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Soft g-closed Sets in Soft Biminimal Spaces

Research Article

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Abstract: The purpose of this paper is to introduce a new class of sets called soft g-closed sets in soft biminimal spaces and its basic properties are investigated.

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1. Introduction

V.Popa and T.Noiri [14] introduced the concept of minimal structure (briefly m-structure). They also introduced the notion of m_X-open set and m_X-closed set and characterize those sets using m_X-closure and m_X-interior operators respectively. C. Boonpok [1] introduced the concept of biminimal structure space and studied $m^1_X m^2_X$-open sets and $m^1_X m^2_X$-closed sets in biminimal structure spaces. R. Gowri and S. Vembu [3] introduced the concept of soft minimal and soft biminimal spaces. In [4] R. Gowri and S. Vembu introduced soft generalized closed sets in soft minimal spaces. C. Viriyapong [15] et.al introduced the concept of generalized m-closed sets in biminimal structure spaces and we obtain some properties of generalized m-closed sets. The concept of soft sets was introduced by Molodtsov [8] in 1999 as a general mathematical tool for dealing with uncertain objects. Later, he applied this theory to several directions (see [9] [10] [11]). In this paper, we introduce soft g-closed sets in soft biminimal spaces which are defined over an initial universe with a fixed set of parameters.

2. Preliminaries

Definition 2.1 ([8])**.** *Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a nonempty subset of E. A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E\}$, where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here, f_A is called approximate function of the soft set F_A. The value of $f_A(x)$ may be arbitrary, some of them may be empty, some may have non empty intersection.*

Note that the set of all soft sets over U will be denoted by S(U).

Definition 2.2 ([3])**.** *Let X be an initial universe set, E be the set of parameters and $A \subseteq E$. Let F_A be a non empty soft set over X and $\tilde{P}(F_A)$ is the soft power set of F_A. A subfamily $\tilde{m}$ of $\tilde{P}(F_A)$ is called a soft minimal set over X if $F_\emptyset \in \tilde{m}$*

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and $F_A \in \tilde{m}$. $(F_A, \tilde{m})$ or $(X, \tilde{m}, E)$ is called a soft minimal space over X. Each member of $\tilde{m}$ is said to be $\tilde{m}$-soft open set and the complement of an $\tilde{m}$-soft open set is said to be $\tilde{m}$-soft closed set over X.

Definition 2.3 ([3]). *Let X be an initial universe set, E be the set of parameters and $A \subseteq E$.Let F_A be a non empty soft set over X. Let $(F_A, \tilde{m_1})$ and $(F_A, \tilde{m_2})$ be the two different soft minimals over X. Then $(X, \tilde{m_1}, \tilde{m_2}, E)$ or $(F_A, \tilde{m_1}, \tilde{m_2})$ is called a soft biminimal spaces.*

Example 2.4. *Let $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. Then*

$$
\begin{aligned}
F_{A_1} &= \{(x_1, \{u_1\})\}, & F_{A_2} &= \{(x_1, \{u_2\})\}, \\
F_{A_3} &= \{(x_1, \{u_1, u_2\})\}, & F_{A_4} &= \{(x_2, \{u_1\})\}, \\
F_{A_5} &= \{(x_2, \{u_2\})\}, & F_{A_6} &= \{(x_2, \{u_1, u_2\})\}, \\
F_{A_7} &= \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, & F_{A_8} &= \{(x_1, \{u_1\}), (x_2, \{u_2\})\}, \\
F_{A_9} &= \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}, & F_{A_{10}} &= \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, \\
F_{A_{11}} &= \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, & F_{A_{12}} &= \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\} \\
F_{A_{13}} &= \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}, & F_{A_{14}} &= \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, \\
F_{A_{15}} &= F_A, & F_{A_{16}} &= F_\emptyset
\end{aligned}
$$

are all soft subsets of F_A. Here, soft biminimals are

$$
\begin{aligned}
\tilde{m_1} &= \{F_\emptyset, F_{A_2}, F_{A_5}, F_{A_7}, F_{A_{11}}, F_A\} \quad \text{and} \\
\tilde{m_2} &= \{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_6}, F_{A_{10}}, F_A\}.
\end{aligned}
$$

