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# On $\beta^* - I_g-$closed Sets in Ideal Topological Spaces

**Research Article**

**Jessie Theodore[1]* and V.Beulah[1]**

1 Department of Mathematics, Sarah Tucker College, Tirunelveli, Tamilnadu, India.

**Abstract:** In this paper some related generalized sets of $\tau^*$ namely $\beta^* - I_g-$closed sets, $I_{gsp}-$closed sets, $\omega - I-$closed sets, $\widehat{\eta}^* - I_g-$closed sets in ideal topological space are introduced the relationship between these sets and some other already existing sets are investigated.

**Keywords:** $I-$open, $\omega - I-$open, semi$-$pre$-I-$open, $\beta^* - I_g-$closed sets, $I_{gsp}-$closed sets, $\omega - I-$closed sets, $\widehat{\eta}^* - I_g-$closed sets.


## 1. Introduction

The notion of generalized closed sets in Ideal topological spaces was studied by J.Dontchev et al [8] in 1999. Further closed sets like $_{rg}$I-closed set, $I_{rw}$-closed set were developed by M. Navaneethakrishnan [24] and A. Vadivel [29] in 2009 and 2013 respectively. The main aim of this paper is to introduce some new related closed sets in the same space and study the relationships between them.

## 2. Preliminaries

An ideal topological space is a topological space $(X, \tau)$ with an ideal I on X, and is denoted by $(X, \tau, I)$. Given a topological space $(X, \tau)$ with an ideal I on X and if P(X) is the set of all subsets of X, a set operator $(\cdot)^* : P(X) \to P(X)$, called a local function [19] of A with respect to $\tau$ and I is defined as follows: for $A \subset X$, $A^*(I, \tau) = \{x \in X / U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \,|x \in U\}$ When there is no chance for confusion $A^*(I, \tau)$ is denoted by $A^*$. For every ideal topological space $(X, \tau, I)$, there exists a topology $\tau^*$ finer than $\tau$, generated by the base $\beta(I, \tau) = \{U \backslash I / U \in \tau \text{ and } I \in I\}$. In general $\beta(I, \tau)$ is not always a topology. We will make use of the basic facts about the local functions [17, Theorem 2.3] without mentioning it explicitly. A Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(I, \tau)$, called the $^*$-topology, finer than $\tau$ is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [30]. When there is no chance for confusion, we will simply write $A^*$ for $A^*(I, \tau)$ and $\tau^*$ or $\tau^*(I)$ for $\tau^*(I, \tau)$. If I is an ideal on X, then $(X, \tau, I)$ is called an ideal space. I is said to be codense [9] if $\tau \cap I = \{\phi\}$. If $A \subset X$, $cl(A)$ and $int(A)$ will, respectively, denote the closure and interior of A in (X, $\tau$) and cl*(A) and int *(A) will, respectively, denote the closure and interior of A in (X, $\tau$*). A subset A of a space (X, $\tau$) is an $\alpha$-open [26] (resp. semi-open [20], pre-open [22], $\beta$-open or semi-pre-open [3]) set if A⊂int(cl(int(A))) (resp. A ⊂ cl(int(A)), A ⊂ int(cl(A)), A ⊂ cl(int(cl(A)))). The complement of an $\alpha$-open [16] (resp. semi-open [11], pre-open [13], $\beta$-open or semi-pre-open [2])

* E-mail: jessielawrence2703@gmail.com

set is $\alpha$-closed (resp. semi-closed, pre-closed, $\beta$-closed or semi-pre-closed). The semi closure [7] of a subset A of X, denoted by sclA is defined to be the intersection of all semi closed sets containing A. The semi-pre closure [3] of a subset A of X, denoted by spclA is defined to be the intersection of all semi-pre-closed sets containing A. A subset A of an ideal space (X, $\tau$, I) is said to be I-open [9] if A $\subset$ int(A*), A subset A of an ideal space (X, $\tau$, I) is *-closed [16] (resp. *-dense in itself [13], *-perfect [13]) if A*$\subset$A (resp. A $\subset$ A*, A = A*). Clearly, A is *perfect if and only if A is *-closed and *-dense in itself.

