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Complements and Relative Complements in Modular Soft Rough Lattice

Research Article

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Abstract: Soft sets and rough sets are both the mathematical tools for dealing the problems of uncertainty. Soft rough set is the connection between these two mathematical approaches to vagueness and it is the generalization of rough set with respect to the soft approximation space. E.K.R.Nagarajan et al. defined the direct product of soft lattices. The complements of soft lattice has been given by E.K.R. Nagarajan et al. [11]. The complements of rough set has been defined by Ameri et al. [1]. In this paper we first define the complements of soft rough set and also the complements and relative complements of modular soft rough lattice. We discuss some theorems related to the complements and relative complements of modular soft rough lattice with an example.

MSC: 06C20.

Keywords: Soft set, rough set, soft rough set, complement of soft rough set, complement of soft rough lattice and relative complement of soft rough lattice.

1. Introduction

In 1999, Molodtsov [8] introduced soft set as a mathematical tool for dealing with uncertainty. Maji et al. [6] discussed the application of soft set theory in a decision making problem. Maji et al. [7] defined the operations of soft set and a theoretical study on soft set. The soft lattice structure has been found in [5, 9, 12]. Rough set theory introduced by Pawlak [13] is another mathematical approach to vagueness. Every rough sets are associated with two crisp sets, called lower and upper approximations and viewed as the set of elements which certainly and possibly belong to the set. Rana and Roy [14] introduced rough set approach on lattice. Soft set theory is a possible way to solve the difficulties of rough set. Thereafter a possible fusion of rough sets and soft set has been proposed by Feng et al. [2].Soft rough set is the generalization of rough set with respect to the soft approximation space. S.K.Roy and S.Bera [15] introduced some operations on soft rough set and defined modular soft rough lattice and distributive soft rough lattice.The properties of soft rough modular and soft rough distributive lattice was discussed in [3, 4]. The direct product of soft lattices was defined in [10]. The complements can be found for soft sets as well as rough sets.The complement of soft lattice has been found in [11]. The complement of rough set has been defined in [1]. In this paper we define the complements of soft rough set and also the complements and relative complements of soft rough lattice and discuss some theorems related to the complements of modular soft rough lattice with an example.

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2. Preliminaries

In this section , we recall some basic definitions and properties related to soft set, soft rough set and soft rough lattices. Let U be an initial universe of objects and E be the set of parameters and $A \subseteq E$. P(U) is the power set of U.

Definition 2.1. *A pair $S = (F, A)$ is called a soft set over U, where $F : A \to P(U)$ is a set valued mapping.*

Definition 2.2. *Let U be the set of universe and ρ an equivalence relation on U. An equivalence class of $x(\in U)$ is denoted by $[x]_\rho$ and defined as follows: $[x]_\rho$=$\{y \in U : x\rho y\}$, where $x \ \rho \ y$ imply (x,y)$\in \rho$.The lower and upper approximations of $X \subseteq U$ are denoted by $A_*(X)$ and $A^*(X)$ respectively and defined as follows: $A_*(X)$= $\left\{x \in U : [x]_\rho \subseteq X\right\}$ and $A^*(X) = \{x \in U : [x]_\rho \cap X \neq \phi\}$. The pair (U,$\rho$) is called an approximation space and is denoted by S. Then $A(X) = (A_*(X), A^*(X))$ is called the rough set of X in S.*

Definition 2.3. *Let $A(X) = (A_*(X), A^*(X))$ and $A(Y) = (A_*(Y), A^*(Y))$ be two rough sets under the approximation space $S = (U, \rho)$ then the rough union is defined by $A(X) \cup A(Y) = (A_*(X) \cup A_*(Y), A^*(X) \cup A^*(Y))$ and the rough intersection is defined by $A(X) \cap A(Y) = (A_*(X) \cap A_*(Y), A^*(X) \cap A^*(Y))$.*

Definition 2.4. *A rough set $A(Y)$ is said to be rough subset of a rough set $A(X)$ if $A_*(Y) \subseteq A_*(X)$ and $A^*(Y) \subseteq A^*(X)$ and it is denoted by $A(Y) \subseteq A(X)$.*

Definition 2.5. *Let S=(F,A) be a soft set over U. Then the pair P=(U,S) is called a soft approximation space. Let $X \subseteq U$. We defined the following operations on P*

$$\underline{apr}(X) = \bigcup_{a \in A} \{F(a) : F(a) \subseteq X\},$$
$$\overline{apr}(X) = \bigcup_{a \in A} \{F(a) : F(a) \cap X \neq \phi\},$$

which are called soft lower and upper approximations respectively of X and the pair $(\underline{apr}(X), \overline{apr}(X))$ is called soft rough set of X with respect to P and is denoted by $S_r(X)$. The set of all soft rough sets over U is denoted by $S_R(U)$ with respect to some soft approximation space P.

