Int. J. Math. And Appl., 9(2)(2021), 141–144
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Two Variable Prime Number Generating Polynomial

Surya Prakash Singh[1,*]

1 ISRO Propulsion Complex, Mahendragiri, Tirunelveli, Tamilnadu, India.

Abstract: Prime generating polynomials gives prime numbers after plugging integer values in the polynomial. Over the years many polynomials have been put forward which can generate prime numbers, and all such papers deals with polynomial having single variable. This paper is about a quadratic polynomial in two variable which can generate a group of 714 prime numbers. Out of 714 prime numbers 287 numbers are distinct.

MSC: 11N05, 11N32

Keywords: Euler Prime series, Prime generating polynomial, Prime numbers.

1. Introduction

"For centuries there has been a fascination with prime producing quadratic polynomials" [1]. The most famous one is the Euler Prime generating polynomial: n^2+n+41 [2] which can generate 40 distinct prime numbers. This polynomial is a single variable polynomial. There are other single variable higher order prime generating polynomial like $\frac{1}{4}(n^5-133n^4+6729n^3-158379n^2+1720294n-6823316)$ discovered by Dress and Landreau [3], Gupta [4] which can generate 57 consecutive prime number. Goldbach [5] have shown that there are no prime number series in single variable with integer coefficient which can generate infinitely many primes. In this paper we are putting forward a prime generating polynomial of two variables which can generate a group of 714 prime numbers.

2. The Prime Producing Polynomial

It was noticed that if we plug integer values of a and b in the polynomial $(a^2+a+41)b^2+(2a+1)b+1$, we will get prime numbers for the successive values of a and b. The first part of the polynomial i,e a^2+a+41 is actually Euler prime generating polynomial.

3. Results

Primes numbers generated by the polynomial $(a^2+a+41)b^2+(2a+1)b+1$ is shown in Figure 1, horizontal axis in the figure is from $a=-39$ to 38 and vertical axis in the figure is from $b=-20$ to 20. Each grid boxes represents value of polynomial for the corresponding a and b, a green grid box represents prime and white represents a composite number. At $b=0$ we will get value of polynomial as 1, but in the figure the number 1 is also considered as a prime.

* *E-mail: surya912003@gmail.com*

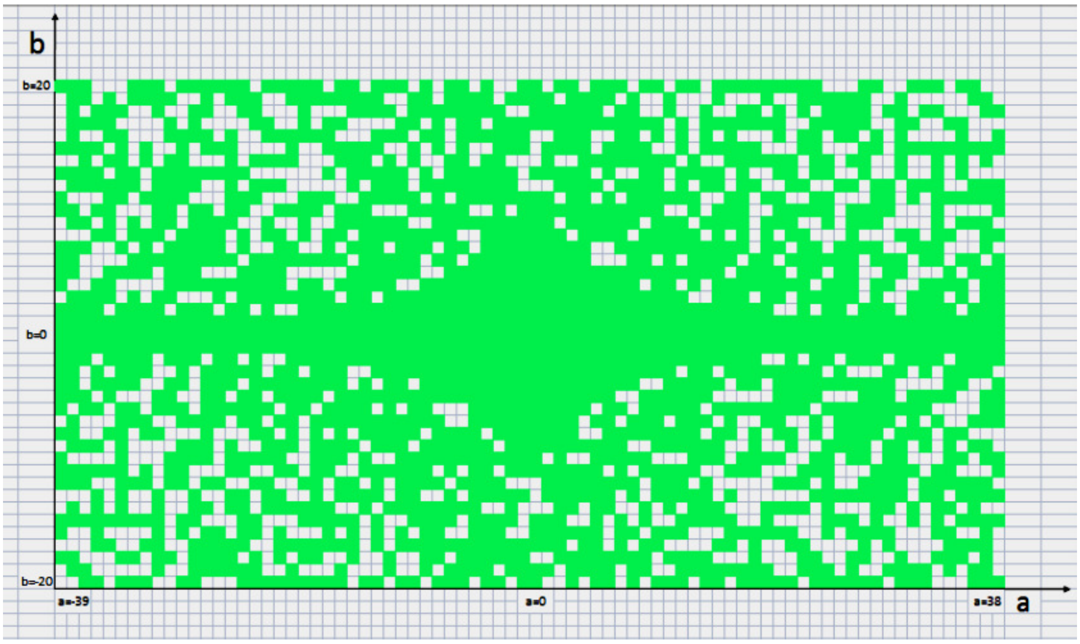

Figure 1. **Primes from the polynomial** $(a^2 + a + 41)b^2 + (2a + 1)b + 1$

The big green blob at the center of figure all prime numbers, this figure also shows that the polynomial is having very high density of prime numbers.

