International Journal of Mathematics And its Applications
Volume 5, Issue 1–C (2017), 349–357.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

# Some Properties of Semi $^{\#}$ Generalized Open Sets in Topological Spaces

**Research Article**

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**Abstract:** In this paper a new class of generalized open sets in topological spaces, namely semi $^{\#}$ generalized open(briefly, $s^{\#}$g-open) sets is introduced. We prove that this class lies between the class of sg-open sets and the class of gs-open sets and we study some basic properties and characterizations of $s^{\#}$g-open sets. Also we introduce $s^{\#}$g-neighbourhood (shortly, $s^{\#}$-neighbourhood), $s^{\#}$g-interior, $s^{\#}$g-closure and $s^{\#}$g-derived set of a set in a topological spaces and investigate some basic properties of these sets.

**MSC:** 54A05.

**Keywords:** $s^{\#}$g-open set, $s^{\#}$g-neighbourhood, $s^{\#}$g-closure, $s^{\#}$g-interior, $s^{\#}$g-derived set.



## 1. Introduction

N. Levine initiated the study of generalized closed sets in topological spaces in [8]. Biswas [5], Njastad [15], Mashhour [12], Robert [17], Bhattacharya [4], Arya and Nour [1], Maki, Devi and Balachandran [10, 11], Sheik John [20], Pushpalatha and Anitha [17], Gnanachandra et.al [7], Veerakumar [21] introduced and investigated semi closed, $\alpha$-open and $\alpha$-closed, pre-open, semi*-open, sg-closed, gs-closed, gp-closed, $\alpha$g-closed, g*s-closed, s*g-closed, w-closed, g*-closed respectively. The authors [18] have recently introduced semi $^{\#}$generalized closed sets. In this paper we introduce a new class of sets called s$^{\#}$g-open sets which is properly placed in between the class of gs-open sets and the class of sg-open sets. We give characterizations of s$^{\#}$g-open sets also investigate many fundamental properties of s$^{\#}$g-open set. We introduce s$^{\#}$g-neighbourhood, s$^{\#}$g-closure, s$^{\#}$g-interior, s$^{\#}$g-derived set and study some basic properties of these sets.

## 2. Preliminaries

Throughout this paper, $(X,\tau)$ represents a topological space on which no separation axiom is assumed unless otherwise mentioned. $(X,\tau)$ will be replaced by X if there is no chance of confusion. For a subset A of a topological space X, cl(A) and int(A) denote the closure of A and the interior of A respectively. We recall the following definitions and results.

**Definition 2.1.** *Let $(X,\tau)$ be a topological space. A subset $A$ of $X$ is said to be generalized closed [8] (briefly g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is an open set in $(X,\tau)$.*

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**Definition 2.2.** *Let $(X, \tau)$ be a topological space and $A \subseteq X$. The generalized closure of A [6], denoted by cl*(A), is defined by the intersection of all g-closed sets containing A and generalized interior of A [6], denoted by int*(A), is defined by the union of all g-open sets contained in A.*

**Definition 2.3.** *Let $(X, \tau)$ be a topological space. A subset A of the space X is said to be*

*(1). semi-open [9] if $A \subseteq cl(int(A))$ and semi-closed [3] if $int(cl(A)) \subseteq A$.*

*(2). α-open [15] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.*

*(3). pre-open [16] if $A \subseteq int(cl(A))$ and pre-closed if $cl(int(A)) \subseteq A$.*

*(4). semi*-open [17] if $A \subseteq cl*(int(A))$ and semi*-closed if $int*(cl(A)) \subseteq A$.*

**Definition 2.4.** *Let $(X, \tau)$ be a topological space and $A \subseteq X$. The semi-closure of A [4], denoted by scl(A), is defined by the intersection of all semi closed sets containing A.*

**Definition 2.5.** *Let $(X, \tau)$ be a topological space. A subset A of X is said to be*

*(1). semi-generalized closed [4] (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in $(X, \tau)$.*