Definition 2.5 ([3]). *Let F_A be a non empty soft set and $\tilde{m}$ be soft minimal over X. For a soft subset F_B of F_A, the $\tilde{m}$-soft closure of F_B and $\tilde{m}$-soft interior of F_B are defined as follows:*

$$
\begin{aligned}
\tilde{m}Cl(F_B) &= \cap \left\{F_\alpha : F_B \tilde{\subseteq} F_\alpha, F_A - F_\alpha \in \tilde{m}\right\}, \\
\tilde{m}Int(F_B) &= \cup \left\{F_\beta : F_\beta \tilde{\subseteq} F_B, F_\beta \in \tilde{m}\right\}.
\end{aligned}
$$

Lemma 2.6 ([3]). *Let F_A be a non empty soft set and $\tilde{m}$ be soft minimal over X. For a soft subset F_B, $F_C \tilde{\subseteq} F_A$, the following properties hold:*

(1). $\tilde{m}Cl(F_A - F_B) = F_A - (\tilde{m}Int(F_B))$ and $\tilde{m}Int(F_A - F_B) = F_A - (\tilde{m}Cl(F_B))$,

(2). If $(F_A - F_B) \in \tilde{m}$, then $\tilde{m}Cl(F_B) = F_B$ and if $F_B \in \tilde{m}$, then $\tilde{m}Int(F_B) = F_B$,

(3). $\tilde{m}Cl(F_\emptyset) = F_\emptyset$, $\tilde{m}Cl(F_A) = F_A$, $\tilde{m}Int(F_\emptyset) = F_\emptyset$ and $\tilde{m}Int(F_A) = F_A$,

(4). If $F_B \tilde{\subseteq} F_C$, then $\tilde{m}Cl(F_B) \tilde{\subseteq} \tilde{m}Cl(F_C)$ and $\tilde{m}Int(F_B) \tilde{\subseteq} \tilde{m}Int(F_C)$,

(5). $F_B \tilde{\subseteq} \tilde{m}Cl(F_B)$ and $\tilde{m}Int(F_B) \tilde{\subseteq} F_B$,

(6). $\tilde{m}Cl(\tilde{m}Cl(F_B)) = \tilde{m}Cl(F_B)$ and $\tilde{m}Int(\tilde{m}Int(F_B)) = \tilde{m}Int(F_B)$.

Lemma 2.7 ([3]). *Let F_A be a non empty soft set and $\tilde{m}$ be soft minimal over X satisfying property B. For a soft subset F_B of F_A, the following properties hold:*

(1). $F_B \in \tilde{m}$ *if and only if* $\tilde{m}Int(F_B) = F_B$,

(2). F_B *is* $\tilde{m}$*-closed if and only if* $\tilde{m}Cl(F_B) = F_B$,

(3). $\tilde{m}Int(F_B) \in \tilde{m}$ *and* $\tilde{m}Cl(F_B) \in \tilde{m}$*-closed.*

Definition 2.8 ([3])**.** *Let* F_A *be a non-empty soft set and* $\tilde{m}$ *on* X *satisfying property* B *if the union of any family of subsets belonging to* $\tilde{m}$ *belongs to* $\tilde{m}$*.*

Definition 2.9 ([3])**.** *Let* F_A *be a non-empty soft set and* $\tilde{m}$ *be soft minimal over* X *satisfying property* B *if the union of any family of subsets belonging to* $\tilde{m}$ *belongs to* $\tilde{m}$*.*

Definition 2.10 ([4])**.** *A soft subset* F_B *of a soft minimal space* $(F_A, \tilde{m})$ *is said to be soft generalized* $\tilde{m}$*-closed sets (briefly* $sg\tilde{m}$*-closed) if* $\tilde{m}Cl(F_B) \tilde{\subseteq} U_B$ *whenever* $F_B \tilde{\subseteq} U_B$ *and* U_B *is soft* $\tilde{m}$*-open set.*

3. Soft Generalized Closed Sets in Soft Biminimal Spaces

In this section, we introduce the concept of soft generalized closed sets in soft biminimal spaces and study some of their properties.