Counting of total number of Primes in the green blob of figure 1:

Counting of total number of Prime in the group is carried out by plugging value of a from -39 to 38, and plugging value of b starting from $b = \pm 1$ till the outcome of polynomial are Prime Numbers. Table I contains primes numbers generated by the polynomial.

Table 1: Table for successive Prime numbers generate by the polynomial $(a^2 + a + 41)b^2 + (2a + 1)b + 1$.

S. No.	a	b (Except $b = 0$)	Prime Numbers after plugging values of a and b (*marked numbers are counted only once throughout the table and are distinct)	Total Number of Primes	Number of distinct Primes
1	-39	-2 to 6	6247*, 1601*, 1447, 5939, 13477, 24061, 37691, 54367	8	2
2	-38	-3 to 5	13249*, 5939*, 1523*, 1373, 5639, 12799, 22853, 35801	8	3
3	-37	-3 to 2	12577*, 5639*, 1447*, 1301, 5347	5	3
4	-36	-4 to 1	21101*, 11923*, 5347*, 1373*, 1231	5	4
5	-35	-1 to 2	1301*, 1163, 4787	3	1
6	-34	-2 to 4	4787*, 1231*, 1097, 4519, 10267, 18341	6	2
7	-33	-3 to 4	10069*, 4519*, 1163*, 1033, 4259, 9679, 17293	7	3
8	-32	-2 to 3	4259*, 1097*, 971, 4007, 9109	5	2
9	-31	-3 to 1	8923*, 4007*, 1033*, 911	4	3
10	-30	-1 to 2	971*, 853, 3527	3	1
11	-29	-2 to 4	3527*, 911*, 797, 3299, 7507, 13421	6	2
12	-28	-2 to 2	3299*, 853*, 743, 3079	4	2
13	-27	-2 to 1	3079*, 797*, 691	3	2
14	-26	-1 to 7	743*, 641, 2663, 6067, 10853, 17021, 24571, 33503	8	1
15	-25	-2 to 4	2663*, 691*, 593, 2467, 5623, 10061	6	2
16	-24	-5 to 1	15061*, 9677*, 5479*, 2467*, 641*, 547	6	5
17	-23	-1 to 8	593*, 503, 2099, 4789, 8573, 13451, 19423, 26489, 34649	9	1
18	-22	-6 to 1	18367*, 12791*, 8221*, 4657*, 2099*, 547*, 461	7	6
19	-21	-1 to 1	503*, 421	2	1
20	-20	-1 to 9	461*, 383, 1607, 3673, 6581, 10331, 14923, 20357, 26633, 33751	10	1
21	-19	-4 to 2	6277*, 3559*, 1607*, 421*, 347, 1459	6	4
22	-18	-4 to 5	5693*, 3229*, 1459*, 383*, 313, 1319, 3019, 5413, 8501	9	4
23	-17	-3 to 4	2917*, 1319*, 347*, 281, 1187, 2719, 4877	7	3
24	-16	-2 to 10	1187*, 313*, 251, 1063, 2437, 4373, 6871, 9931, 13553, 17737, 22483, 27791	12	2