*(2). generalized semi-closed [1] (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X, \tau)$.*

*(3). α-generalized closed [10] (briefly αg-closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X, \tau)$.*

*(4). g*s-closed set [15] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in $(X, \tau)$.*

*(5). semi*generalized closed [7] (briefly semi*g-closed) if $s^*cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi*-open in $(X, \tau)$.*

*(6). w-closed [20] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in $(X, \tau)$.*

*(7). *g-closed [21] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is w-open in $(X, \tau)$.*

*(8). wg-closed [2] if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in $(X, \tau)$.*

*(9). wgα-closed [14] if $\alpha cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is α-open in $(X, \tau)$.*

*(10). wα-closed [3] if$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is w-open in $(X, \tau)$.*

*(11). gwα-closed [2] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is wα-open in $(X, \tau)$.*

*The complements of the above mentioned closed sets are their respective open sets.*

**Theorem 2.6** ([16]). *Arbitrary intersection of semi*-closed sets is a semi*-closed set.*

**Definition 2.7** ([18]). *A subset A of a space $(X, \tau)$ is called a semi #generalized closed set (briefly, $s^{\#}g$-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi*-open in $(X, \tau)$.*

**Remark 2.8.** *$scl(X \setminus A) = X \setminus sint(A)$.*

**Theorem 2.9** ([18]). *Let A be any $s^{\#}g$-closed set in $(X, \tau)$. If $A \subseteq B \subseteq scl(A)$, then B is also a $s^{\#}g$-closed set.*

**Theorem 2.10** ([18]). *For every element x in a space X, $X \setminus \{x\}$ is $s^{\#}g$-closed or semi*-open.*

## 3. Semi $^{\#}$generalized Open Sets

In this section we introduce the notion of s$^{\#}$g-open sets, and study some of their basic properties.

**Definition 3.1.** *A subset A of $(X, \tau)$ is said be semi $^{\#}$generalized open set if its complement $X \setminus A$ is s$^{\#}$g-closed in X. The family of all s$^{\#}$g-open sets in X is denoted by s$^{\#}$gO(X).*

**Theorem 3.2.**

*(1). Every open set is s$^{\#}$g-open.*

*(2). Every semi-open set is s$^{\#}$g-open.*

*(3). Every semi*-open set is s$^{\#}$g-open.*

**Theorem 3.3.**

*(1). Every sg-open set is s$^{\#}$g-open.*

*(2). Every s$^{\#}$g-open set is gs-open.*

**Remark 3.4.** *From the Theorem 3.3, we conclude that s$^{\#}$g-open sets properly placed in between the class of gs-closed sets and the class of sg-closed sets.*

**Theorem 3.5.**

*(1). Every w-open set is s$^{\#}$g-open.*

*(2). Every g*s-open set is s$^{\#}$g-open.*

*(3). Every semi*g-open set is s$^{\#}$g-open.*

**Remark 3.6.** *The converse of the above theorem is not true as seen from the following examples.*

**Example 3.7.** *Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b, c\}\}$. In the space $(X, \tau)$, the sets $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$ are s$^{\#}$g-open sets but not open(semi-open, semi*-open).*

**Example 3.8.** *Let $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. In the space $(X, \tau)$, the sets $\{a, b\}$, $\{b, c\}$ are s$^{\#}$g-open but not semi*g-open.*

**Example 3.9.** *Let $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. In the space $(X, \tau)$, $\{b\}$ is a gs-open set which is not s$^{\#}$g-open.*

**Example 3.10.** *Let $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. In the space $(X, \tau)$, the sets $\{b, c\}$, $\{a, b\}$ are s$^{\#}$g-open which are not w-open.*

**Example 3.11.** *Let $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a\}\}$. In the space $(X, \tau)$, the sets $\{c\}$ is s$^{\#}$g-open but not g*s- open.*

