_L(b) \\
&= (\tilde{A} \curlyvee f_L(a)) \curlywedge f_L(b) \\
&= \tilde{A} \curlywedge f_L(b) \\
f_L(x) \curlyvee f_L(z) &= f_L(b)
\end{aligned}$$

Here $f_L(z)$ is a fuzzy soft relative complement of $f_L(x)$ in $[f_L(a), f_L(b)]$. $\therefore$ $[f_L(a), f_L(b)]$ is fuzzy soft complemented. Thus every interval of $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ is fuzzy soft complemented. Hence, a complemented distributive fuzzy soft lattice is relatively complemented distributive fuzzy soft lattice. $\square$

Remark 3.11. *The converse of above theorem is obviously true only when the distributive fuzzy soft lattice* $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ *is bounded.*

Theorem 3.12. *If* $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ *be a distributive bounded fuzzy soft lattice, then the complemented elements of* $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ *form a sublattice of* $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$.

Proof. Let $(f_M, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft subset of $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$ where $f_M = \{f_L(x) \in f_L; f_L(x) \text{ is fuzzy soft complemented}\}$. Then $f_M \neq \phi$. Let $f_L(x), f_L(y) \in (f_L, \curlyvee, \curlywedge, \preccurlyeq)$ be any elements. Then $f_L(x), f_L(y)$ are fuzzy soft complemented. Let${f_L}^c(x), {f_L}^c(y)$ be fuzzy soft complements of $f_L(x)$ and $f_L(y)$ respectively. Then,

$$f_L(x) \curlywedge {f_L}^c(x) = \tilde{\phi} = f_L(y) \curlywedge {f_L}^c(y)$$
$$f_L(x) \curlyvee {f_L}^c(x) = \tilde{A} = f_L(y) \curlyvee {f_L}^c(y)$$

Now,

$$\begin{aligned}(f_L(x) \curlywedge f_L(y)) \curlywedge (f_L(x) \curlywedge f_L(y))^c &= (f_L(x) \curlywedge f_L(y)) \curlywedge ({f_L}^c(x) \curlyvee {f_L}^c(y)) \\ &= ((f_L(x) \curlywedge f_L(y)) \curlywedge {f_L}^c(x)) \curlyvee ((f_L(x) \curlywedge f_L(y)) \curlywedge {f_L}^c(y)) \\ &= ((f_L(x) \curlywedge {f_L}^c(x)) \curlywedge f_L(y)) \curlyvee (f_L(x) \curlywedge (f_L(y) \curlywedge {f_L}^c(y))) \\ &= (\tilde{\phi} \curlywedge f_L(y)) \curlyvee (f_L(x) \curlywedge \tilde{\phi}) \\ &= (\tilde{\phi} \curlyvee \tilde{\phi}) \\ &= \tilde{\phi} \\ (f_L(x) \curlywedge f_L(y)) \curlyvee (f_L(x) \curlywedge f_L(y))^c &= (f_L(x) \curlywedge f_L(y)) \curlyvee ({f_L}^c(x) \curlyvee {f_L}^c(y)) \\ &= ({f_L}^c(x) \curlyvee {f_L}^c(y) \curlyvee (f_L(x) \curlywedge f_L(y)) \\ &= (({f_L}^c(x) \curlyvee {f_L}^c(y)) \curlyvee f_L(x)) \curlywedge (({f_L}^c(x) \curlyvee {f_L}^c(y)) \curlyvee f_L(y)) \\ &= (({f_L}^c(x) \curlyvee f_L(x)) \curlyvee {f_L}^c(y)) \curlywedge ({f_L}^c(x) \curlyvee ({f_L}^c(y) \curlyvee f_L(y))) \\ &= (\tilde{A} \curlyvee {f_L}^c(y)) \curlywedge ({f_L}^c(x) \curlyvee \tilde{A}) \\ &= (\tilde{A} \curlywedge \tilde{A}) \\ &= \tilde{A}\end{aligned}$$