Definition 3.1. *A soft subset* F_B *of a soft biminimal space* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is called a soft generalized* $\tilde{m}_i\tilde{m}_j$*-closed sets (briefly* $sg\tilde{m}_{(i,j)}$*-closed) if* $\tilde{m}_jCl(F_B) \tilde{\subseteq} U_B$ *whenever* $F_B \tilde{\subseteq} U_B$ *and* U_B *is soft* $\tilde{m}_i$*-open, where* $i, j = 1, 2$ *and* $i \neq j$*. The complement of a* $sg\tilde{m}_{(i,j)}$*-closed set is called a* $sg\tilde{m}_{(i,j)}$*-open. The family of all* $sg\tilde{m}_{(i,j)}$*-closed(*$sg\tilde{m}_{(i,j)}$*-open) sets of* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is denoted by* $sg\tilde{m}_{(i,j)}C(F_A)$ *(reps.* $sg\tilde{m}_{(i,j)}O(F_A)$*), where* $i, j = 1, 2$ *and* $i \neq j$

Definition 3.2. *A soft subset* F_B *of a soft biminimal spaces* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is said to be pairwise* $sg\tilde{m}$*-closed (briefly* $p-sg\tilde{m}$*-closed) if* F_B *is* $sg\tilde{m}_{(1,2)}$*-closed and* $sg\tilde{m}_{(2,1)}$*-closed. The complement of a pairwise* $sg\tilde{m}$*-closed set is said to be pairwise* $sg\tilde{m}$*-open (briefly* $p - sg\tilde{m}$*-open).*

Example 3.3. *Let* $X = \{u_1, u_2\}$*,* $E = \{x_1, x_2, x_3\}$*,* $A = \{x_1, x_2\} \subseteq E$ *and* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$*. Then*

$$F_{A_1} = \{(x_1, \{u_1\})\}, \qquad F_{A_2} = \{(x_1, \{u_2\})\},$$
$$F_{A_3} = \{(x_1, \{u_1, u_2\})\}, \qquad F_{A_4} = \{(x_2, \{u_1\})\},$$
$$F_{A_5} = \{(x_2, \{u_2\})\}, \qquad F_{A_6} = \{(x_2, \{u_1, u_2\})\},$$
$$F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, \qquad F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\},$$
$$F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}, \qquad F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\},$$
$$F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, \qquad F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}$$
$$F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}, \qquad F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\},$$
$$F_{A_{15}} = F_A, \qquad F_{A_{16}} = F_\emptyset$$
$$\tilde{m}_1 = \{F_\emptyset, F_{A_6}, F_{A_7}, F_{A_{13}}, F_A\}, \qquad \tilde{m}_2 = \{F_\emptyset, F_{A_3}, F_{A_6}, F_{A_8}, F_{A_{12}}, F_A\},$$

$sg\tilde{m}_{(1,2)}C(F_A) = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_8}, F_{A_9}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}, F_A\}$.

Remark 3.4. *By setting* $\tilde{m}_1 = \tilde{m}_2$ *in Definition 3.1, a* $sg\tilde{m}_{(i,j)}$*-closed becomes* $sg\tilde{m}$*-closed set.*

Remark 3.5. *Every soft* $\tilde{m_{(i,j)}}$*-closed set is* $sg\tilde{m}_{(i,j)}$*-closed set but the converse is not true as seen from the following example.*

Example 3.6. *Let* $X = \{u_1, u_2\}$*,* $E = \{x_1, x_2, x_3\}$*,* $A = \{x_1, x_2\} \subseteq E$ *and* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$*. Then* $\tilde{m_1} = \{F_\emptyset, F_{A_3}, F_{A_4}, F_{A_{11}}, F_A\}$*,and* $\tilde{m_2} = \{F_\emptyset, F_{A_5}, F_{A_6}, F_{A_9}, F_{A_{10}}, F_A\}$*. Here* F_{A_1} *is* $sg\tilde{m}_{(1,2)}$*-closed set but not soft* $\tilde{m_{(1,2)}}$ *closed set.*

Proposition 3.7. *Let* $(F_A, \tilde{m_1}, \tilde{m_2})$ *be a soft biminimal spaces and let* F_B *be soft subset of* $(F_A, \tilde{m_1}, \tilde{m_2})$*. If* F_B *is both soft* $\tilde{m_i}$*-open and* $sgm_{(i,j)}$*-closed, then* F_B *is soft* $\tilde{m_j}$*-closed.*

Proof. Let F_B be soft $\tilde{m_i}$-open and $sgm_{(i,j)}$-closed, we have $\tilde{m}Cl(F_B) = F_B$. Hence, F_B is soft $\tilde{m_j}$-closed. □

Theorem 3.8. *Union of two* $sg\tilde{m}_{(i,j)}$*-closed sets is* $sg\tilde{m}_{(i,j)}$*-closed set.*