**Remark 3.12.** *The following example shows that the concept of s$^{\#}$g-open sets is independent of each of the concepts of g-open sets, wg-open sets, g*-open sets, wg$\alpha$-open sets, gw$\alpha$-open sets and $\alpha$g-open sets.*

**Example 3.13.** *Let $X = \{a, b, c\}$ and $Y = \{a, b, c, d\}$ be the topological spaces.*

*(1). Consider the topology $\tau = \{\phi, X, \{a\}, \{a, b\}\}$. Then the set $\{b\}$ is an $\alpha$g-open set but not s$^{\#}$g-open in $(X, \tau)$.*

*(2). Consider the topology $\tau = \{\phi, Y, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$. Then the sets $\{a\}$, $\{c\}$, $\{a,b\}$, $\{b,c\}$, $\{a,c\}$ are $s^{\#}g$-open sets but not $\alpha g$-open in $(Y,\tau)$.*

**Example 3.14.** *Let $X = \{a,b,c\}$ be the topological spaces with the topology $\tau = \{\phi, X, \{a\}, \{a,b\}\}$. Then the set $\{a,c\}$ is $s^{\#}g$-open set but not g-open, also the set $\{b\}$ is g-open set but not $s^{\#}g$-open in $(X,\tau)$.*

**Example 3.15.** *Let $Y = \{a,b,c,d\}$ be the topological spaces with the topology $\tau = \{\phi, Y, \{c\}, \{a,b\}, \{a,b,c\}\}$.Then the set $\{a,c\}$ is wg$\alpha$-open and gw$\alpha$-open but not $s^{\#}g$-open set also $\{c\}$, $\{a,b\}$ are $s^{\#}g$-open sets but not wg$\alpha$-open and gw$\alpha$-open.*

**Example 3.16.** *Let $X = \{a,b,c\}$ and be the topological spaces.*

*(1). Consider the topology $\tau = \{\phi, X, \{a\}, \{a,b\}\}$. Then the set $\{b\}$ is wg-open set but not $s^{\#}g$-open in $(X,\tau)$.*

*(2). Consider the topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$. Then the sets $\{b,c\}$, $\{a,c\}$ are $s^{\#}g$-open set but not wg-open in $(X,\tau)$.*

**Example 3.17.** *Let $X = \{a,b,c\}$ and $Y = \{a,b,c,d\}$ be the topological spaces.*

*(1). Consider the topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a,b\}\}$. Then the sets $\{b,c\}$, $\{a,c\}$ are $s^{\#}g$-open but not *g-open in $(X,\tau)$.*

*(2). Consider the topology $\tau = \{\phi, X, \{a\}, \{a,b\}\}$. Then the sets $\{b\}$, $\{c\}$ are *g-open but not $s^{\#}g$-open in $(X,\tau)$.*

**Theorem 3.18.** *Let (X, $\tau$) be a topological space and $A \subseteq X$. A is an $s^{\#}g$ open if and only if $F \subseteq$ sint(A), whenever $F \subseteq A$ and F is semi*-closed.*

*Proof.* **Necessity:** Let A be an s$^{\#}$g-open set in (X, $\tau$). Let $F \subseteq A$ and F is semi*-closed. Then X\A is s$^{\#}$g-closed and it is contained in the semi*-open set X\F. Therefore scl(X\A) $\subseteq$ X $\setminus$ F. This implies that X $\setminus$ sint(A) $\subseteq$ X $\setminus$ F. Hence F $\subseteq$ sint(A).

