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Average Distance of Certain Graphs

Research Article

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Abstract: Mean distance or average distance is used in studying the efficiency of networks or more generally 'good networks' which are often characterized by small distance. It is also used as a tool in analytic networks where the performance time is proportional to the distance between any two nodes. In this paper, we compute the Wiener index and the average distance of generalised prisms, uniform n-wheel split graph, uniform n-star split graph and cyclic split graph.

MSC: 05C69.

Keywords: Wiener index, average distance, generalized prisms, uniform n-wheel split graph, uniform n-star split graph, cyclic split graph.

1. Introduction

The distance between two nodes is defined as the number of edges along the shortest path connecting them. If the two nodes are disconnected, the distance is infinity. The average distance of a graph $G = (V, E)$ with $|V| = n$, denoted by $\mu(G)$ is the expected distance between a randomly chosen pair of distinct vertices; that is,

$$\mu(G) = \frac{2W(G)}{n(n-1)}$$

where $W(G)$ is the Wiener index which is the sum of the shortest path between any two vertices of the graph G. Doyle and Graver [7] were the first to define $\mu(G)$ as a graph parameter. The study of the average distance began with the chemist Wiener [25], who noticed that the boiling point of certain hydrocarbons is proportional to the sum of all distances between unordered pairs of vertices of the corresponding graph. This sum is now called the Wiener number or Wiener index of the graph and is denoted by $W(G)$. The average distance of a graph is used for comparing the compactness of architectural plans [18].

2. Applications and Survey

The shortest path problem finds its application in various domains like preparing travel time and distance charts, in telecommunications and transportation industries where message or vehicles must be sent between two geographical locations as

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quickly or as cheaply as possible. Other examples are complex traffic flow simulations and planning tools which rely on a large number of individual shortest path problems. Further applications include many practical integer programming problems. Shortest path computations are used as subroutines in the solution procedure for computational biology(DNA) sequence alignment, VLSI design, knapsack packing problems, traveling salesman problems and for many other problems.
If we consider a network like the World Wide Web, then the short average path length facilitates the quick transfer of information and reduces costs. In 1999, Barabasi et al. [2] observed that in certain portions of the internet any two webpages are at most 19 clicks away from one another. The efficiency of mass transfer in a metabolic network can be judged by studying its average path length. A power grid network will have less losses if its average path length is minimized.
A diverse set of shortest path models and algorithms have been developed to accommodate these various applications. Average distance can be used as a tool in analytic networks where the performance time is proportional to the distance between any two nodes. It is a measure of the time needed in the average case, as opposed to the diameter, which indicates the maximum performance time [3]. Mean distance or average distance is used in studying the efficiency of networks or more generally 'good networks' which are often characterized by small distance [9].
Wiener index is used to study the relation between molecular structure, physical and chemical properties of certain hydrocarbon compounds. In the initial applications, the Wiener index is employed to predict physical parameters such as boiling points, heats of vaporization, molar volumes and more refractions of alkanes. The study of Wiener index is one of the current areas of research in Mathematical Chemistry. In theoretical computer science, Wiener index is considered as one of the basic descriptors of fixed interconnection network because it provides the average distance between any two nodes of the network [21]. The mean Wiener index is nowadays known as the average shortest-path distance and it has been instrumental in the definition of the concept of 'small-world' networks where everyone is connected to everyone else through a very short path [9]. Many works on average distance in graphs are available in the literature [1, 4–6, 8, 10–12, 20, 24, 26, 27].

3. Preliminaries

The graphs considered in this paper are finite, simple and undirected and we will use the standard graph-theoretic terminologies. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$.

Definition 3.1. *The distance $d_G(u,v)$ between two vertices $u, v \in V(G)$ is the minimum number of edges on a path in G between u and v [27].*

Definition 3.2. The average distance $\mu(G)$ [27] between the vertices of G is defined as follows:

$$\mu(G) = \frac{2W(G)}{n(n-1)}$$

where $W(G)$ is the Wiener index of a graph G.