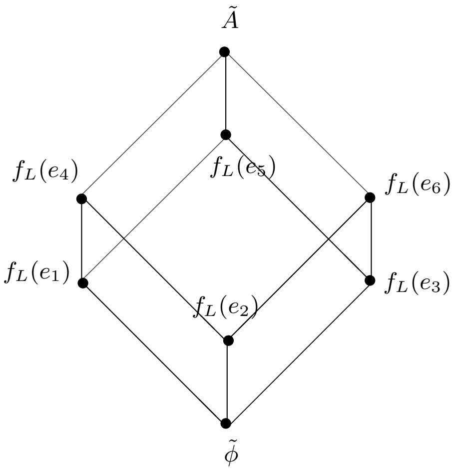
$\therefore$ ${f_L}^c(x) \curlyvee {f_L}^c(y)$ is a fuzzy soft complement of $f_L(x) \curlywedge f_L(y)$. (ie) $f_L(x) \curlywedge f_L(y) \in (f_M, \curlyvee, \curlywedge, \preccurlyeq)$. Similarly, $f_L(x) \curlyvee f_L(y) \in (f_M, \curlyvee, \curlywedge, \preccurlyeq)$. Hence $(f_M, \curlyvee, \curlywedge, \preccurlyeq)$ forms a sublattice of $(f_L, \curlyvee, \curlywedge, \preccurlyeq)$. $\square$

Example 3.13. *Let* $U = \{x_1, x_2, x_3, x_4\}$ *be the universal set,* L=$\{e_1, e_2, e_3, e_4, e_5, e_6\}$ *be the parameters. Assume that*

$$\tilde{\phi} = \{(x_1, 0.8, 0.3), (x_2, 0.8, 0.6)\}$$
$$f_L(e_1) = \{(x_1, 0.8, 0.3), (x_2, 1, 0.6)\}$$
$$f_L(e_2) = \{(x_1, 1, 0.3), (x_2, 0.8, 0), (x_3, 1, 0)\}$$
$$f_L(e_3) = \{(x_1, 0.8, 0), (x_2, 0.8, 0.6), (x_4, 1, 0)\}$$
$$f_L(e_4) = \{(x_1, 1, 0.3), (x_2, 1, 0), (x_3, 1, 0)\}$$

$$f_L(e_5) = \{(x_1, 0.8, 0), (x_2, 1, 0), (x_4, 1, 0)\}$$
$$f_L(e_6) = \{(x_1, 1, 0), (x_2, 0.8, 0), (x_3, 1, 0), (x_4, 1, 0)\}$$
$$\tilde{A} = \{(x_1, 1, 0), (x_2, 1, 0), (x_3, 1, 0), (x_4, 1, 0)\}$$

Then $(f_L, \cup, \cap, \subseteq)$ is a relatively complemented distributive fuzzy soft lattice. Hence, $(f_L, \cup, \cap, \subseteq)$ is a complemented distributive fuzzy soft lattice.



Relatively Complemented Distributive Fuzzy Soft Lattice

Theorem 3.14. *If $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ are two relatively complemented distributive fuzzy soft lattice, then $(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ is relatively complemented distributive fuzzy soft lattice.*

Proof. Let $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ be relatively complemented distributive fuzzy soft lattices and also bounded. Let$[f_A(x_1), f_A(x_2)]$ and $[f_B(y_1), f_B(y_2)]$ be an interval in $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ respectively. Let $f_A(x) \in [f_A(x_1), f_A(x_2)]$ and $f_B(y) \in [f_B(y_1), f_B(y_2)]$. Then

$$f_A(x_1) \preccurlyeq f_A(x) \preccurlyeq f_A(x_2) \quad \text{and} \quad f_B(y_1) \preccurlyeq f_B(y) \preccurlyeq f_B(y_2) \tag{1}$$

Since $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ are relatively complemented distributive fuzzy soft lattices. By Remark 3.11, $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ are complemented distributive fuzzy soft lattices. Let ${f_A}^c(x)$ and ${f_A}^c(y)$ be a fuzzy soft complements of $f_A(x)$ and $f_B(y)$ in $[f_A(x_1), f_A(x_2)] \in (f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $[f_B(y_1), f_B(y_2)] \in (f_B, \curlyvee, \curlywedge, \preccurlyeq)$ respectively. Then

$$f_A(x) \curlywedge {f_A}^c(x) = \tilde{\phi},\ f_A(x) \curlyvee {f_A}^c(x) = \tilde{A} \ \text{ and } \ f_B(y) \curlywedge {f_B}^c(y) = \tilde{\phi},\ f_B(y) \curlyvee {f_B}^c(y) = \tilde{A} \tag{2}$$