Definition 2.6. *Let $S_r(X)$ =$(\underline{apr}(X), \overline{apr}(X))$ and $S_r(Y)$ =$(\underline{apr}(Y), \overline{apr}(Y))$ be two soft rough set. Then soft rough union and soft rough intersection of $S_r(X)$ and $S_r(Y)$ are defined by $S_r(X) \sqcup S_r(Y) = (\underline{apr}(X) \cup \underline{apr}(Y), \overline{apr}(X) \cup \overline{apr}(Y))$ and $S_r(X) \sqcap S_r(Y) = (\underline{apr}(X) \cap \underline{apr}(Y), \overline{apr}(X) \cap \overline{apr}(Y))$ respectively, where the symbols $\sqcup$ and $\sqcap$ stand for soft rough union and intersection respectively.*

Definition 2.7. *Let $S_r(X) = (\underline{apr}(X), \overline{apr}(X))$ and $S_r(Y) = (\underline{apr}(Y), \overline{apr}(Y))$ be two soft rough set. Then $S_r(Y)$ is said to be soft rough subset of $S_r(X)$, denoted by $S_r(Y) \sqsubseteq S_r(X)$ if $\underline{apr}(Y) \subseteq \underline{apr}(X)$ and $\overline{apr}(Y) \subseteq \overline{apr}(X)$, where $\sqsubseteq$ stands for soft rough inclusion relation.*

Let $S = (F, A)$ be a soft set over U and $P = (U, S)$ be a soft approximation space and $S_R(U)$ be the set of all soft rough sets with respect to P.

Definition 2.8. *Let $\mathcal{L} \subseteq S_R(U), \vee$ and $\wedge$ be two binary operations on $\mathcal{L}$. The algebraic structure ($\mathcal{L}$,$\vee, \wedge, \preceq$) is said to be soft rough lattice if*

(i). $\vee$ and $\wedge$ are associative,

(ii). $\vee$ and $\wedge$ are commutative,

(iii). $\vee$ and $\wedge$ satisfied absorption laws.

Definition 2.9. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice and $\mathcal{K} \subseteq \mathcal{L}$. Then $(\mathcal{K}, \vee, \wedge, \preceq)$ is said to be soft rough sublattice of $(\mathcal{L}, \vee, \wedge, \preceq)$ if and only if is closed under both operations $\vee$ and $\wedge$. (i.e) If $S_r(X), S_r(Y) \in \mathcal{K}$ then $S_r(X) \wedge S_r(Y) \in \mathcal{K}$ and $S_r(X) \vee S_r(Y) \in \mathcal{K}$*

Definition 2.10. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be distributive soft rough lattice if for every $S_r(X), S_r(Y), S_r(Z) \in \mathcal{L}$, then $S_r(X) \wedge (S_r(Y) \vee S_r(Z)) = (S_r(X) \wedge S_r(Y)) \vee (S_r(X) \wedge S_r(Z))$*

Definition 2.11. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be modular soft rough lattice if for every $S_r(X), S_r(Y), S_r(Z) \in \mathcal{L}$, with $S_r(X) \succeq S_r(Y)$ the following inequality holds $S_r(X) \wedge (S_r(Y) \vee S_r(Z)) = S_r(Y) \vee (S_r(X) \wedge S_r(Z))$.*