Definition 3.3 ([16])**.** *For a graph G, let $d_G(u,v)$ be the number of edges on any shortest path joining vertex u to vertex v. The Wiener index of the graph G is defined as*

$$W(G) = \frac{1}{2}\sum_{u,v\in V(G)} d_G(u,v)$$

where the sum runs over all ordered pairs of vertices. The factor (1/2) is needed in order to count each pair exactly once. If the vertex set is linearly ordered, we can write

$$W(G) = \sum_{u,v\in V(G)} d_G(u,v)$$

3.1. Cut Method

The cut method plays a vital role in calculating the topological indices of the chemical graphs without using the distance matrix in the field of chemistry. Mathematically, it is used to calculate the shortest path between any two vertices without using the brute-force method.

Proposition 3.4 ([16]). *Let G be a connected graph. Then G admits a partition of $E(G)$ into convex cuts if and only if G is a partial cube.*

Definition 3.5 ([16]). *Let $G = (V(G), E(G))$ be a connected graph. A subgraph H of a graph G is convex if for any vertices u, v of H, any shortest path in G between u and v lies completely in H.*

Theorem 3.6 ([16]). *Let G be a partial cube. Then relation Θ partitions the edge set $E(G)$ into Θ-classes $F_1, ..., F_k$, where edges e and f lie in a common class F_i if and only if $e\Theta f$. Moreover, for any index i, the graph $G - F_i$ consists of precisely two connected components. Let $n_1(F_i)$ and $n_2(F_i)$ be the number of vertices in the two connected components of $G - F_i$. Then*

$$W(G) = \sum_{i=1}^{k} n_1(F_i) \times n_2(F_i)$$

3.2. Extended Cut Method

It is the cut method which is applied to the classes which are larger than the partial cubes. The first extension of the standard cut method beyond partial cubes are ℓ_1-graphs. In the bipartite case, ℓ_1-graphs coincide with partial cubes and hence their generalization is important in the non-bipartite case [16].

Definition 3.7 ([16]). *ℓ_1-graphs are graphs whose shortest-path metric can be isometrically embedded into an ℓ_1 -space. Let $\lambda \in N$ and let G and H be two graphs. Then H is scale λ-embeddable into G if there exists a mapping $\alpha : V(H) \to V(G)$ such that for all vertices $u, v \in V(H)$, $d_G(\alpha(u), \alpha(v)) = \lambda.d_H(u, v)$.*

3.3. Characterization of ℓ_1-graphs [16]

- A graph G is an ℓ_1-graph if and only if G is scale λ embeddable into a hypercube for some $\lambda \geq 1$.
- A graph G is an ℓ_1 -graph if and only if G admits a collection $\mathcal{C}(G)$ of (not necessarily different) convex cuts of G such that every edge of G is cut by precisely λ cuts from $\mathcal{C}(G)$).

Theorem 3.8 ([16, 19]). *Let G be a scale λ embeddable into a hypercube and let $\mathcal{C}(G)$ be the family of convex cuts defined in the embedding. Then*

$$W(G) = \frac{1}{\lambda} \sum_{\{F\} \in \mathcal{C}(G)} |n_1(F_i)| \times |n_2(F_i)|$$

where F_i is an edge cut of the graph G such that $G - F_i$ consists of two components $n_1(F_i)$ and $n_2(F_i)$, then F_i is called a convex cut if both $n_1(F_i)$ and $n_2(F_i)$ are convex subgraphs of G. $\lambda \geq 1$, denotes the collection of edges of a graph G in which each edge in G is repeated exactly λ times.

In this paper, we compute the average distance of generalised prisms $C_m \times P_n$ by the standard cut method and average distances of the uniform n-wheel split graph K_nW_r, uniform n-star split graph ST_r^n and cyclic split graph $C_nK_r^k$ by the extended cut method.

4. Average Distance of Generalized Prisms

Distance behaves nicely in cartesian products of graphs.