**Definition 2.1.** *A subset A of a topological space $(X, \tau)$ is said to be*

*(1) a generalized closed set [21] (g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*

*(2) a generalized semi-closed set [5] (gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*

*(3) a generalized semi-pre-closed set [5] (gsp-closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.*

*(4) a $\omega$-closed set [15] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semiopen.*

*(5) an $\widehat{\eta}^*$-closed set [27] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\omega$-open.*

*(6) a $\beta$*-closed set [4] if $spcl(A) \subseteq intU$ whenever $A \subseteq U$ and U is $\omega$-open.*

**Definition 2.2.** *A subset A of an ideal space $(X, \tau, I)$ is called pre-I-open [10] (resp. $\alpha$-I-open [12], semi-I-open [12], semi-pre-I-open [13]) if $A \subseteq int(cl^*(A))$ (resp. $A \subseteq int(cl^*(int(A)))$, $A \subseteq cl^*(int(A))$, $A \subseteq cl^*(int(cl^*(A)))$. The complement of an pre-I-open (resp. $\alpha$-I-open semi-I-open, semi-pre-I-open) set is pre-I-closed (resp. $\alpha$-I-closed ,semi-I-closed, semi-pre-I-closed). A subset A of a space (X, $\tau$) is an I-open [1] if $A \subseteq int(A^*)$ and rI-open [18] if A =int(cl*(A))*

**Definition 2.3.** *A subset A of an ideal space (X, $\tau$, I) is said to be*

*(1) $I_g$-closed set [8] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is open.*

*(2) ${}_{sg}I$-closed set [6] if $scl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-I-open.*

*(3) $I_{*g}$-closed set [26] if $A^* \subseteq U$ whenever $A \subseteq U$ and U is $\omega$-open.*

*(4) ${}_{rg}I$-closed set if [24] $cl^*A \subseteq U$ whenever $A \subseteq U$ and U is rI-open.*

*(5) pre generalized pre regular I-closed set [30] (${}_{pgpr}I$-closed) if $pcl^*A \subseteq U$ whenever $A \subseteq U$ and U is ${}_{rg}I$-open.*

*(6) regular pre semi I-closed set [28] (${}_{rps}I$-closed) if $spcl^*A \subseteq U$ whenever $A \subseteq U$ and U is ${}_{rg}I$-open.*

**Lemma 2.4.** *Let (X, $\tau$, I) be an ideal topological space and $A \subseteq X$. If $A \subseteq A^*$, then $A^* = Cl(A^*) = Cl(A) = Cl^*(A)$ [17, Theorem 5].*

**Definition 2.5.** *A subset A of an ideal space (X, $\tau$, I) is called $\omega$-I-closed set if $A^* \subseteq U$ whenever $A \subseteq U$ and U is semi-I-open.*

**Definition 2.6.** *A subset A of an ideal space (X, $\tau$, I) is called $\widehat{\eta}^* - I_g$-closed set if $spcl^*A \subseteq U$ whenever $A \subseteq U$ and U is $\omega$-I-open.*

**Definition 2.7.** *A subset A of an ideal space (X,$\tau$, I) is called $I_{gsp}$-closed set if $spcl^*A \subseteq U$ whenever $A \subseteq U$ and U is I-open.*

**Definition 2.8.** *A space X is called a $sp\omega^*$- space if the intersection of every semi-pre-I-closed set of X with every $\omega$-I-closed set of X is $\omega$-I-closed.*

## 3. $\beta^* - I_g$-closed Sets

**Definition 3.1.** *A subset A of an ideal space (X,$\tau$, I) is called $\beta * - I_g -$closed set if $spcl^* A \subseteq int^* U$ whenever A$\subseteq$U and U is $\omega$-I-open.*

**Theorem 3.2.** *A set A is $\omega$-I-open iff $F \subseteq int^* A$ whenever F is semi-I-closed and F$\subseteq$A.*

*Proof.* Suppose that A is $\omega$-I-open, F$\subseteq$A and F is semi-I-closed. Then $A^C \subseteq F^C$, therefore $(X - A) * \subseteq F^C$, $cl^* (X - A) \subseteq F^C$ and so $F \subseteq (cl^* (X - A))^C = int^* A$. □

**Theorem 3.3.** *Every *-closed set is $\beta^* - I_g -$closed set but not conversely.*

*Proof.* Let A be a *-closed set, then A*$\subseteq$A. Let A$\subseteq$U where U is $\omega$-I-open. Then $cl^* A \subseteq U$ which implies $scl^* A \subseteq U$. Then by theorem 3.2 $scl^* A \subseteq int^* U$ so $spcl^* A \subseteq int^* U$. □