Definition 2.12. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ be two soft rough lattices. Consider the cartesian product $\mathcal{L} \times \mathcal{M} = \{(S_r(X), S_r(Y))/S_r(X) \in \mathcal{L}, S_r(Y) \in \mathcal{M}\}$. Define the binary operations $\vee$ and $\wedge$ on $\mathcal{L}\times\mathcal{M}$ as follows: For all $(S_r(X_1), S_r(Y_1))$ and $(S_r(X_2), S_r(Y_2)) \in \mathcal{L} \times \mathcal{M}$,*

$$\begin{aligned}
(S_r(X_1), S_r(Y_1)) \vee (S_r(X_2), S_r(Y_2)) &= (S_r(X_1) \vee_1 S_r(X_2), S_r(Y_1) \vee_2 S_r(Y_2)) \\
&= (\overline{apr}(X_1) \vee_1 \overline{apr}(X_2), \overline{apr}(Y_1) \vee_2 \overline{apr}(Y_2)) \quad \text{and} \\
(S_r(X_1), S_r(Y_1)) \wedge (S_r(X_2), S_r(Y_2)) &= (S_r(X_1) \wedge_1 S_r(X_2), S_r(Y_1) \wedge_2 S_r(Y_2)) \\
&= (\overline{apr}(X_1) \wedge_1 \overline{apr}(X_2), \overline{apr}(Y_1) \wedge_2 \overline{apr}(Y_2)).
\end{aligned}$$

3. Complements of Soft Rough Set

In this section, we introduce the definition of complement on soft rough sets and illustrate it with an example. Let U be the initial universe of objects and E be the set of parameters and $A \subseteq E$. Let $S = (F, A)$ be a soft set over U and $P = (U, S)$ be a soft approximation space and $S_R(U)$ be the set of all soft rough sets with respect to P.

Definition 3.1. *If $S_r(X) = (\underline{apr}(X), \overline{apr}(X)) = (\phi, \phi) = S_r(\phi)$ for all $x \in E$, then $S_r(\phi)$ is called a null soft rough set.*

Definition 3.2. *If $S_r(X) = (\underline{apr}(X), \overline{apr}(X)) = (U, U) = S_r(U)$ for all $x \in A$, then $S_r(U)$ is called a A-universal soft rough set.*

Definition 3.3. *If A=E and $S_r(X)$=$(\underline{apr}(X), \overline{apr}(X)) = (U, U) = S_r(U)$ for all $x \in E$, then $S_r(U)$ is called a universal soft rough set.*

Definition 3.4. *The complement of the soft rough set $S_r(X)$ over the soft approximation space (U,S)is defined by $S_r^c(X) = (U \setminus \overline{apr}(X), U \setminus \underline{apr}(X)) = (\overline{apr}^c(X), \underline{apr}^c(X))$ and it is denoted by $S_r^c(X)$.*

Example 3.5. *Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ and $A = \{e_1, e_2, e_3, e_4, e_5\}$. Let $S = (F, A)$ be a soft set over U given by $F(e_1) = \{u_2, u_4\}, F(e_2) = \{u_1, u_3\}, F(e_3) = \{u_2, u_4, u_5\}, F(e_4) = \{u_1, u_5\}, F(e_5) = \{u_3\}$. Let $X_1 = \phi, X_2 = \{u_1\}, X_3 = \{u_3\}, X_4 = \{u_2, u_3\}, X_5 = \{u_1, u_2, u_3, u_5\}$. The soft rough sets on the soft approximation space P=(U,S) are given by $S_r(X_1) = (\phi, \phi), S_r(X_2) = (\phi, u_1u_3u_5), S_r(X_3) = (u_3, u_1u_3),$ $S_r(X_4) = (\phi, u_1u_2u_3u_4u_5), S_r(X_5) = (u_1u_3u_5, u_1u_2u_3u_4u_5)$. The complements of the soft rough sets on the soft approximation space P=(U,S) of $S_r(X_1), S_r(X_2), S_r(X_3), S_r(X_4), S_r(X_5)$ are given by $S_r^c(X_1) = (u_1u_2u_3u_4u_5, u_1u_2u_3u_4u_5)$, $S_r^c(X_2) = (u_2u_4, u_1u_2u_3u_4u_5), S_r^c(X_3) = (u_2u_4u_5, u_1u_2u_4u_5), S_r^c(X_4) = (\phi, u_1u_2u_3u_4u_5), S_r^c(X_5) = (\phi, u_2u_4)$ respectively.*

4. Complements and Relative Complements of Modular Soft Rough Lattice

In this section, we define the complements and relative complements on soft rough lattice and discuss some theorems related to the complements of modular soft rough lattice with an example. Let $\mathcal{L} \subseteq S_R(U)$, the set of all soft rough sets over U with respect to the soft approximation space $P = (U, S)$.