Definition 4.1 ([23])**.** The generalized prism can be defined as the cartesian product $C_m \times P_n$ of a cycle on m vertices with a path on n vertices. Let $V(C_m \times P_n) = \{v_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertex set and $E(C_m \times P_n) = \{v_{i,j}v_{i+1,j} : 1 \leq i \leq m-1, 1 \leq j \leq n\} \bigcup \{v_{m,j}v_{1,j} : 1 \leq j \leq n\} \bigcup \{v_{i,j}v_{i,j+1} : 1 \leq i \leq m, 1 \leq j \leq n-1\}$ be the edge set. Clearly $|V(C_m \times P_n)| = mn$ and $|E(C_m \times P_n)| = m(2n-1)$. See Figure 1.

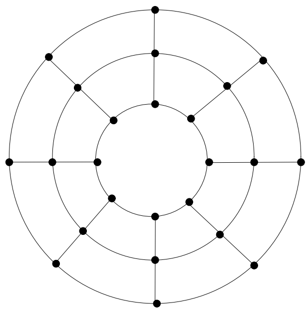

Figure 1. **The graph $C_8 \times P_3$**

Note: We restrict our proof to the case when m is even and $m > 2$.

Theorem 4.2. *The Wiener index of the generalized prism $C_m \times P_n$ is given by $W(C_m \times P_n) = \frac{m^2 n}{24}\left[3mn + 4(n^2-1)\right]$.*

Proof. Let $\{S_i : 1 \leq i \leq \frac{m}{2}\}$ and $\{S_i^{'} : 1 \leq i \leq n-1\}$ be the edge cuts of $C_m \times P_n$. See Figure 2. We observe that $\{S_i : 1 \leq i \leq \frac{m}{2}\}$ and $\{S_i^{'} : 1 \leq i \leq n-1\}$ forms a partition of the edge set of $C_m \times P_n$. For $1 \leq i \leq \frac{m}{2}$, the removal of S_i leaves $C_m \times P_n$ into two components G_{S_i} and $G_{S_i}^{'}$ where $|V(G_{S_i})| = \frac{mn}{2}$ and $|V(G_{S_i}^{'})| = \frac{mn}{2}$. For $1 \leq i \leq n-1$, the removal of $S_i^{'}$ leaves $C_m \times P_n$ into two components $G_{S_i^{'}}$ and $G_{S_i^{'}}^{'}$ where $|V(G_{S_i^{'}})| = mi$ and $|V(G_{S_i^{'}}^{'})| = m(n-i)$. Hence,

$$\begin{aligned} W(C_m \times P_n) &= \frac{m}{2}\left[\frac{m^2 n^2}{4}\right] + \sum_{i=1}^{n-1}[(mi)(m(n-i))] \\ &= \frac{m^2 n}{24}[3mn + 4(n^2-1)] \end{aligned}$$

□

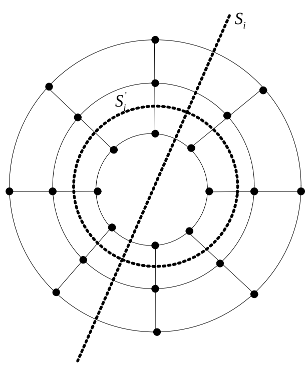

Figure 2. **Cuts of $C_8 \times P_3$**

Theorem 4.3. *The average distance of the generalized prism $C_m \times P_n$ is given by* $\mu(C_m \times P_n) = \frac{m[3mn+4(n^2-1)]}{12(mn-1)}$.

Proof. We know from Theorem 4.2. that $W(C_m \times P_n) = \frac{m^2 n}{24}[3mn + 4(n^2 - 1)]$. Hence the average distance of $C_m \times P_n$ is

$$\mu(C_m \times P_n) = \frac{m[3mn + 4(n^2 - 1)]}{12(mn - 1)}$$

□

5. Average Distance of Uniform n-wheel Split Graph $(K_n W_r)$

Definition 5.1 ([14]). *The n-wheel split graph is defined as follows: Let $u_i, 1 \leq i \leq n$ be the vertices of the complete graph graph K_n. For $1 \leq i \leq n$, let $W^i_{r+1} = C^i_r + K_1$, where r is a positive integer, be wheels with hubs w_i and let u_i be adjacent to w_i. The graph thus constructed is called an uniform n-wheel split graph and is denoted by $K_n W_r$. The number of vertices in $K_n W_r$ is $n(2+r)$, $r \geq 3$, $n \geq 3$. See Figure 3.*