Continuation of Table I.					
S. No.	a	b (Except $b = 0$)	Prime Numbers after plugging values of a and b (*marked numbers are counted only once throughout the table and are distinct)	Total Number of Primes	Number of distinct Primes
25	-15	-5 to 2	6421*, 4133*, 2347*, 1063*, 281*, 223, 947	7	5
26	-14	-11 to 2	27281*, 22571*, 18307*, 14489*, 11117*, 8191*, 5711*, 3677*, 2089*, 947*, 251*, 197, 839	13	11
27	-13	-2 to 3	839*, 223*, 173, 739, 1699	5	2
28	-12	-6 to 9	6367*, 4441*, 2861*, 1627*, 739*, 197*, 151, 647, 1489, 2677, 4211, 6091, 8317, 10889, 13807	15	6
29	-11	-3 to 5	1423*, 647*, 173*, 131, 563, 1297, 2333, 3671	8	3
30	-10	-3 to 3	1237*, 563*, 151*, 113, 487, 1123	6	3
31	-9	-4 to 7	1877*, 1069*, 487*, 131*, 97, 419, 967, 1741, 2741, 3967, 5419	11	4
32	-8	-4 to 5	1613*, 919*, 419*, 113*, 83, 359, 829, 1493, 2351	9	4
33	-7	-10 to 5	8431*, 6841*, 5417*, 4159*, 3067*, 2141*, 1381*, 787*, 359*, 97*, 71, 307, 709, 1277, 2011	15	10
34	-6	-5 to 5	1831*, 1181*, 673*, 307*, 83*, 61, 263, 607, 1093, 1721	10	5
35	-5	-6 to 19	2251*, 1571*, 1013*, 577*, 263*, 71*, 53, 227, 523, 941, 1481, 2143, 2927, 3833, 4861, 6011, 7283, 8677, 10193, 11831, 13591, 15473, 17477, 19603, 21851	25	6
36	-4	-9 to 7	4357*, 3449*, 2647*, 1951*, 1361*, 877*, 499*, 227*, 61*, 47, 199, 457, 821, 1291, 1867, 2549	16	9
37	-3	-18 to 8	15319*, 13669*, 12113*, 10651*, 9283*, 8009*, 6829*, 5743*, 4751*, 3853*, 3049*, 2339*, 1723*, 1201*, 773*, 439*, 199*, 53*, 43, 179, 409, 733, 1151, 1663, 2269, 2969	26	18
38	-2	-9 to 11	3511*, 2777*, 2129*, 1567*, 1091*, 701*, 397*, 179*, 47*, 41, 167, 379, 677, 1061, 1531, 2087, 2729, 3457, 4271, 5171	20	9
39	-1	-11 to 11	4973*, 4111*, 3331*, 2633*, 2017*, 1483*, 1031*, 661*, 373*, 167*, 43*, 41, 163, 367, 653, 1021, 1471, 2003, 2617, 3313, 4091, 4951	22	11
40	0	-11 to 11	4951*, 4091*, 3313*, 2617*, 2003*, 1471*, 1021*, 653*, 367*, 163*, 41*, 43, 167, 373, 661, 1031, 1483, 2017, 2633, 3331, 4111, 4973	22	11
41	1	-11 to 9	5171*, 4271*, 3457*, 2729*, 2087*, 1531*, 1061*, 677*, 379*, 167, 41, 47, 179, 397, 701, 1091, 1567, 2129, 2777, 3511	20	9
42	2	-8 to 18	2969*, 2269*, 1663*, 1151*, 733*, 409*, 179, 43, 53, 199, 439, 773, 1201, 1723, 2339, 3049, 3853, 4751, 5743, 6829, 8009, 9283, 10651, 12113, 13669, 15319	26	6
43	3	-7 to 9	2549*, 1867*, 1291*, 821*, 457*, 199, 47, 61, 227, 499, 877, 1361, 1951, 2647, 3449, 4357	16	5
44	4	-19 to 6	21851*, 19603*, 17477*, 15473*, 13591*, 11831*, 10193*, 8677*, 7283*, 6011*, 4861*, 3833*, 2927*, 2143*, 1481*, 941*, 523*, 227, 53, 71, 263, 577, 1013, 1571, 2251	25	17
45	5	-5 to 5	1721*, 1093*, 607*, 263, 61, 83, 307, 673, 1181, 1831	10	3
46	6	-5 to 10	2011*, 1277*, 709*, 307*, 71*, 97, 359, 787, 1381, 2141, 3067, 4159, 5417, 6841, 8431	15	3
47	7	-5 to 4	2351*, 1493*, 829*, 359, 83, 113, 419, 919, 1613	9	3
48	8	-7 to 4	5419*, 3967*, 2741*, 1741*, 967*, 419, 97, 131, 487, 1069, 1877	11	5
49	9	-3 to 3	1123*, 487, 113, 151, 563, 1237	6	1
50	10	-5 to 3	3671*, 2333*, 1297*, 563, 131, 173, 647, 1423	8	3
51	11	-9 to 6	13807*, 10889*, 8317*, 6091*, 4211*, 2677*, 1489*, 647, 151, 197, 739, 1627, 2861, 4441, 6367	15	7
52	12	-3 to 2	1699*, 739, 173, 223, 839	5	1
53	13	-2 to 11	839, 197, 251, 947, 2089, 3677, 5711, 8191, 11117, 14489, 18307, 22571, 27281	13	0
54	14	-2 to 5	947, 223, 281, 1063, 2347, 4133, 6421	7	0
55	15	-10 to 2	27791*, 22483*, 17737*, 13553*, 9931*, 6871*, 4373*, 2437*, 1063, 251, 313, 1187	12	8
56	16	-4 to 3	4877*, 2719*, 1187, 281, 347, 1319, 2917	7	2
57	17	-5 to 4	8501*, 5413*, 3019*, 1319, 313, 383, 1459, 3229, 5693	9	3
58	18	-2 to 4	1459, 347, 421, 1607, 3559, 6277	6	0
59	19	-9 to 1	33751*, 26633*, 20357*, 14923*, 10331*, 6581*, 3673*, 1607, 383, 461	10	7
60	20	-1 to 1	421, 503	2	0
61	21	-1 to 6	461, 547, 2099, 4657, 8221, 12791, 18367	7	0
62	22	-8 to 1	34649*, 26489*, 19423*, 13451*, 8573*, 4789*, 2099, 503, 593	9	6
63	23	-1 to 5	547, 641, 2467, 5479, 9677, 15061	6	0
64	24	-4 to 2	10061*, 5623*, 2467, 593, 691, 2663	6	2
65	25	-7 to 1	33503*, 24571*, 17021*, 10853*, 6067*, 2663, 641, 743	8	5
66	26	-1 to 2	691, 797, 3079	3	0
67	27	-2 to 2	3079, 743, 853, 3299	4	0
68	28	-4 to 2	13421*, 7507*, 3299, 797, 911, 3527	6	2
69	29	-2 to 1	3527, 853, 971	3	0
70	30	-1 to 3	911, 1033, 4007, 8923	4	0