Definition 4.1. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice and $S_r(I), S_r(J) \in \mathcal{L}$. Suppose $S_r(I) \preceq S_r(J), [S_r(I), S_r(J)] = \{S_r(X) \in \mathcal{L}/S_r(I) \preceq S_r(X) \preceq S_r(J)\}$. Then $[S_r(I), S_r(J)]$ is said to be soft rough complemented if for every $S_r(X) \in [S_r(I), S_r(J)]$ there exists $S_r(Y) \in [S_r(I), S_r(J)]$ such that $S_r(X) \wedge S_r(Y) = S_r(I)$ and $S_r(X) \vee S_r(Y) = S_r(J)$.Here $S_r(Y)$ is said to be the soft rough relative complement of $S_r(X)$.*

Definition 4.2. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be relatively complemented if every interval of the form $[S_r(I), S_r(J)]$ (with $S_r(I) \preceq S_r(J)$) is soft rough complemented.*

Definition 4.3. *Let $\mathcal{L} \subseteq S_R(U)$. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice, if there exists a soft rough set $S_r(X) \in \mathcal{L}$ such that $S_r(X) \preceq S_r(Y)$ for all $S_r(Y) \in \mathcal{L}$, then $S_r(X)$ is called the least element of $(\mathcal{L}, \vee, \wedge, \preceq)$.*

Definition 4.4. *Let $\mathcal{L} \subseteq S_R(U)$ Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a soft rough lattice, if there exists a soft rough set $S_r(X) \in \mathcal{L}$ such that $S_r(X) \succeq S_r(Y)$ for all $S_r(Y) \in \mathcal{L}$, then $S_r(X)$ is called the greatest element of $(\mathcal{L}, \vee, \wedge, \preceq)$.*

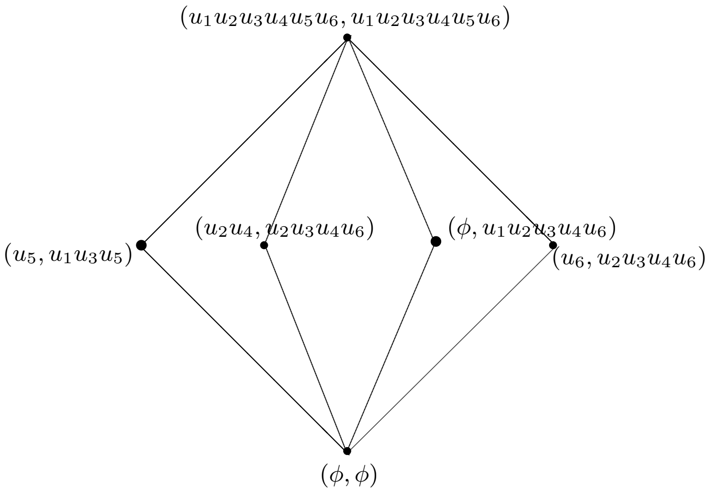
Definition 4.5. *A soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be a bounded soft rough lattice if it has both the greatest element $S_r(U)$ and least element $S_r(\phi)$.*

Definition 4.6. *Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a bounded soft rough lattice, for any $S_r(X) \in \mathcal{L}$, there exists $S_r^c(X) \in \mathcal{L}$ such that $S_r(X) \vee S_r^c(X) = S_r(U)$ and $S_r(X) \wedge S_r^c(X) = S_r(\phi)$. Then $S_r^c(X)$ is called the soft rough complement of $S_r(X)$.*

Remark 4.7. *The complement of the crisp set is not valid for the complement of the soft rough set in a soft rough lattice. i.e.,$S_r(X) \vee S_r^c(X) = (\underline{apr}(\widehat{X}), \overline{apr}(U)) \sqsubseteq S_r(U)$ and $S_r(X) \wedge S_r^c(X) = (\underline{apr}(\phi), \overline{apr}(\widehat{X})) \sqsupseteq S_r(\phi)$ where $(\underline{apr}(\widehat{X}))^c = \overline{apr}(\widehat{X}))$*