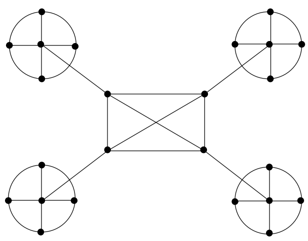

Figure 3. **The graph** $K_4 W_4$

Theorem 5.2. The Wiener index of the uniform n-wheel split graph $K_n W_r$ is given by

$$W(K_n W_r) = \frac{1}{2}\left[4n^2r^2 + 11n^2r - 2nr^2 - 10nr + 6n^2 - 5n\right]$$

Proof. Let $\{S_i : 1 \leq i \leq r\}$, $\{S_i^{'} : 1 \leq i \leq n\}$, $\{S_i^{''} : 1 \leq i \leq n\}$ be the edge cuts of $K_n W_r$. See Figure 4. We observe that $\{S_i : 1 \leq i \leq r\}$, $\{S_i^{'} : 1 \leq i \leq n\}$ and $\{S_i^{''} : 1 \leq i \leq n\}$ forms a partition of the edge set of $K_n W_r$.
For $1 \leq i \leq r$, the removal of S_i leaves $K_n W_r$ into two components G_{S_i} and $G^{'}_{S_i}$ where $|V(G_{S_i})| = 2$ and $|V(G^{'}_{S_i})| = n(2+r) - 2$. For $1 \leq i \leq n$, the removal of $S_i^{'}$ leaves $K_n W_r$ into two components $G_{S_i^{'}}$ and $G^{'}_{S_i^{'}}$ where $|V(G_{S_i^{'}})| = r + 1$ and $|V(G^{'}_{S_i^{'}})| = n(2+r) - (r+1)$. For $1 \leq i \leq n$, the removal of $S_i^{''}$ leaves $K_n W_r$ into two components $G_{S_i^{''}}$ and $G^{'}_{S_i^{''}}$ where $|V(G_{S_i^{''}})| = r + 2$ and $|V(G^{'}_{S_i^{''}})| = n(2+r) - (r+2)$. Hence,

$$\begin{aligned} W(K_n W_r) &= \frac{1}{2}\{nr[2(n(r+2) - 2)] + n[(r+1)(n(r+2) - (r+1))] + n[(r+2)(n(r+2) - (r+2))]\} \\ &= \frac{1}{2}\{4n^2r^2 + 11n^2r - 10nr - 2nr^2 + 6n^2 - 5n\} \end{aligned}$$

□

Theorem 5.3. *The average distance of the uniform n-wheel split graph $K_n W_r$ is given by*

$$\mu(K_n W_r) = \frac{4n^2r^2 + 11n^2r - 10nr - 2nr^2 + 6n^2 - 5n}{4n^2 + 2n^2r - 2n + 2nr^2 + nr^3 - r^2}$$

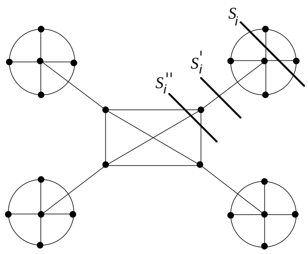

Figure 4. **Cuts of K_4W_4**

Proof. We know from Theorem 5.2. that $W(K_nW_r) = \frac{1}{2}\{4n^2r^2 + 11n^2r - 10nr - 2nr^2 + 6n^2 - 5n\}$. Hence the average distance of the uniform n-wheel split graph K_nW_r is

$$\mu(K_nW_r) = \frac{4n^2r^2 + 11n^2r - 10nr - 2nr^2 + 6n^2 - 5n}{4n^2 + 2n^2r - 2n + 2nr^2 + nr^3 - r^2}$$

□

6. Average Distance of Uniform n-star Split Graph

Definition 6.1 ([14]). *An uniform n-star split graph ST_r^n contains a clique K_n such that the deletion of the nC_2 edges of K_n partitions the graph into n independent star graphs S_{r+1}. The number of vertices in ST_r^n is $n(r+1)$. See Figure 5.*