**Sufficiency:** If F is semi*-closed set such that F $\subseteq$ sint(A) whenever F $\subseteq$ A. It follows that X $\setminus$ A $\subseteq$ X $\setminus$ F and X $\setminus$ sint (A) $\subseteq$ X $\setminus$ F. Therefore scl(X $\setminus$ A) $\subseteq$ X $\setminus$ F. Hence X $\setminus$ A is s$^{\#}$g-closed and hence A is s$^{\#}$g-open. □

**Theorem 3.19.** *If a set A is $s^{\#}g$-open and $B \subseteq X$ such that sint(A) $\subseteq B \subseteq A$, then B is $s^{\#}g$-open.*

*Proof.* If sint(A) $\subseteq$ B $\subseteq$ A then, X $\setminus$ A $\subseteq$ X $\setminus$ B $\subseteq$ X $\setminus$ sint(A). That is, X\A $\subseteq$ X $\setminus$ B $\subseteq$ scl(X $\setminus$ A). Observe X $\setminus$ A is s$^{\#}$g-closed. Therefore by Theorem 2.10, X $\setminus$ B is s$^{\#}$g-closed and hence B is s$^{\#}$g-open. □

**Theorem 3.20.** *If $A \subseteq X$ is $s^{\#}g$-closed, then scl(A) $-$ A is $s^{\#}g$-open.*

*Proof.* Let A be s$^{\#}$g-closed. Then by Theorem 4.1[18], scl(A) $-$ A contains no non-empty semi*-closed set. Thus $\phi$ is the only semi*-closed set contained in scl(A)\A. Hence by Theorem 3.12, scl(A) $-$ A is s$^{\#}$g-open. □

**Remark 3.21.**

*(1). Intersection of any two $s^{\#}g$-open sets need not be $s^{\#}g$-open.*

*(2). Union of any two $s^{\#}g$-open sets need not be $s^{\#}g$-open, as seen from the following example.*

**Example 3.22.** *Consider the topological spaces $(X,\tau)$ where $X = \{a,\ b,\ c\}$ and $\tau = \{\phi,\ X,\ \{a\}\}$. The sets $\{b\}$ and $\{c\}$ are $s^{\#}g$-open. But their union $\{b,\ c\}$ is not $s^{\#}g$-open.*

**Example 3.23.** *Consider the topological spaces $(X,\tau)$ where $X = \{a,\ b,\ c\}$ and $\tau = \{\phi,\ X,\ \{a\},\ \{b\},\ \{a,\ b\}\}$. The sets $\{b,\ c\}$ and $\{a,\ c\}$ are $s^{\#}g$-open sets. But their intersection $\{c\}$ is not $s^{\#}g$-open.*

**Theorem 3.24.** *Every singleton set in a topological space is either $s^{\#}g$-open or semi*-closed.*

*Proof.* Let X be a topological space. Let x ∈ X. Then by Theorem 2.11, $X - \{x\}$ is either s#g-closed or semi*-open. Hence {x} is either s#g-open or semi*-closed. □

## 4. s#g-neighborhoods and s#g-limit Points

In this section we define the notions of s#g-neighborhood, s#g-limit point and s#g-derived set of a set and discuss some of their basic properties and analogous to those for open sets.

**Definition 4.1.** *Let X be a topological space and let $x \in X$. A subset N of X is said to be an $s^{\#}g$-neighborhood (shortly, $s^{\#}g$-neighbourhood) of x iff there exists an $s^{\#}g$-open set U such that $x \in U \subseteq N$.*

**Definition 4.2.** *A subset N of a space X, is called a $s^{\#}g$-neighbourhood of $A \subset X$ iff there exists an $s^{\#}g$-open set U such that $A \subseteq U \subseteq N$.*

**Theorem 4.3.** *Every neighbourhood N of $x \in X$ is an $s^{\#}g$-neighbourhood of x.*

*Proof.* Let N be a neighbourhood of point $x \in X$. Then there exists an open set U such that $x \in U \subseteq N$. Since every open set is s#g-open, U is an s#g-open set such that $x \in U \subseteq N$. This implies N is an s#g-neighbourhood of x. □

**Remark 4.4.** *The converse of the above theorem is not true as seen from the following example.*

**Example 4.5.** *Let X = {a, b, c, d} with topology $\tau = \{\phi, \{c\}, \{a,b\}, \{a, b, c\}, X\}$. In this topological space $(X, \tau)$, $s^{\#}gO(X) = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b,c\}, \{a, b, d\}, \{b, c\ d\}\}$. The set {b, d} is the $s^{\#}g$-neighbourhood of b, since {b} is $s^{\#}g$-open set such that $b \in \{b\} \subset \{b, d\}$. However, the set {b, d} is not a neighbourhood of the point b.*