Note 4.8. *A bounded soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is said to be soft rough complemented lattice if every soft rough set of $(\mathcal{L}, \vee, \wedge, \preceq)$ has atleast one soft rough complement.*

Example 4.9. *Let U=$\{u_1, u_2, u_3, u_4, u_5, u_6\}, A = \{e_1, e_2, e_3, e_4, e_5\}$.Let S=(F,A) be a soft set over U given by $F(e_1) = \{u_1, u_3\}, F(e_2) = \{u_5\}, F(e_3) = \{u_2, u_4\}, F(e_4) = \{u_6\}, F(e_5) = \{u_2, u_3, u_4, u_6\}$. Let $X_1 = \phi, X_2 = \{u_1, u_5\}, X_3 = \{u_2, u_4\}, X_4 = \{u_3\}, X_5 = \{u_6\}, X_6 = \{u_1, u_2, u_3, u_4, u_5, u_6\}$. The soft rough sets on the soft approximation space P=(U,S) are given by $S_r(X_1) = (\phi, \phi), S_r(X_2) = (u_5, u_1u_3u_5), S_r(X_3) = (u_2u_4, u_2u_3u_4u_6), S_r(X_4) = (\phi, u_1u_2u_3u_4u_6), S_r(X_5) = (u_6, u_2u_3u_4u_6)$ and $S_r(X_6) = (u_1u_2u_3u_4u_5u_6, u_1u_2u_3u_4u_5u_6)$. Then the lattice $\mathcal{L}$=$\{S_r(X_1), S_r(X_2), S_r(X_3),$ $S_r(X_4), S_r(X_5), S_r(X_6)\}$ is a soft rough relatively complemented lattice with the operations $\sqcup, \sqcap, \sqsubseteq$. Hence $(\mathcal{L}, \sqcup, \sqcap, \sqsubseteq)$ is also a soft rough complemented lattice.*



Complemented modular soft rough lattice

Theorem 4.10. *A complemented modular soft rough lattice is a relatively complemented soft rough lattice.*

Proof. Let $(\mathcal{L}, \vee, \wedge, \preceq)$ be a complemented modular soft rough lattice. Let $[S_r(I), S_r(J)]$ be any interval in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $S_r(X) \in [S_r(I), S_r(J)]$. Since $(\mathcal{L}, \vee, \wedge, \preceq)$ is a complemented modular soft rough lattice, $S_r(X)$ has a complement say$S_r^c(X)$. Then, $S_r(X) \wedge S_r^c(X) = S_r(\phi)$ and $S_r(X) \vee S_r^c(X) = S_r(U)$. Also,$S_r(I) \preceq S_r(X) \preceq S_r(J)$. Now, Let us take $S_r(Y) = S_r(I) \vee (S_r(J) \wedge S_r^c(X))$ where $S_r(Y) \in [S_r(I), S_r(J)]$

$$\begin{aligned}
S_r(X) \wedge S_r(Y) &= S_r(X) \wedge (S_r(I) \vee (S_r(J) \wedge S_r^c(X))) \\
&= S_r(I) \vee (S_r(X) \wedge (S_r(J) \wedge S_r^c(X))) \\
&= S_r(I) \vee (S_r(J) \wedge (S_r^c(X) \wedge S_r(X))) \\
&= S_r(I) \vee (S_r(J) \wedge (S_r(\phi)) \\
&= S_r(I) \vee (S_r(\phi) \\
&= S_r(I). \\
S_r(X) \vee S_r(Y) &= S_r(X) \vee (S_r(I) \vee (S_r(J) \wedge S_r^c(X))) \\
&= (S_r(X) \vee (S_r(I)) \vee (S_r(J) \wedge S_r^c(X)) \\
&= S_r(X) \vee (S_r(J) \wedge S_r^c(X)) \\
&= S_r(J) \wedge (S_r(X) \vee S_r^c(X))) \\
&= S_r(J) \vee (S_r(X) \wedge (S_r(\phi)) \\
&= S_r(J) \vee (S_r(\phi)) \\
&= S_r(J).
\end{aligned}$$

$\therefore$ $S_r(J)$ is a soft rough relative complement of $S_r(X)$ in $[S_r(I), S_r(J)]$. Hence $(\mathcal{L}, \vee, \wedge, \preceq)$ is a relatively complemented soft rough lattice. □