**Example 3.4.** *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, X, \{a\}, \{b, c\}\}$ and the ideal $I = \{\phi, \{c\}\}$. It is clear that A=\{b\} is $\beta^* - I_g -$closed set but A is not a $^*$-closed set.*

**Theorem 3.5.** *Every semi-I-closed set and hence I-closed and $\alpha$-I-closed sets are $\beta^* - I_g -$closed set but not conversely.*

*Proof.* Let A $\subseteq$U be a semi-I-closed set where U is $\omega$-I-open set. Then $scl^* A = A \subseteq U$ which implies $scl^* A \subseteq int^* U$. Therefore $spcl^* A \subseteq scl^* A \subseteq int^* U$. □

**Example 3.6.** *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, X, \{c\}\}$ and the ideal $I = \{\phi\}$. Clearly $A = \{a, c\}$ is $\beta^* - I_g -$closed set but A is not a semi-I- closed set.*

**Theorem 3.7.** *Every semi-closed set and hence closed set and $\alpha$-closed sets are is $\beta^* - I_g -$closed set but not conversely.*

*Proof.* Let A be a semi-closed set. Since every semi-closed set is a semi-I-closed set and follows from Theorem 3.6; A is $\beta^* - I_g -$closed set. □

**Example 3.8.** *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, X, \{c\}\}$ and the ideal $I = \{\phi\}$. Clearly $A = \{a, c\}$ is $\beta^* - I_g -$closed set but A is not a semi-closed set.*

**Remark 3.9.** *$\beta * - I_g -$closed set and $\beta*$-closed sets are independent of each other, as seen from the following examples.*

**Example 3.10.** *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and the ideal $I = \{\phi, \{a\}\}$. It is clear that $A = \{a\}$ is a $\beta^* - I_g -$closed set but A is not a $\beta^*$-closed set.*

**Example 3.11.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. Clearly, the set A=\{a, b, d\}is $\beta^*$-closed set but A is not a $\beta^* - I_g -$closed set.*

**Remark 3.12.** *$\beta * - I_g -$closed set and $I_g$-closed sets are independent of each other, as seen from the following examples.*

**Example 3.13.** *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and the ideal $I = \{\phi, \{a\}\}$. It is clear that $A = \{c\}$ is a $\beta^* - I_g -$closed set but A is not a $I_g$-closed set.*

**Example 3.14.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. Clearly, the set A=\{a, b, d\} is $I_g$-closed set but A is not a $\beta * - I_g -$closed set.*

**Remark 3.15.** *$\beta^* - I_g -$closed set and $I_{*g}$-closed sets are independent to each other, as seen from the following examples.*

**Example 3.16.** *Let $X = \{a, b, c\}$ with the topology $\tau = \{\phi, X, \{a\}, \{a, c\}\}$ and the ideal $I = \{\phi, \{a\}\}$. It is clear that $A = \{c\}$ is a $\beta^* - I_g-$closed set but A is not a $I_{*g}$-closed set.*

**Example 3.17.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. Clearly, the set A={a, b, d} is $I_{*g}$-closed set but A is not a $\beta^* - I_g-$closed set.*

**Remark 3.18.** *$\beta^* - I_g-$closed set and semi-pre-I-closed sets are independent to each other, as seen from the following example.*

**Example 3.19.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. It is clear that $A_1 = \{a\}$ is semi-pre-I-closed set but not a $\beta^* - I_g-$closed set. Also $A_2 = \{a, b, c\}$ is $\beta^* - I_g-$closed set but not a semi-pre-I-closed set.*

**Remark 3.20.** *$\beta^* - I_g-$closed set and pre-I-closed sets are independent to each other, as seen from the following example.*

**Example 3.21.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. We see that $A_1 = \{a\}$ is pre-I-closed set but not $\beta * -I_g-$closed set. Also $A_2 = \{a, b, c\}$ is $\beta^* - I_g-$closed set but not pre-I-closed set.*