Proof. Suppose F_B and G_B are sg$\tilde{m}_{(i,j)}$-closed set. Let U_B be soft $\tilde{m_i}$-open set such that $F_B \cup G_B \tilde{\subseteq} U_B$. Since $F_B \cup G_B \tilde{\subseteq} U_B$, we have $F_B \tilde{\subseteq} U_B$, $G_B \tilde{\subseteq} U_B$. Since U_B is soft $\tilde{m_i}$-open and F_B and G_B are sg$\tilde{m}_{(i,j)}$-closed sets, we have $\tilde{m_j}Cl(F_B) \tilde{\subseteq} U_B$ and $\tilde{m_j}Cl(G_B) \tilde{\subseteq} U_B$. Therefore, $\tilde{m_j}Cl(F_B \cup G_B) \tilde{\subseteq} \tilde{m_j}Cl(F_B) \cup \tilde{m_j}Cl(G_B) \tilde{\subseteq} U_B$. Hence, $F_B \cup G_B$ is sg$\tilde{m}_{(i,j)}$-closed set. □

Remark 3.9. *The intersection of two* $sg\tilde{m}_{(i,j)}$*-closed sets need not be* $sg\tilde{m}_{(i,j)}$*-closed as seen from the following example*

Example 3.10. *Let* $X = \{u_1, u_2\}$*,* $E = \{x_1, x_2, x_3\}$*,* $A = \{x_1, x_2\} \subseteq E$ *and* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$*. Then* $\tilde{m_1} = \{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_7}, F_{A_{12}}, F_A\}$*, and* $\tilde{m_2} = \{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_7}, F_{A_{12}}, F_{A_{14}}, F_A\}$*. Then* $F_{A_{13}}$ *and* $F_{A_{14}}$ *are* $sg\tilde{m}_{(1,2)}$*-closed but* $F_{A_{13}} \cap F_{A_{14}} = F_{A_3}$ *is not* $sg\tilde{m}_{(1,2)}$*-closed.*

Theorem 3.11. *Let* $(F_A, \tilde{m_1}, \tilde{m_2})$ *be a soft biminimal space. If* F_B *is* $sg\tilde{m}_{(i,j)}$*-closed and* H_B *is soft* $\tilde{m_j}$*-closed, then* $F_B \cap H_B$ *is* $sg\tilde{m}_{(i,j)}$*-closed.*

Proof. Let U_B be soft $\tilde{m_i}$-open such that $F_B \cap H_B \tilde{\subseteq} U_B$. Then $F_B \tilde{\subseteq} U_B \cup (H_B)^c$ and so $\tilde{m_j}Cl(F_B) \tilde{\subseteq} U_B \cup (H_B)^c$. Therefore $\tilde{m_j}Cl(F_B) \cap H_B \tilde{\subseteq} U_B$. Since H_B is soft $\tilde{m_j}$-closed. Therefore, $\tilde{m_j}Cl(F_B \cap H_B) \tilde{\subseteq} U_B$. Hence, $F_B \cap H_B$ is sg$\tilde{m}_{(i,j)}$-closed. □

Remark 3.12. $sg\tilde{m}_{(1,2)}C(F_A)$ *is generally not equal to* $sg\tilde{m}_{(2,1)}C(F_A)$ *as can be seen from the following example.*

Example 3.13. *Let* $X = \{u_1, u_2\}$*,* $E = \{x_1, x_2, x_3\}$*,* $A = \{x_1, x_2\} \subseteq E$ *and* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$*. Then* $\tilde{m_1} = \{F_\emptyset, F_{A_3}, F_{A_4}, F_{A_{11}}, F_A\}$*, and* $\tilde{m_2} = \{F_\emptyset, F_{A_5}, F_{A_6}, F_{A_9}, F_{A_{10}}, F_A\}$*. Then* $sg\tilde{m}_{(1,2)}C(F_A) = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_8}, F_{A_9}, F_{A_{10}}, F_{A_{11}}, F_{A_{12}}, F_{A_{14}}, F_A\}$*,* $sg\tilde{m}_{(2,1)}C(F_A) = \{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_4}, F_{A_5}, F_{A_6}, F_{A_7}, F_{A_{11}}, F_{A_{12}}, F_{A_{13}}, F_{A_{14}}, F_A\}$*. Thus* $sg\tilde{m}_{(1,2)}C(F_A) \neq sg\tilde{m}_{(2,1)}C(F_A)$*.*