Note 4.11. *The converse of the above theorem is obviously true only when the modular soft rough lattice $(\mathcal{L}, \vee, \wedge, \preceq)$ is bounded.*

Theorem 4.12. *If $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are relatively complemented modular soft rough lattice then $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is relatively complemented modular soft rough lattice.*

Proof. Let$(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are relatively complemented modular soft rough lattice. Also $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are bounded modular soft rough lattice. Let $[S_r(X_1), S_r(X_2)]$ and $[S_r(Y_1), S_r(Y_2)]$ be intervals in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ respectively. Let $S_r(X) \in [S_r(X_1), S_r(X_2)]$ and $S_r(Y) \in [S_r(Y_1), S_r(Y_2)]$

$$\Rightarrow S_r(X_1) \preceq S_r(X) \preceq S_r(X_2) \quad \text{and} \quad S_r(Y_1) \preceq S_r(Y) \preceq S_r(Y_2) \tag{1}$$

Since $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are relatively complemented modular soft rough lattice. By the Note 4.11, $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$ are complemented modular soft rough lattice. Hence $S_r(X)$ and $S_r(Y)$ has complements $S_r^c(X)$ and $S_r^c(Y)$ in $[S_r(\phi), S_r(U)] \in (\mathcal{L}, \vee, \wedge, \preceq)$ and $[S_r(\phi), S_r(U)] \in (\mathcal{M}, \vee, \wedge, \preceq)$ respectively. Then $S_r(X) \wedge S_r^c(X) = S_r(\phi)$ and $S_r(X) \vee S_r^c(X) = S_r(U)$ and

$$S_r(Y) \wedge S_r^c(Y) = S_r(\phi) S_r(Y) \vee S_r^c(Y) = S_r(U) \tag{2}$$

Let $(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))$ and $(S_r(X), S_r(Y)) \in (\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ with $(S_r(X_1), S_r(Y_1)) \preceq (S_r(X_2), S_r(Y_2))$. Let $[(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))]$ be an interval in $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ and $(S_r(X), S_r(Y)) \in [(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))]$. From (1) we get $(S_r(X_1), S_r(Y_1)) \preceq (S_r(X), S_r(Y)) \preceq (S_r(X_2), S_r(Y_2))$. Now,(2) $\Rightarrow$

$$\begin{aligned}
(S_r(X), S_r(Y)) \wedge (S_r^c(X), S_r^c(Y)) &= (S_r(X) \wedge S_r^c(X), S_r(Y) \wedge S_r^c(Y)) \\
&= (S_r\phi), S_r(\phi)) \\
(S_r(X), S_r(Y)) \vee (S_r^c(X), S_r^c(Y)) &= (S_r(X) \vee S_r^c(X), S_r(Y) \vee S_r^c(Y)) \\
&= (S_r(U), S_r(U))
\end{aligned}$$

$\Rightarrow (\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is a complement soft rough modular lattice. Let $(S_r(X'), S_r(Y'))$ be an element in $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ where $(S_r(X'), S_r(Y')) \in [(S_r(X_1), S_r(Y_1)), (S_r(X_2), S_r(Y_2))]$. Take $(S_r(X'), S_r(Y')) = (S_r(X_1), S_r(Y_1)) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y)))$

$$\begin{aligned}
(S_r(X), S_r(Y)) \wedge (S_r(X'), S_r(Y')) &= (S_r(X), S_r(Y)) \wedge ((S_r(X_1), S_r(Y_1)) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y)))). \\
&= (S_r(X_1), S_r(Y_1)) \vee ((S_r(X), S_r(Y)) \wedge ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y)))). \\
&= (S_r(X_1), S_r(Y_1)) \vee ((S_r^c(X), S_r^c(Y)) \wedge ((S_r(X), S_r(Y)) \wedge (S_r(X_2), S_r(Y_2)))). \\
&= (S_r(X_1), S_r(Y_1)) \vee ((S_r(\phi), S_r(\phi)) \wedge (S_r(X_2), S_r(Y_2))). \\
&= (S_r(X_1), S_r(Y_1)) \vee ((S_r(\phi), S_r(\phi)). \\
&= (S_r(X_1), S_r(Y_1)) \\
(S_r(X), S_r(Y)) \vee (S_r(X'), S_r(Y')) &= (S_r(X), S_r(Y)) \vee ((S_r(X_1), S_r(Y_1)) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y)))). \\
&= ((S_r(X), S_r(Y)) \vee (S_r(X_1), S_r(Y_1))) \vee ((S_r(X_2), S_r(Y_2)) \wedge (S_r^c(X), S_r^c(Y))) \\
&= (S_r(X), S_r(Y)) \vee ((S_r(X_2), S_r(Y_2)) \wedge ((S_r^c(X), S_r^c(Y))). \\
&= (S_r(X_2), S_r(Y_2)) \wedge ((S_r(X), S_r(Y)) \vee ((S_r^c(X), S_r^c(Y))). \\
&= (S_r(X_2), S_r(Y_2)) \wedge ((S_r(U), S_r(U)) \\
&= (S_r(X_2), S_r(Y_2)).
\end{aligned}$$