**Remark 4.6.** *Every $s^{\#}g$-open set is a $s^{\#}g$-neighbourhood of each of its points.*

**Remark 4.7.** *The converse of the above theorem need not be true in general as seen from the following example.*

**Example 4.8.** *Let X = {a, b, c, d} with the topology $\tau = \{\phi, \{a, b\}, \{a, b, d\}, X\}$. In this topological spaces $s^{\#}gO(X) = \{\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X\}$. The set {b, c} is a $s^{\#}g$-neighbourhood of c. However {b, c} is not $s^{\#}g$-open.*

**Theorem 4.9.** *If F is an $s^{\#}g$-closed subset of X and $x \in X \backslash F$, then there exists an $s^{\#}g$-neighbourhood N of x such that $N \cap F = \phi$.*

*Proof.* Let F be s#g-closed subset of X and $x \in F^c$. Then $F^c$ is s#g-open set of X. By Theorem 4.6, $F^c$ contains a s#g-neighbourhood of each of its points. Hence there exists a s#g-neighbourhood N of x such that $N \subseteq F^c$. Hence $N \cap F = \phi$. □

**Definition 4.10.** *The collection of all $s^{\#}g$-neighborhoods of $x \in X$ is called the $s^{\#}g$-neighborhood system at x and is denoted by $s^{\#}g$-N(x).*

**Theorem 4.11.** *Let $(X, \tau)$ be a topological space and $x \in X$. Then*

*(1). $s^{\#}g$-N(x) $\neq \phi$ and $x \in$ each member of $s^{\#}g$-N(x)*

*(2). If $N \in s^{\#}g$-N(x) and $N \subseteq M$, then $M \in s^{\#}g$-N(x).*

*(3). Each member $N \in s^{\#}g$-$N(x)$ is a superset of a member $G \in s^{\#}g$-$N(x)$ where $G$ is a $s^{\#}g$-open set.*

*Proof.*

(1). Since X is $s^{\#}$g-open set containing x, it is an $s^{\#}$g-neighbourhood of every $x \in X$. Thus for each $x \in X$, there exists atleast one $s^{\#}$g-neighbourhood, namely X. That is, $s^{\#}$g-N(x) $\neq \phi$. Let N$\in s^{\#}$g-N(x). Then N is a $s^{\#}$g-neighbourhood of x. Hence there exists a $s^{\#}$g-open set G such that $x \in G \subseteq N$, so $x \in N$. Therefore $p \in$ every member N of $s^{\#}$g-N(x).

(2). If $N \in s^{\#}$g-N(x), then there is an $s^{\#}$g-open set G such that $x \in G \subseteq N$. Since $N \subseteq M$, M is $s^{\#}$g-neighbourhood of x. Hence $M \in s^{\#}$g-N(x).

(3). Let $N \in s^{\#}$g-N(x). Then there is an $s^{\#}$g-open set G, such that $x \in G \subseteq N$. Since G is $s^{\#}$g-open and $x \in G$, G is $s^{\#}$g-neighbourhood of x. Therefore $G \in s^{\#}$g-N(x) and also $G \subseteq N$.