Remark 3.14. *Let* $\tilde{m_1}$ *and* $\tilde{m_2}$ *be two soft minimals on* F_A*. If* $\tilde{m_1} \tilde{\subseteq} \tilde{m_2}$*, then* $sg\tilde{m}_{(2,1)}C(F_A) \tilde{\subseteq} sg\tilde{m}_{(1,2)}C(F_A)$

Example 3.15. *Let* $X = \{u_1, u_2\}$*,* $E = \{x_1, x_2, x_3\}$*,* $A = \{x_1, x_2\} \subseteq E$ *and* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$*. Then* $\tilde{m_1} = \{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_7}, F_{A_{12}}, F_A\}$*,and* $\tilde{m_2} = \{F_\emptyset, F_{A_1}, F_{A_3}, F_{A_7}, F_{A_{12}}, F_{A_{14}}, F_A\}$*. Thus* $sg\tilde{m}_{(2,1)}C(F_A) \tilde{\subseteq} sg\tilde{m}_{(1,2)}C(F_A)$*.*

Theorem 3.16. *Let* F_B *be a soft subset of soft minimal space* $(F_A, \tilde{m_1}, \tilde{m_2})$*. If* F_B *be* $sg\tilde{m}_{(i,j)}$*-closed, then* $\tilde{m_j}Cl(F_B) - F_B$ *contains no non empty soft* $\tilde{m_i}$*-closed set.*

Proof. Let F_B be sg$\tilde{m}_{(i,j)}$-closed subset of $\tilde{m_j}Cl(F_B) - F_B$. Now $H_B \tilde{\subseteq} \tilde{m_j}Cl(F_B) - F_B$ and $F_B \tilde{\subseteq} (H_B)^c$ where F_B is sg$\tilde{m}_{(i,j)}$-closed and $(H_B)^c$ is soft $\tilde{m_i}$-open. Thus $\tilde{m_j}Cl(F_B) \tilde{\subseteq} (H_B)^c$ or equivalently $H_B \tilde{\subseteq} [\tilde{m_j}Cl(F_B)]^c$. By assumption, $H_B \tilde{\subseteq} [\tilde{m_j}Cl(F_B)]$ and so $H_B \tilde{\subseteq} [\tilde{m_j}Cl(F_B)] \cap [\tilde{m_j}Cl(F_B)]^c = F_\emptyset$. Therefore, $H_B = F_\emptyset$. Hence, $\tilde{m_j}Cl(F_B) - F_B$ contains no non empty soft $\tilde{m_i}$-closed set. □

Remark 3.17. *The converse of the above Theorem 3.16 is not true as seen from the following example*

Example 3.18. *Let* $X = \{u_1, u_2\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\} \subseteq E$. *Then* $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}$. *Then* $\tilde{m}_1 = \{F_\emptyset, F_{A_2}, F_{A_5}, F_{A_{10}}, F_{A_{12}}, F_A\}$ *and* $\tilde{m}_2 = \{F_\emptyset, F_{A_1}, F_{A_7}, F_{A_{13}}, F_A\}$. *Take* $F_B = F_{A_2}$. *Then* $\tilde{m}_2Cl(F_B) - F_B = \tilde{m}_2Cl(F_{A_2}) - F_{A_2} = F_{A_5}$ *which does not contain any non empty soft* $\tilde{m}_1$*-closed set. But* F_{A_2} *is not* $sg\tilde{m}_{(1,2)}$*-closed.*

Corollary 3.19. *If* F_B *is* $sg\tilde{m}_{(i,j)}$*-closed set in* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *then* F_B *is soft* $\tilde{m}_j$*-closed if and only if* $\tilde{m}_jCl(F_B) - F_B$ *is soft* $\tilde{m}_i$*-closed.*

Proof. Assume that F_B is sg$\tilde{m}_{(i,j)}$-closed set and soft $\tilde{m}_j$-closed. Then $\tilde{m}_jCl(F_B) = F_B$. That is $\tilde{m}_jCl(F_B) - F_B = F_\emptyset$ and hence $\tilde{m}_jCl(F_B) - F_B$ is soft $\tilde{m}_i$-closed. Conversely, suppose $\tilde{m}_jCl(F_B) - F_B$ is soft $\tilde{m}_i$-closed, then by Theorem 3.16, F_B is $sg\tilde{m}_{(i,j)}$-closed, then $\tilde{m}_jCl(F_B) - F_B$ contains no non empty soft $\tilde{m}_i$-closed set. That implies $\tilde{m}_jCl(F_B) - F_B = F_\emptyset$. Hence, F_B is soft $\tilde{m}_j$-closed. □