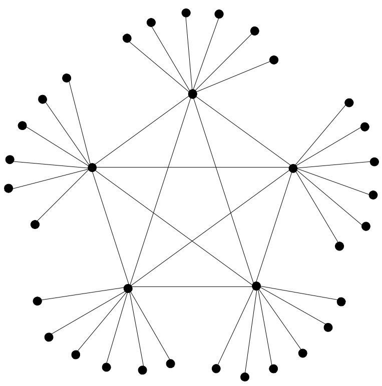

Figure 5. **The graph ST_5^6**

Theorem 6.2. *The Wiener index of the uniform n-star split graph ST_r^n is given by*

$$W(ST_r^n) = \frac{1}{2}\{2n^2r^2 + n^2r - 2nr + nr^2 - r\}$$

Proof. Let $\{S_i : 1 \leq i \leq nr\}$ and $\{S_i' : 1 \leq i \leq r\}$ be the edge cuts of ST_r^n. See Figure 6. We observe that $\{S_i : 1 \leq i \leq nr\}$ and $\{S_i' : 1 \leq i \leq r\}$ forms a partition of the edge set of ST_r^n. For $1 \leq i \leq nr$, the removal of S_i leaves ST_r^n into two components G_{S_i} and G_{S_i}' where $|V(G_{S_i})| = 1$ and $|V(G_{S_i}')| = n(r+1) - 1$. For $1 \leq i \leq r$, the removal of S_i' leaves ST_r^n into two components $G_{S_i'}$ and $G_{S_i'}'$ where $|V(G_{S_i'})| = n + 1$ and $|V(G_{S_i'}')| = [n(r+1) - (n+1)]$. Hence,

$$\begin{aligned} W(ST_r^n) &= \frac{1}{2}\{nr[(n(r+1) - 1)] + r[(n+1)(n(r+1)) - (n+1)^2]\} \\ &= \frac{1}{2}\{2n^2r^2 + n^2r - 2nr + nr^2 - r\} \end{aligned}$$

□

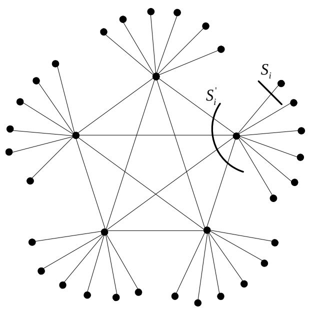

Figure 6. **Cuts of ST_5^6**

Theorem 6.3. *The average distance of the uniform n-star split graph ST_r^n is given by*

$$\mu(ST_r^n) = \frac{2n^2r^2 + n^2r - 2nr + nr^2 - r}{n(r+1)[n(r+1)-1]}.$$

Proof. We know from Theorem 5.2. that $W(ST_r^n) = \frac{1}{2}\{2n^2r^2 + n^2r - 2nr + nr^2 - r\}$. Hence the average distance of ST_r^n is

$$\mu(ST_r^n) = \frac{2n^2r^2 + n^2r - 2nr + nr^2 - r}{n(r+1)[n(r+1)-1]}.$$

□

7. Average Distance of Cyclic Split Graph

Definition 7.1. A cyclic split graph [17] is a split graph in which the vertices can be partitioned into a clique and an independent set of cycles. Thus, we consider a cyclic split graph $C_nK_r^k$ which has a complete graph K_r with vertices $v_1, v_2, \ldots, v_r$ and kr wheels $W_{i,j}$ attached at the each vertex v_i in K_r, such that $W_{i,j} = v_i + C_{n,i,j}, 1 \leq i \leq r$ and $1 \leq j \leq k$ (A wheel graph $W_{i,j}$ is obtained from a cycle $C_{n,i,j}$ by adding new vertex v_i and joining it to all the n vertices of the cycle by an edge. The new edges are called the spokes of the wheel). The deletion of the spokes of the wheel results in the disjoint union of the complete graph K_r and kr independent cycles $C_{n,i,j}, 1 \leq i \leq r$ and $1 \leq j \leq k$, where each cycle has n vertices which are labelled as $a_{n,i,j}$. See Figure 7. The number of vertices in $C_nK_r^k$ is $r(nk+1)$.