□

**Definition 4.12.** *Let $(X, \tau)$ be a topological space and $A$ be a subset of $X$. Then a point $x \in X$ is called a $s^{\#}g$-limit point of $A$ iff every $s^{\#}g$-neighbourhood of $x$ contains a point of $A$ disjoint from $x$. That is, $A \cap (N \backslash \{x\}) \neq \phi$ for each $s^{\#}g$-neighbourhood $N$ of $x$. Equivalently iff every $s^{\#}g$-open set $G$ containing $x$ intersects $A$ at a point other than $x$.*

**Definition 4.13.** *In a topological space $(X, \tau)$ the set of all $s^{\#}g$-limit points of a given subset $A$ of $X$ is called the $s^{\#}g$-derived set of $A$ and is denoted by $s^{\#}g$-$d(A)$.*

**Theorem 4.14.** *Let $A$ and $B$ be subset of a topological space $(X, \tau)$. Then*

*(1). $s^{\#}g\text{-}d(\phi) = \phi$*

*(2). If $A \subseteq B$, then $s^{\#}g\text{-}d(A) \subseteq s^{\#}g\text{-}d(B)$*

*(3). If $x \in s^{\#}g\text{-}d(A)$, then $x \in s^{\#}g\text{-}d(A \setminus \{x\})$*

*(4). $s^{\#}g\text{-}d(A \cup B) \supseteq s^{\#}g\text{-}d(A) \cup s^{\#}g\text{-}d(B)$*

*(5). $s^{\#}g\text{-}d(A \cap B) \subseteq s^{\#}g\text{-}d(A) \cap s^{\#}g\text{-}d(B)$.*

*Proof.* (1) and (2) is trivial.

(3) If $x \in s^{\#}$g-d(A), then by Definition 4.12 every $s^{\#}$g-open set G containing x contains atleast one point of A other than x. Hence x is $s^{\#}$g-limit point of $A \setminus \{x\}$ and it belongs to $s^{\#}$g-d$[A \setminus \{x\}]$. Therefore $x \in s^{\#}$g-d(A) implies $x \in s^{\#}$g-d$[A \setminus \{x\}]$.

(4) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, $s^{\#}$g-d$(A \cup B) \supseteq s^{\#}$g-d(A) $\cup\, s^{\#}$g-d(B).

(5) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$ and by (ii) $s^{\#}$g-d$(A \cap B) \subseteq s^{\#}$g-d(A) and $s^{\#}$g-d$(A \cap B) \subseteq s^{\#}$g-d(B). Consequently $s^{\#}$g-d$(A \cap B) \subseteq s^{\#}$g-d(A) $\cap\, s^{\#}$g-d(B). □

**Theorem 4.15.** *Let $(X, \tau)$ be a topological space and $A$ be a subset of $X$. If $A$ is $s^{\#}g$-closed, then $s^{\#}g$-$d(A) \subseteq A$.*

*Proof.* Let A be a $s^{\#}$g-closed. If $x \notin A$, then $X \backslash A$ is $s^{\#}$g-open set containing x which does not intersect A. Therefore x $\notin s^{\#}$g-d(A). □

# 5. $s^{\#}$g-closure and $s^{\#}$g-interior

In this section we define the notions of $s^{\#}$g-closure and $s^{\#}$g-interior and discuss some of their basic properties.

**Definition 5.1.** *Let $(X, \tau)$ be a topological space. Let $A \subseteq X$. Then*

*(1). The union of all $s^{\#}$g-open sets of X contained in A is called the $s^{\#}$g-interior of A and is denoted by $s^{\#}$g-int(A).*

*(2). The intersection of all $s^{\#}$g-closed sets of X containing A is called the $s^{\#}$g-closure of A and is denoted by $s^{\#}$g-cl(A).*

**Remark 5.2.** *Since intersection of $s^{\#}$g-closed sets need not be $s^{\#}$g-closed , $s^{\#}$g-cl(A) is not necessarily a $s^{\#}$g-closed set.*

**Example 5.3.** *Let $X = \{a,b,c,d\}$, $\tau = \{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,b,c\}, X\}$. Then the sets $A = \{b,c,d\}$ and $B = \{a,b,d\}$ are $s^{\#}$g -closed sets. But $A \cap B = \{b,d\}$ is not $s^{\#}$g-closed. Consequently , $s^{\#}$g-cl($\{b\}$) $= \{b\}$ is not $s^{\#}$g-closed.*