Theorem 3.20. *For each* $(x, u) \in (F_A, \tilde{m}_1, \tilde{m}_2)$*, the singleton* $\{(x, u)\}$ *is soft* $\tilde{m}_i$*-closed or* $\{(x, u)\}^c$ *is* $sg\tilde{m}_{(i,j)}$*-closed set.*

Proof. Suppose that $\{(x, u)\}$ is not soft $\tilde{m}_i$-closed, then $\{(x, u)\}^c$ is not soft $\tilde{m}_i$-open. Then F_A is the only soft $\tilde{m}_i$-open set which contains $\{(x, u)\}^c$ and $\{(x, u)\}^c$ is sg$\tilde{m}_{(i,j)}$-closed set. □

Theorem 3.21. *If* F_B *be* $sg\tilde{m}_{(i,j)}$*-closed set in* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *and* $F_B \tilde{\subseteq} G_B \tilde{\subseteq} \tilde{m}_jCl(F_B)$*, then* G_B *is also* $sg\tilde{m}_{(i,j)}$*-closed set*

Proof. Assume F_B is sg$\tilde{m}_{(i,j)}$-closed set and $F_B \tilde{\subseteq} G_B \tilde{\subseteq} \tilde{m}_jCl(F_B)$. Let $G_B \tilde{\subseteq} U_B$ and U_B is soft $\tilde{m}_i$-open. Given $F_B \tilde{\subseteq} G_B$. Then $F_B \tilde{\subseteq} U_B$. Since F_B is sg$\tilde{m}_{(i,j)}$-closed set, we have $\tilde{m}_jCl(F_B) \tilde{\subseteq} U_B$. Since $G_B \tilde{\subseteq} \tilde{m}_jCl(F_B)$, $\tilde{m}_jCl(G_B) \tilde{\subseteq} \tilde{m}_jCl(F_B) \tilde{\subseteq} U_B$. Hence G_B is sg$\tilde{m}_{(i,j)}$-closed set. □

Theorem 3.22. *A soft subset* F_B *of soft biminimal space in* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *is* $sg\tilde{m}_{(i,j)}$*-open set if and only if* $H_B \tilde{\subseteq} \tilde{m}_jInt(F_B)$ *whenever* H_B *is soft* $\tilde{m}_i$*-closed and* $H_B \tilde{\subseteq} F_B$ *where* $i, j = 1, 2$ *and* $i \neq j$*.*

Proof. Let F_B be sg$\tilde{m}_{(i,j)}$-open set. Let H_B be a soft $\tilde{m}_i$-closed set such that $H_B \tilde{\subseteq} F_B$. Let $F_B \tilde{\subseteq} H_B$ and H_B is soft $\tilde{m}_i$-closed. Then $(F_B)^c \tilde{\subseteq} (H_B)^c$ and $(H_B)^c$ is soft $\tilde{m}_i$-open, we have $(F_B)^c$ is sg$\tilde{m}_{(i,j)}$-closed. Hence, $[\tilde{m}_jCl(F_B)^c] \tilde{\subseteq} (H_B)^c$. Consequently, $[\tilde{m}_jInt(F_B)]^c \tilde{\subseteq} (H_B)^c$. Therefore, $H_B \tilde{\subseteq} \tilde{m}_jInt(F_B)$. Conversely, suppose $H_B \tilde{\subseteq} \tilde{m}_jInt(F_B)$ whenever $H_B \tilde{\subseteq} F_B$ and H_B is soft $\tilde{m}_i$-closed. Let $(F_B)^c \tilde{\subseteq} U_B$ and U_B is soft $\tilde{m}_i$-open. Then $(U_B)^c \tilde{\subseteq} F_B$ and $(U_B)^c$ is soft $\tilde{m}_i$-closed. By hypothesis $(U_B)^c \tilde{\subseteq} \tilde{m}_jInt(F_B)$. Hence, $[\tilde{m}_jInt(F_B)]^c \tilde{\subseteq} U_B$. (i.e) $[\tilde{m}_jCl(F_B)^c] \tilde{\subseteq} U_B$. Consequently, $(F_B)^c$ is sg$\tilde{m}_{(i,j)}$-closed set. Hence, F_B is sg$\tilde{m}_{(i,j)}$-open. □