Let $(f_A(x_1), f_B(y_1)), (f_A(x_2), f_B(y_2)), (f_A(x), f_B(y)) \in (f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ with $(f_A(x_1), f_B(y_1)) \preccurlyeq (f_A(x_2), f_B(y_2))$. Let$[(f_A(x_1), f_B(y_1)), (f_A(x_2), f_B(y_2))]$ be an interval in $(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_A(x), f_B(y)) \in [(f_A(x_1), f_B(y_1)), (f_A(x_2), f_B(y_2))]$ be any element. From (1), we get $(f_A(x_1), f_B(y_1)) \preccurlyeq (f_A(x), f_B(y)) \preccurlyeq (f_A(x_2), f_B(y_2))$. Now, (2) $\Rightarrow$

$$\begin{aligned}(f_A(x), f_B(y)) \curlywedge (f_A(x), f_B(y))^c &= (f_A(x), f_B(y)) \curlywedge ({f_A}^c(x), {f_B}^c(y)) \\ &= (f_A(x) \curlywedge {f_A}^c(x), f_B(y) \curlywedge {f_B}^c(y)) \\ &= (\tilde{\phi}, \tilde{\phi})\end{aligned}$$

$$\begin{aligned}(f_A(x), f_B(y)) \curlyvee (f_A(x), f_B(y))^c &= (f_A(x), f_B(y)) \curlyvee ({f_A}^c(x), {f_B}^c(y)) \\ &= (f_A(x) \curlyvee {f_A}^c(x), f_B(y) \curlyvee {f_B}^c(y)) \\ &= (\tilde{A}, \tilde{A})\end{aligned}$$

Let $(f_A(a), f_B(b)) \in (f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ where $f_A(a) \in (f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $f_B(b) \in (f_B, \curlyvee, \curlywedge, \preccurlyeq)$. Take$(f_A(a), f_B(b)) = (f_A(x_1), f_B(y_1)) \curlyvee ((f_A(x_2), f_B(y_2)) \curlywedge ({f_A}^c(x), {f_B}^c(y)))$ where $(f_L(a), f_L(b)) \in [(f_L(x_1), f_L(y_1)), (f_L(x_2), f_L(y_2))]$. Now,

$$\begin{aligned}(f_A(x), f_B(y)) \curlywedge (f_A(a), f_B(y)) &= (f_A(x), f_B(y)) \curlywedge ((f_A(x_1), f_B(y_1)) \curlyvee ((f_A(x_2), f_B(y_2)) \curlywedge ({f_A}^c(x), {f_B}^c(y)))) \\ &= ((f_A(x), f_B(y)) \curlywedge (f_A(x_1), f_B(y_1))) \curlyvee ((f_A(x), f_B(y)) \curlywedge ((f_A(x_2), f_B(y_2)) \\ &\quad \curlywedge ({f_A}^c(x), {f_B}^c(y)))) \\ &= (f_A(x_1), f_B(y_1)) \curlyvee ((f_A(x), f_B(y)) \curlywedge (({f_A}^c(x), {f_B}^c(y)) \curlywedge (f_A(x_2), f_B(y_2)))) \\ &= (f_A(x_1), f_B(y_1)) \curlyvee (((f_A(x), f_B(y)) \curlywedge ({f_A}^c(x), {f_B}^c(y))) \curlywedge (f_A(x_2), f_B(y_2))) \\ &= (f_A(x_1), f_B(y_1)) \curlyvee ((\tilde{\phi}, \tilde{\phi}) \curlywedge (f_A(x_2), f_B(y_2))) \\ &= (f_A(x_1), f_B(y_1)) \curlyvee (\tilde{\phi}, \tilde{\phi}) \\ &= (f_A(x_1), f_B(y_1)) \\ \therefore\ (f_A(x), f_B(y)) \curlywedge (f_A(a), f_B(b)) &= (f_A(x_1), f_B(y_1)) \\ (f_A(x), f_B(y)) \curlyvee (f_A(a), f_B(y)) &= (f_A(x), f_B(y)) \curlyvee ((f_A(x_1), f_B(y_1)) \curlyvee ((f_A(x_2), f_B(y_2)) \curlywedge ({f_A}^c(x), {f_B}^c(y)))) \\ &= ((f_A(x), f_B(y)) \curlyvee (f_A(x_1), f_B(y_1))) \curlyvee ((f_A(x_2), f_B(y_2)) \curlywedge ({f_A}^c(x), {f_B}^c(y))) \\ &= (f_A(x), f_B(y)) \curlyvee ((f_A(x_2), f_B(y_2)) \curlywedge ({f_A}^c(x), {f_B}^c(y))) \\ &= ((f_A(x), f_B(y)) \curlyvee (f_A(x_2), f_B(y_2))) \curlywedge ((f_A(x), f_B(y)) \curlyvee ({f_A}^c(x), {f_B}^c(y))) \\ &= (f_A(x_2), f_B(y_2)) \curlywedge (\tilde{A}, \tilde{A}) \\ &= (f_A(x_2), f_B(y_2)) \\ \therefore\ (f_A(x), f_B(y)) \curlyvee (f_A(a), f_B(y)) &= (f_A(x_2), f_B(y_2))\end{aligned}$$