**Theorem 5.4.** *For a subset A of $(X, \tau)$ and $x \in X$, $x \in s^{\#}$g-cl(A) if and only if $V \cap A \neq \phi$ for every $s^{\#}$g-open set V containing x.*

*Proof.* Let A $\subseteq$ X and x $\in$ X. Let us prove the contra positive. Suppose that there exists an $s^{\#}$g-open set V containing x such that V $\cap$ A = $\phi$. Since A$\subset$ X $\setminus$ V, $s^{\#}$g-cl(A) $\subset$ X $\setminus$ V and then x $\notin$ $s^{\#}$g-cl(A). Conversely, suppose x $\notin$ $s^{\#}$g-cl(A). Then there exists an $s^{\#}$g -closed set F containing A such that x$\notin$ F. Since x $\in$ X $\setminus$ F and X $\setminus$ F is $s^{\#}$g-open, (X $\setminus$ F) $\cap$ A = $\phi$. That is, there exists an $s^{\#}$g-open set X$\setminus$F containing x such that (X $\setminus$ F) $\cap$ A = $\phi$. Hence x$\in$ $s^{\#}$g-cl(A) if and only if V $\cap$ A $\neq \phi$ for every $s^{\#}$g-open set V containing x. □

**Theorem 5.5.** *Let A and B be two subsets of a topological space $(X, \tau)$. Then the followings are hold*

*(1). $s^{\#}$g-cl(X) $= X$ and $s^{\#}$g-cl($\phi$) $= \phi$*

*(2). $A \subseteq s^{\#}$g-cl(A)*

*(3). If $A \subseteq B$, then $s^{\#}$g-cl(A) $\subseteq s^{\#}$g-cl(B)*

*(4). $x \in s^{\#}$g-cl(A) iff for each a $s^{\#}$g-open set U containing x, $U \cap A \neq \phi$*

*(5). If A is $s^{\#}$g closed set then $A = s^{\#}$g-cl(A)*

*(6). $s^{\#}$g-cl(A) $\subseteq s^{\#}$g-cl($s^{\#}$g-cl(A)).*

*(7). $s^{\#}$g-cl(A) $\cup$ $s^{\#}$g-cl(B) $\subseteq s^{\#}$g-cl(A$\cup$B)*

*(8). $s^{\#}$g-cl($A \cap B$) $\subseteq s^{\#}$g-cl(A) $\cap$ $s^{\#}$g-cl(B).*

**Theorem 5.6.** *Let A and B be two subsets of a topological space $(X, \tau)$. Then the followings are hold*

*(1). $s^{\#}$g-int(X) $= X$ and $s^{\#}$g-int($\phi$) $= \phi$*

*(2). $s^{\#}$g-int(A) $\subseteq A$*

*(3). If $A \subseteq B$, then $s^{\#}$g-int(A) $\subseteq s^{\#}$g-int(B)*

*(4). $x \in s^{\#}$g-int(A) iff for each a $s^{\#}$g-closed set U containing x, $U \cap A \neq \phi$*

*(5). If A is $s^{\#}$g-open set then $A = s^{\#}$g-int(A)*

*(6).* $s^{\#}g\text{-}int(s^{\#}g\text{-}int(A)) \subseteq s^{\#}g\text{-}int(A)$

*(7).* $s^{\#}g\text{-}int(A \cap B) \supseteq s^{\#}g\text{-}int(A) \cap s^{\#}g\text{-}int(B)$

*(8).* $s^{\#}g\text{-}int(A) \cup s^{\#}g\text{-}int(B) \subseteq s^{\#}g\text{-}int(A \cup B)$.