**Theorem 3.22.** *Every $\beta^* - I_g-$closed set is $\widehat{\eta}^*$-$I_g$-closed set but not conversely.*

*Proof.* Let A be a $\beta^* - I_g-$closed set. Then $spcl^*A \subseteq int^*U$ whenever A⊂U and U is $\omega$-I-open which implies $spcl^*A \subseteq U$ whenever A⊂U and U is $\omega$-I-open. Hence A is an $\widehat{\eta}^*$-$I_g$-closed set. □

**Example 3.23.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. It is clear that $A = \{a\}$ is a $\widehat{\eta}^*$-$I_g$-closed set but A is not an $\beta^* - I_g-$closed set.*

**Theorem 3.24.** *Every $\beta^* - I_g-$closed set is $I_{gsp}$-closed set but not conversely.*

*Proof.* Let A⊂U be a $\beta^* - I_g-$closed set and U is I-open. Since every I-open set is $\omega$-I-open, we have $spcl^*A \subseteq int^*U$ and therefore $spcl^*A \subseteq U$. Hence A is $I_{gsp}$-closed set. □

**Remark 3.25.** *$\beta^* - I_g-$closed set and $_{rg}I$-closed sets are independent to each other, as seen from the following examples.*

**Example 3.26.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. It is clear that $A = \{a\}$ is a $_{rg}I$-closed set but A is not a $\beta^* - I_g-$closed set.*

**Example 3.27.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and the ideal $I = \{\phi, \{a\}\}$. It is clear that $A = \{c\}$ is a $\beta^* - I_g-$closed set but A is not a $_{rg}I$ -closed set.*

**Remark 3.28.** *$\beta^* - I_g-$closed set and $_{pgpr}I$-closed sets are independent to each other, as seen from the following examples.*

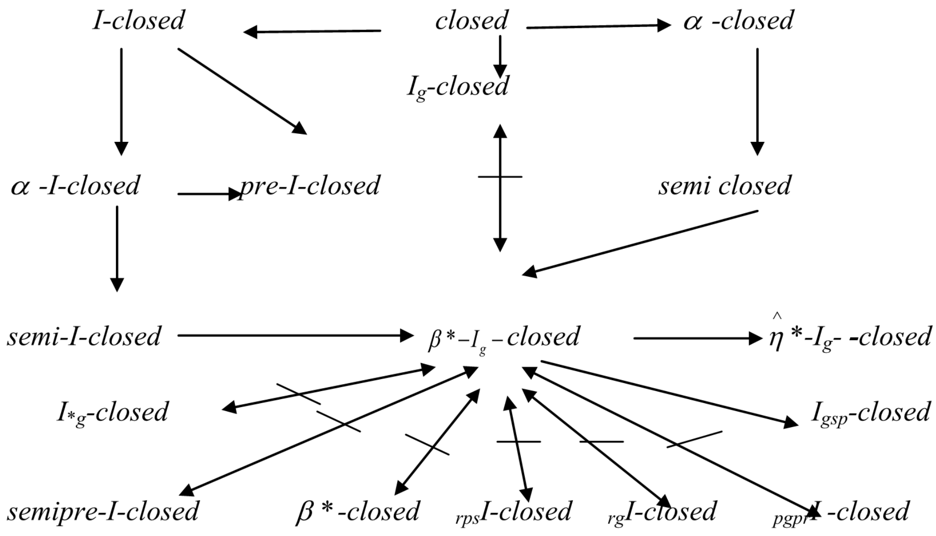
**Example 3.29.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. It is clear that $A = \{a\}$ is a $_{pgpr}I$-closed set but A is not a $\beta^* - I_g-$closed set.*

**Example 3.30.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and the ideal $I = \{\phi, \{a\}\}$. It is clear that $A = \{b, c\}$ is a $\beta^* - I_g-$closed set but A is not a $_{pgpr}I$ -closed set.*

**Remark 3.31.** *$\beta^* - I_g-$closed set and $_{rps}I$-closed sets are independent to each other, as seen from the following examples.*

**Example 3.32.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a, b\}\}$ and the ideal $I = \{\phi, \{c\}\}$. It is clear that $A = \{a\}$ is a $_{rps}I$-closed set but A is not a $\beta^* - I_g-$closed set.*

**Example 3.33.** *Let $X = \{a, b, c, d\}$ with the topology $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and the ideal $I = \{\phi, \{a\}\}$. It is clear that $A = \{b, c\}$ is a $\beta^* - I_g-$closed set but A is not a $_{rps}I$ -closed set.*

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