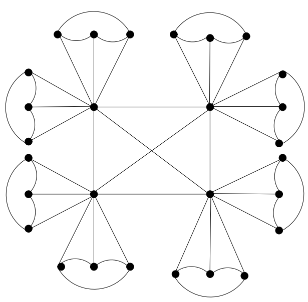

Figure 7. **The graph $C_3K_4^2$**

Theorem 7.2. *The Wiener index of the cyclic split graph $C_nK_r^k$ is given by*

$$W(C_nK_r^k) = \frac{1}{2}\{3n^2r^2k^2 + 4nr^2k - n^2rk - n^2k^2r - 3nrk + r^2 - r\}$$

Proof. Let $\{S_i : 1 \leq i \leq rk\}$, $\{S_i^{'} : 1 \leq i \leq r\}$ and $\{S_i^{''} : 1 \leq i \leq nrk\}$ be the edge cuts of $C_nK_r^k$. See Figure 8. We observe that $\{S_i : 1 \leq i \leq rk\}$, $\{S_i^{'} : 1 \leq i \leq r\}$ and $\{S_i^{''} : 1 \leq i \leq nrk\}$ forms a partition of the edge set of $C_nK_r^k$. For $1 \leq i \leq rk$, the removal of S_i leaves $C_nK_r^k$ into two components G_{S_i} and $G_{S_i}^{'}$ where $|V(G_{S_i})| = n$ and $|V(G_{S_i}^{'})| = r(nk+1) - n$.
For $1 \leq i \leq r$, the removal of $S_i^{'}$ leaves $C_nK_r^k$ into two components $G_{S_i^{'}}$ and $G_{S_i^{'}}^{'}$ where $|V(G_{S_i^{'}})| = (nk+1)$ and $|V(G_{S_i^{'}}^{'})| = r(nk+1) - (nk+1) = (nk+1)(r-1)$. For $1 \leq i \leq nrk$, the removal of $S_i^{''}$ leaves $C_nK_r^k$ into two components $G_{S_i^{''}}$ and $G_{S_i^{''}}^{'}$ where $|V(G_{S_i^{''}})| = 1$ and $|V(G_{S_i^{''}}^{'})| = r(nk+1) - 1$.

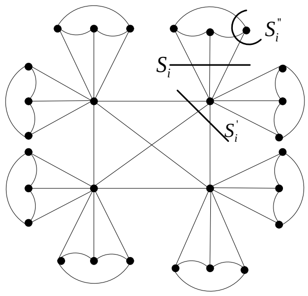

Figure 8. **Cuts of** $C_3K_4^2$

Hence,

$$\begin{aligned} W(C_nK_r^k) &= \frac{1}{2}\{rk[n(r(nk+1)-n)] + r[(nk+1)^2(r-1)] + nrk[r(nk+1)-1]\} \\ &= \frac{1}{2}\{3n^2r^2k^2 + 4nr^2k - n^2rk - n^2k^2r - 3nrk + r^2 - r\} \end{aligned}$$

□

Theorem 7.3. *The average distance of the cyclic split graph $C_nK_r^k$ is given by*

$$\mu(C_nK_r^k) = \frac{3n^2r^2k^2 + 4nr^2k - n^2rk - n^2k^2r - 3nrk + r^2 - r}{r(nk+1)[r(nk+1)-1]}.$$

Proof. We know from Theorem 5.2. that $W(C_nK_r^k) = \frac{1}{2}\{3n^2r^2k^2 + 4nr^2k - n^2rk - n^2k^2r - 3nrk + r^2 - r\}$. Hence the average distance of $C_nK_r^k$ is

$$\mu(C_nK_r^k) = \frac{3n^2r^2k^2 + 4nr^2k - n^2rk - n^2k^2r - 3nrk + r^2 - r}{r(nk+1)[r(nk+1)-1]}.$$

□

8. Concluding Remark

In this paper, the Wiener index and the average distance of generalised prisms, uniform n-wheel split graph, uniform n-star split graph and cyclic split graph were calculated. It is to be noted that the cut method was employed in order to find the Wiener index of these graphs. The problem of computing the average distance of other interconnection networks and finding new techniques to calculate the Wiener index is under investigation.

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