**Remark 5.7.** *By the following example we show that the inclusion relation in the theorem (above two) parts (7) & (8) cannot be replaced by equality.*

**Example 5.8.** *Let* $X = \{a, b, c, d\}$ *with* $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$. *Here* $s^{\#}g$*-closed sets are* $\{\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. *Consider* $A = \{c, d\}$ *and* $B = \{a, b, d\}$ *so that* $s^{\#}g\text{-}cl(A) = \{c, d\}$ *and* $s^{\#}g\text{-}cl(B) = X$. *Therefore* $s^{\#}g\text{-}cl(A) \cap s^{\#}g\text{-}cl(B) = \{c, d\}$. *Also* $A \cap B = \{d\}$ *and* $s^{\#}g\text{-}cl(A \cap B) = \{d\}$. *Hence* $s^{\#}g\text{-}cl(A \cap B) \subsetneq s^{\#}g\text{-}cl(A) \cap s^{\#}g\text{-}cl(B)$.

**Remark 5.9.** *The following example shows that for any two subsets A and B of X,* $A \subsetneq B$ *implies* $s^{\#}g\text{-}cl(A) = s^{\#}g\text{-}cl(B)$.

**Example 5.10.** *Let* $X = \{a, b, c, d\}$ *with* $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$. *In this topological spaces,* $s^{\#}g$*-closed sets are* $\{\phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. *Consider* $A = \{a, b\}$ *and* $B = \{a, b, d\}$ *so that* ${}^{\#}g\text{-}cl(A) = \{a, b, d\} = s^{\#}g\text{-}cl(B)$, *whenever* $A \subsetneq B$.

**Theorem 5.11.** *If* $A \subseteq X$, *then* $A \subseteq s^{\#}g\text{-}cl(A) \subseteq s^{*}cl(A) \subseteq cl(A)$.

*Proof.* Since every closed set is $s^{\#}$g-closed, the proof follows. □

**Remark 5.12.** *The inclusions in the theorem (above) may be proper as seen from the following example.*

**Example 5.13.** *Let* $X = \{a, b, c, d\}$ *with* $\tau = \{\phi, \{a\}, \{a, b, c\}, X\}$. *Let* $A = \{a, b\}$. *Then* $s^{\#}g\text{-}cl(A) = \{a, b, d\}$ & $cl(A) = X$ *and so* $A \subsetneq s^{\#}g\text{-}cl(A) \subsetneq cl(A)$.

**Theorem 5.14.** *For a subset A in a topological space* $(X, \tau)$, *the following statements are true.*

*(1).* $s^{\#}g\text{-}cl(X \backslash A) = X \backslash s^{\#}g\text{-}int(A)$

*(2).* $s^{\#}g\text{-}int(X \backslash A) = X \backslash s^{\#}g\text{-}cl(A)$

*Proof.*

(1). Let $x \in X \backslash s^{\#}g\text{-}int(A)$. Then $x \notin s^{\#}g\text{-}int(A)$. This implies that x does not belong to any $s^{\#}$g-open subset of A. Let F be any $s^{\#}$g-closed set containing $X \backslash A$. Then $X \backslash F$ is a $s^{\#}$g-open set contained in A. Therefore $x \notin X \backslash F$ and so $x \in F$. Hence $x \in s^{\#}g\text{-}cl(X \backslash A)$. Hence $X \backslash s^{\#}g\text{-}int(A) \subseteq s^{\#}g\text{-}cl(X \backslash A)$.

Conversely, let $x \in s^{\#}g\text{-}cl(X \backslash A)$. Then x belongs to every $s^{\#}$g-closed set containing $X \backslash A$. Hence x does not belong to any $s^{\#}$g-open subset of A. Therefore $x \notin s^{\#}g\text{-}int(X \backslash A)$. Hence $x \in X \backslash s^{\#}g\text{-}int(A)$. Thus $s^{\#}g\text{-}cl(X \backslash A) = X \backslash s^{\#}g\text{-}int(A)$.

(2). can be proved from (i) by replacing A by $X \backslash A$.

□

## 6. Conclusion

In this paper, $s^{\#}$g-open sets, $s^{\#}$g-closure, $s^{\#}$g-interior, $s^{\#}$g-neighborhood, $s^{\#}$g-derived sets are newly defined. This study can be extended to other topological spaces in future.

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