Here $(S_r(X'), S_r(Y'))$ is a soft rough relative complement of $(S_r(X), S_r(Y))$ in $[(S_r(X_1), S_r(X_2)), (S_r(Y_1), S_r(Y_2))]$. Hence the $[(S_r(X_1), S_r(X_2)), (S_r(Y_1), S_r(Y_2))]$ is soft rough complemented. Similarly, every interval in $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is soft rough complemented. Hence $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ is a soft rough relatively complemented modular soft rough lattice. □

Theorem 4.13. *If* $(\mathcal{L} \times \mathcal{M}, \vee, \wedge, \preceq)$ *is a relatively complemented modular modular soft rough lattice. Then* $(\mathcal{L}, \vee, \wedge, \preceq)$ *and* $(\mathcal{M}, \vee, \wedge, \preceq)$ *are relatively complemented modular soft rough lattice.*

Proof. Let $(\mathcal{L}\times\mathcal{M}, \vee, \wedge, \preceq)$ is relatively complemented modular soft rough lattice and also $(\mathcal{L}\times\mathcal{M}, \vee, \wedge, \preceq)$ is bounded soft rough lattice. Let $[(S_r(X_1), S_r(X_2)), (S_r(Y_1), S_r(Y_2))]$ be any interval in $\mathcal{L}\times\mathcal{M}$. Then by the Note 4.11 $\Rightarrow (\mathcal{L} \times \mathcal{M}., \vee, \wedge, \preceq)$ is complemented modular soft rough lattice. $\therefore$ For an element $(S_r(X), S_r(Y)) \in [(S_r(\phi), S_r(\phi)), (S_r(U), S_r(U))]$ there exists a complement $(S_r^c(X), S_r^c(Y))$ relative to $[(S_r(\phi), S_r(\phi)), (S_r(U), S_r(U))]$. So, $(S_r(X), S_r(Y)) \wedge (S_r^c(X), S_r^c(Y)) = (S_r(\phi), S_r(\phi))$ and

$$(S_r(X), S_r(Y)) \vee (S_r^c(X), S_r^c(Y)) = (S_r(U), S_r(U)) \tag{3}$$

Let $[S_r(X_1), S_r(X_2)]$ and $[S_r(Y_1), S_r(Y_2)]$ be any interval in $(\mathcal{L}, \vee, \wedge, \preceq)$ and $(\mathcal{M}, \vee, \wedge, \preceq)$. Let $S_r(X) \in [S_r(X_1), S_r(X_2)], S_r(Y) \in [S_r(X_1), S_r(X_2)]$ be any elements in $\mathcal{L}$ and $\mathcal{M}$. Then $S_r(X_1) \preceq S_r(X) \preceq S_r(X_2)$ and $S_r(Y_1) \preceq S_r(X) \preceq S_r(Y_2)$. From (3) we have, $(S_r(X)\wedge S_r^c(X), S_r(Y)\wedge S_r^c(Y)) = (S_r(\phi), S_r(\phi))$ and $(S_r(X)\vee S_r^c(X), S_r(Y)\vee S_r^c(Y)) = (S_r(U), S_r(U)) \Rightarrow S_r(X) \wedge S_r^c(X) = S_r(\phi)$, $S_r(Y) \wedge S_r^c(Y)) = S_r(\phi)$ and $S_r(X) \vee S_r^c(X) = S_r(U)$, $S_r(Y) \vee S_r^c(Y)) = S_r(U)$. Let $S_r(X') \in (\mathcal{L}, \vee, \wedge, \preceq)$ where $S_r(X') \in [S_r(X_1), S_r(X_2)]$. Take $S_r(Y) = S_r(X_1) \vee (S_r(X_2) \wedge S_r^c(X))$