Hence $(f_A(a), f_B(b))$ is a fuzzy soft relative complement of $(f_A(x), f_B(y))$ in interval $[(f_A(x_1), f_B(y_1)), (f_A(x_2), f_B(y_2))]$. $\therefore$ $[(f_A(x_1), f_B(y_1)), (f_A(x_2), f_B(y_2))]$ is fuzzy soft complemented. Since every interval of $(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ is fuzzy soft complemented,$(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ is a relatively complemented distributive fuzzy soft lattice. □

Theorem 3.15. *If $(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ is a relatively complemented distributive fuzzy soft lattice. Then $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ are relatively complemented distributive fuzzy soft lattice.*

Proof. Let $(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ be a fuzzy soft relatively complemented and bounded lattice. Let $[(f_A(x_1), f_B(y_1)), (f_A(x_2), f_B(y_2))]$ be any interval in $(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$. By Remark 3.11, $(f_A \times f_B, \curlyvee, \curlywedge, \preccurlyeq)$ is fuzzy soft complemented. Thus for an element $(f_A(x), f_B(y)) \in [(\tilde{\phi}, \tilde{\phi}), (\tilde{A}, \tilde{A})]$ there exist a fuzzy soft complement $({f_A}^c(x), {f_B}^c(y))$ relative to $[(\tilde{\phi}, \tilde{\phi}), (\tilde{A}, \tilde{A})]$. So,

$$(f_A(x), f_B(y)) \curlywedge ({f_A}^c(x), {f_B}^c(y)) = (\tilde{\phi}, \tilde{\phi}) \ \text{ and } \ (f_A(x), f_B(y)) \curlyvee ({f_A}^c(x), {f_B}^c(y)) = (\tilde{A}, \tilde{A}) \tag{3}$$

Let $[f_A(x_1), f_A(x_2)], [f_B(y_1), f_B(y_2)]$ be any interval in $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$. Let $f_A(x) \in [f_A(x_1), f_A(x_2)]$, $f_B(y) \in [f_B(y_1), f_B(y_2)]$ be any elements in $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$. Then,

$$f_A(x_1) \preccurlyeq f_A(x) \preccurlyeq f_A(x_2) \ \text{ and } \ f_B(y_1) \preccurlyeq f_B(y) \preccurlyeq f_B(y_2) \tag{4}$$

From (3), $(f_A(x) \curlywedge {f_A}^c(x), f_B(y) \curlywedge {f_B}^c(y)) = (\tilde{\phi}, \tilde{\phi})$ and $(f_A(x) \curlyvee {f_A}^c(x), f_B(y) \curlyvee {f_B}^c(y)) = (\tilde{A}, \tilde{A}) \Rightarrow f_A(x) \curlywedge {f_A}^c(x) = \tilde{\phi}$, $f_B(y) \curlywedge {f_B}^c(y) = \tilde{\phi}$ and $f_A(x) \curlyvee {f_A}^c(x) = \tilde{A}$, $f_B(y) \curlyvee {f_B}^c(y) = \tilde{A}$. $\therefore$ $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ are fuzzy soft complemented lattices. Let $f_A(x) \in (f_A, \curlyvee, \curlywedge, \preccurlyeq)$ and $f_B(y) \in (f_B, \curlyvee, \curlywedge, \preccurlyeq)$ where $f_A(z) \in [f_A(x_1), f_B(x_2)]$. Take $f_A(z) = f_A(x_1) \curlyvee (f_A(x_2) \curlywedge {f_A}^c(x))$. Now,