Theorem 3.23. *A soft subset* F_B *is* $sg\tilde{m}_{(i,j)}$*-closed set then* $\tilde{m}_jCl(F_B) - F_B$ *is* $sg\tilde{m}_{(i,j)}$*-open set.*

Proof. Let F_B is sg$\tilde{m}_{(i,j)}$-closed set. Let $H_B \tilde{\subseteq} \tilde{m}_jCl(F_B) - F_B$ where H_B is soft $\tilde{m}_i$-closed set. Since F_B is sg$\tilde{m}_{(i,j)}$-closed, we have $\tilde{m}_jCl(F_B) - F_B$ does not contain nonempty soft $\tilde{m}_i$-closed by Theorem 3.16. Consequently, $H_B = F_\emptyset$. Therefore, $F_\emptyset \tilde{\subseteq} \tilde{m}_jCl(F_B) - F_B$, $F_\emptyset \tilde{\subseteq} \tilde{m}_jInt(\tilde{m}_jCl(F_B) - F_B)$, we obtain $H_B \tilde{\subseteq} \tilde{m}_jInt(\tilde{m}_jCl(F_B) - F_B)$. Hence, $\tilde{m}_jCl(F_B) - F_B$ is sg$\tilde{m}_{(i,j)}$-open. □

Theorem 3.24. *Let* F_B *and* G_B *be a soft subset of soft biminimal space* $(F_A, \tilde{m}_1, \tilde{m}_2)$ *such that* $\tilde{m}_jInt(F_B) \tilde{\subseteq} G_B \tilde{\subseteq} F_B$*. If* F_B *is* $sg\tilde{m}_{(i,j)}$*-open, then* G_B *is* $sg\tilde{m}_{(i,j)}$*-open where* $i, j = 1, 2$ *and* $i \neq j$*.*

Proof. Let F_B is sg$\tilde{m}_{(i,j)}$-open. Let H_B be a soft $\tilde{m}_i$-closed such that $H_B \tilde{\subseteq} G_B$. Since $H_B \tilde{\subseteq} G_B$ and $G_B \tilde{\subseteq} F_B$, we have $H_B \tilde{\subseteq} F_B$. Therefore, $H_B \tilde{\subseteq} \tilde{m}_jInt(F_B)$. Since $\tilde{m}_jInt(F_B) \tilde{\subseteq} G_B$, we have $\tilde{m}_jInt(\tilde{m}_jInt(F_B)) \tilde{\subseteq} \tilde{m}_jInt(G_B)$. Therefore, $\tilde{m}_jInt(F_B) \tilde{\subseteq} \tilde{m}_jInt(G_B)$. Consequently, $H_B \tilde{\subseteq} \tilde{m}_jInt(G_B)$. Hence, G_B is sg$\tilde{m}_{(i,j)}$-open. □

Theorem 3.25. *If F_B and G_B are two sg$\tilde{m}_{(i,j)}$-open subsets of soft biminimal space $(F_A, \tilde{m}_1, \tilde{m}_2)$, then $F_B \cap G_B$ is also sg$\tilde{m}_{(i,j)}$-open.*

Proof. Suppose H_B is soft $\tilde{m}_j$-closed set contained in $F_B \cap G_B$. Since F_B and G_B are sg$\tilde{m}_{(i,j)}$-open sets. Since, $H_B \tilde{\subseteq} F_B \cap G_B$, we have $H_B \tilde{\subseteq} F_B$ and $H_B \tilde{\subseteq} G_B$. Since F_B and G_B are two sg$\tilde{m}_{(i,j)}$-open sets, we have $H_B \tilde{\subseteq} \tilde{m}_j Int(F_B)$ and $H_B \tilde{\subseteq} \tilde{m}_j Int(G_B)$. Therefore, $H_B \tilde{\subseteq} \tilde{m}_j Int(F_B) \cap \tilde{m}_j Int(G_B) \tilde{\subseteq} \tilde{m}_j Int(F_B \cap G_B)$. Hence, $F_B \cap G_B$ is sg$\tilde{m}_{(i,j)}$-open. □

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