$$\begin{aligned}
S_r(X) \wedge S_r(X') &= S_r(X) \wedge (S_r(X_1) \vee (S_r(X_2) \wedge S_r^c(X))) \\
&= S_r(X_1) \vee (S_r(X) \wedge (S_r(X_2) \wedge S_r^c(X))) \\
&= S_r(X_1) \vee (S_r(X) \wedge S_r^c(X) \wedge (S_r(X_2)) \\
&= S_r(X_1) \vee (S_r(\phi) \wedge S_r(X_2)) \\
&= S_r(X_1) \vee S_r(\phi) \\
&= S_r(X_1). \\
S_r(X) \vee S_r(X') &= S_r(X) \vee (S_r(X_1) \vee (S_r(X_2) \wedge S_r^c(X))) \\
&= (S_r(X) \vee S_r(X_1)) \vee (S_r(X_2) \wedge S_r^c(X)) \\
&= S_r(X) \vee (S_r(X_2) \wedge S_r^c(X)) \\
&= S_r(X_2) \wedge (S_r(X) \vee S_r^c(X)) \\
&= S_r(X_2) \wedge S_r(U) \\
&= S_r(X_2).
\end{aligned}$$

$\therefore$ $S_r(X')$ is the soft rough relative complement of $S_r(X)$. $\Rightarrow [S_r(X_1), S_r(X_2)]$ is soft rough complemented. Similarly every interval of $(\mathcal{L}, \vee, \wedge, \preceq)$ is soft rough complemented. $\therefore$ $(\mathcal{L}, \vee, \wedge, \preceq)$ is a relatively complemented lattice. Let $S_r(Y') \in (\mathcal{M}, \vee, \wedge, \preceq)$ where $S_r(Y') \in [S_r(Y_1), S_r(Y_2)]$. Take $S_r(Y') = S_r(Y_1) \vee (S_r(Y_2) \wedge S_r^c(Y))$

$$\begin{aligned}
S_r(Y) \wedge S_r(Y') &= S_r(Y) \wedge (S_r(Y_1) \vee (S_r(Y_2) \wedge S_r^c(Y))) \\
&= S_r(Y_1) \vee (S_r(Y) \wedge (S_r(Y_2) \wedge S_r^c(Y))) \\
&= S_r(Y_1) \vee (S_r(Y) \wedge S_r^c(Y) \wedge (S_r(Y_2)) \\
&= S_r(Y_1) \vee (S_r(\phi) \wedge S_r(Y_2)) \\
&= S_r(Y_1) \vee S_r(\phi) \\
&= S_r(Y_1).
\end{aligned}$$

$$\begin{aligned} S_r(Y) \vee S_r(Y') &= S_r(Y) \vee (S_r(Y_1) \vee (S_r(Y_2) \wedge S_r^c(Y))) \\ &= (S_r(Y) \vee S_r(Y_1)) \vee (S_r(Y_2) \wedge S_r^c(y)) \\ &= S_r(Y) \vee (S_r(Y_2) \wedge S_r^c(Y)) \\ &= S_r(Y_2) \wedge (S_r(Y) \vee S_r^c(Y)) \\ &= S_r(Y_2) \wedge S_r(U) \\ &= S_r(Y_2). \end{aligned}$$

$\therefore S_r(Y')$ is the soft rough relative complement of $S_r(Y)$. $\Rightarrow [S_r(Y_1), S_r(Y_2)]$ is soft rough complemented. Similarly every interval of $\mathcal{M}$ is soft rough complemented. $\therefore (\mathcal{M},\vee,\wedge,\preceq)$ is a relatively complemented modular soft rough lattice. □

5. Conclusion

The complements can be found for soft sets as well as rough sets. In this paper, we have first defined the complements of soft rough set and also the complements and relative complements of soft rough lattice. We have discussed some theorems related to the complements and relative complements of modular soft rough lattice with an example. We are studying about these modular soft rough lattices and are expected to give some more results in our future study.

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