$$\begin{aligned}
f_A(x) \curlywedge f_A(z) &= f_A(x) \curlywedge (f_A(x_1) \curlyvee (f_A(x_2) \curlywedge {f_A}^c(x))) \\
&= (f_A(x) \curlywedge f_A(x_1)) \curlyvee (f_A(x) \curlywedge (f_A(x_2) \curlywedge {f_A}^c(x))) \\
&= f_A(x_1) \curlyvee (f_A(x) \curlywedge (f_A(x_2) \curlywedge {f_A}^c(x))) \\
&= f_A(x_1) \curlyvee (f_A(x) \curlywedge ({f_A}^c(x) \curlywedge f_A(x_2))) \\
&= f_A(x_1) \curlyvee ((f_A(x) \curlywedge {f_A}^c(x)) \curlywedge f_A(x_2)) \\
&= f_A(x_1) \curlyvee (\tilde{\phi} \curlywedge f_A(x_2)) \\
&= f_A(x_1) \curlyvee \tilde{\phi} \\
\therefore \ f_A(x) \curlywedge f_A(z) &= f_A(x_1) \\
f_A(x) \curlyvee f_A(z) &= f_A(x) \curlyvee (f_A(x_1) \curlyvee (f_A(x_2) \curlywedge {f_A}^c(x))) \\
&= (f_A(x) \curlyvee f_A(x_1)) \curlyvee (f_A(x_2) \curlywedge {f_A}^c(x)) \\
&= f_A(x) \curlyvee (f_A(x_2) \curlywedge {f_A}^c(x)) \\
&= (f_A(x) \curlyvee f_A(x_2)) \curlywedge ((f_A(x) \curlyvee {f_A}^c(x)) \\
&= f_A(x_2) \curlywedge (f_A(x) \curlyvee {f_A}^c(x)) \\
&= f_A(x_2) \curlywedge \tilde{A} \\
f_A(x) \curlyvee f_A(z) &= f_A(x_2)
\end{aligned}$$

Here $f_A(z)$ is a fuzzy soft relative complement of $f_A(x)$ in $[f_A(x_1), f_A(x_2)]$. $\therefore$ $[f_A(x_1), f_A(x_2)]$ is fuzzy soft complemented. Since every interval of $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ is fuzzy soft complemented, $(f_A, \curlyvee, \curlywedge, \preccurlyeq)$ is a fuzzy soft relatively complemented distributive fuzzy soft lattice. Let $f_B(y) \in f_B$ where $f_B(z) \in [f_B(y_1), f_B(y_2)]$. Take $f_B(z) = f_B(y_1) \curlyvee (f_B(y_2) \curlywedge {f_B}^c(y))$. Now,

$$\begin{aligned}
f_B(y) \curlywedge f_B(z) &= f_B(y) \curlywedge (f_B(y_1) \curlyvee (f_B(y_2) \curlywedge {f_B}^c(y))) \\
&= (f_B(y) \curlywedge f_B(y_1)) \curlyvee (f_B(y) \curlywedge (f_B(y_2) \curlywedge {f_B}^c(y))) \\
&= f_B(y_1) \curlyvee (f_B(y) \curlywedge (f_B(y_2) \curlywedge {f_B}^c(y))) \\
&= f_B(y_1) \curlyvee (f_B(y) \curlywedge ({f_B}^c(y) \curlywedge f_B(y_2))) \\
&= f_B(y_1) \curlyvee ((f_B(y) \curlywedge {f_B}^c(y)) \curlywedge f_B(y_2)) \\
&= f_B(y_1) \curlyvee (\tilde{\phi} \curlywedge f_B(y_2)) \\
&= f_B(y_1) \curlyvee \tilde{\phi} \\
\therefore \ f_B(y) \curlywedge f_B(z) &= f_B(y_1)
\end{aligned}$$

$$
\begin{aligned}
f_B(y) \curlyvee f_B(z) &= f_B(y) \curlyvee (f_B(y_1) \curlyvee (f_B(y_2) \curlywedge {f_B}^c(y))) \\
&= (f_B(y) \curlyvee f_B(y_1)) \curlyvee (f_B(y_2) \curlywedge {f_B}^c(y)) \\
&= f_B(y) \curlyvee (f_B(y_2) \curlywedge {f_B}^c(y)) \\
&= (f_B(y) \curlyvee f_B(y_2)) \curlywedge ((f_B(y) \curlyvee {f_B}^c(y)) \\
&= f_B(y_2) \curlywedge (f_B(y) \curlyvee {f_B}^c(y)) \\
&= f_B(y_2) \curlywedge \tilde{A} \\
f_B(y) \curlyvee f_B(z) &= f_B(y_2)
\end{aligned}
$$

Here $f_B(z)$ is a fuzzy soft relative complement of $f_B(y)$ in $[f_B(y_1), f_B(y_2)]$. $\therefore$ $[f_B(y_1), f_B(y_2)]$ is fuzzy soft complemented. Since every interval of $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ is fuzzy soft complemented, $(f_B, \curlyvee, \curlywedge, \preccurlyeq)$ is a fuzzy soft relatively complemented distributive fuzzy soft lattice. □

4. Conclusion

In this paper, we have given an definition of the complemented fuzzy soft lattice and relatively complemented fuzzy soft lattice. We also prove some theorems on complemented and relatively complemented distributive fuzzy soft lattice and illustrate them with an example. Now, we are studying about these fuzzy soft lattices and are expected to give some more results in our future study.

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