Continuation of Table I.					
S. No.	a	b (Except $b = 0$)	Prime Numbers after plugging values of a and b (*marked numbers are counted only once throughout the table and are distinct)	Total Number of Primes	Number of distinct Primes
71	31	-3 to 2	9109*, 4007, 971, 1097, 4259	5	1
72	32	-4 to 3	17293*, 9679*, 4259, 1033, 1163, 4519, 10069	7	2
73	33	-4 to 2	18341*, 10267*, 4519*, 1097*, 1231, 4787	6	2
74	34	-2 to 1	4787, 1163, 1301	3	0
75	35	-1 to 4	1231, 1373, 5347, 11923, 21101	5	0
76	36	-2 to 3	5347, 1301, 1447, 5639, 12577	5	0
77	37	-5 to 3	35801*, 22853*, 12799*, 5639, 1373, 1523, 5939, 13249	8	3
78	38	-6 to 2	54367*, 37691*, 24061*, 13477*, 5939, 1447, 1601, 6247	8	4
			Total	714	287

4. Conclusion

The two variable quadratic polynomial $(a^2 + a + 41)b^2 + (2a + 1)b + 1$ is having very high density of prime numbers. This polynomial has generated continuously 714 prime numbers and out of which 287 numbers are distinct.

Acknowledgement

I would like to thank Shri. N Murugesan, Group Head, SR-CSS, Dr. DP Sudhakar, Group Director, SRG and members of Academic Selection Committee, ISRO Propulsion complex, Mahendragiri, India for reviewing the paper and approving this for further processing for publishing.

References

[1] R. Mollin, *Quadratic polynomials producing consecutive, distinct primes and class groups of complex quadratic fields*, Acta Arithmetica, 74(1996) 17–30.

[2] L. Euler, *Extrait d'un lettre de m. euler le pere 'a m. bernoulli concernant le m ́emoire imprim ́e parmi ceux de 1771*, 318(1774).

[3] L. Bernard, *Polyn\ ˆ omes de degr\ 'e sup\ 'erieur\a 2 prenant beaucoup de valeurs premi\eres*, arXiv preprint arXiv:1402.7312, (2014).

[4] J. Maloth, *Prime Number Generating Polynomial* $3n^2+3n+23$, International Journal of Mathematics And its Application, 4(1-C)(2016) 149-150.

[5] E. W. Weisstein, *Prime-generating polynomial*, https://mathworld.wolfram.com, (2005).